OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4761

Mechanics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 21 January 2009
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

1 A particle is travelling in a straight line. Its velocity v m s^{-1} at time t seconds is given by

$$v = 6 + 4t \quad \text{for } 0 \leqslant t \leqslant 5.$$

(i) Write down the initial velocity of the particle and find the acceleration for $0 \leqslant t \leqslant 5$. **[2]**

(ii) Write down the velocity of the particle when $t = 5$. Find the distance travelled in the first 5 seconds. **[3]**

For $5 \leqslant t \leqslant 15$, the acceleration of the particle is 3 m s^{-2}.

(iii) Find the total distance travelled by the particle during the 15 seconds. **[3]**

2 Fig. 2 shows an acceleration-time graph modelling the motion of a particle.

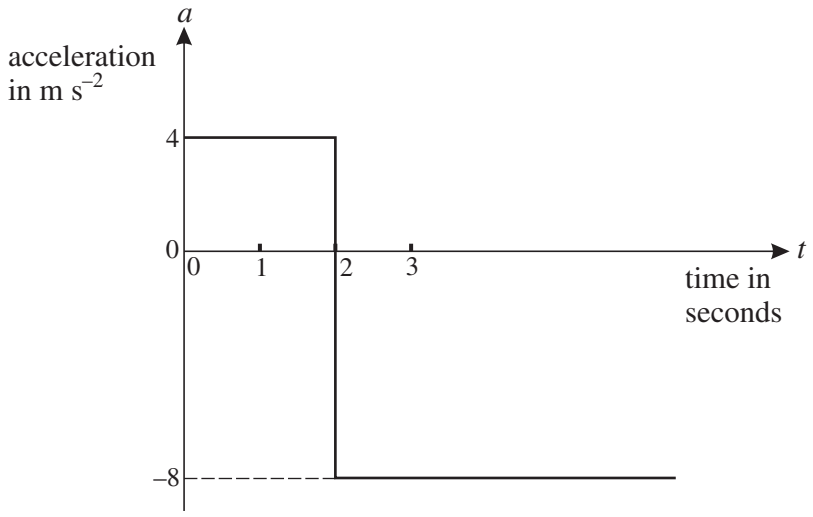

Fig. 2

At $t = 0$ the particle has a velocity of 6 m s^{-1} in the positive direction.

(i) Find the velocity of the particle when $t = 2$. **[2]**

(ii) At what time is the particle travelling in the negative direction with a speed of 6 m s^{-1}? **[2]**

3 The resultant of the force $\begin{pmatrix} -4 \\ 8 \end{pmatrix}$ N and the force **F** gives an object of mass 6 kg an acceleration of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ m s^{-2}.

(i) Calculate **F**. **[4]**

(ii) Calculate the angle between **F** and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. **[2]**

4 Sandy is throwing a stone at a plum tree. The stone is thrown from a point O at a speed of $35\,\text{m s}^{-1}$ at an angle of α to the horizontal, where $\cos\alpha = 0.96$. You are *given* that, t seconds after being thrown, the stone is $(9.8t - 4.9t^2)$ m higher than O.

When descending, the stone hits a plum which is 3.675 m higher than O. Air resistance should be neglected.

Calculate the horizontal distance of the plum from O. **[6]**

5 A man of mass 75 kg is standing in a lift. He is holding a parcel of mass 5 kg by means of a light inextensible string, as shown in Fig. 5. The tension in the string is 55 N.

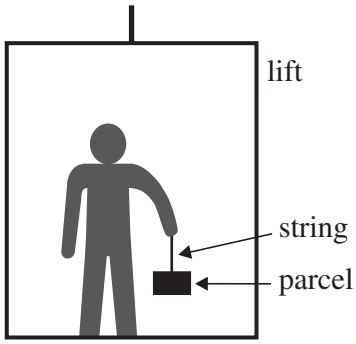

Fig. 5

(i) Find the upward acceleration. **[3]**

(ii) Find the reaction on the man of the lift floor. **[2]**

6 Small stones A and B are initially in the positions shown in Fig. 6 with B a height H m directly above A.

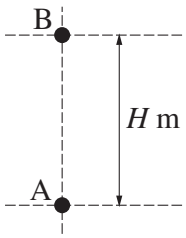

Fig. 6

At the instant when B is released from rest, A is projected vertically upwards with a speed of $29.4\,\text{m s}^{-1}$. Air resistance may be neglected.

The stones collide T seconds after they begin to move. At this instant they have the same speed, $V\,\text{m s}^{-1}$, and A is still rising.

By considering when the speed of A upwards is the same as the speed of B downwards, or otherwise, show that $T = 1.5$ and find the values of V and H. **[7]**

Section B (36 marks)

7 An explorer is trying to pull a loaded sledge of total mass 100 kg along horizontal ground using a light rope. The only resistance to motion of the sledge is from friction between it and the ground.

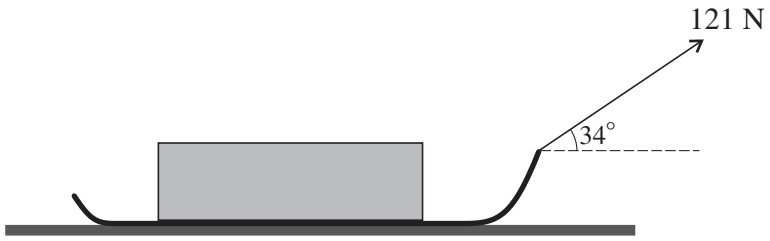

Fig. 7

Initially she pulls with a force of 121 N on the rope inclined at 34° to the horizontal, as shown in Fig. 7, but the sledge does not move.

(i) Draw a diagram showing all the forces acting on the sledge.

Show that the frictional force between the ground and the sledge is 100 N, correct to 3 significant figures.

Calculate the normal reaction of the ground on the sledge. **[7]**

The sledge is given a small push to set it moving at $0.5\,\mathrm{m\,s^{-1}}$. The explorer continues to pull on the rope with the same force and the same angle as before. The frictional force is also unchanged.

(ii) Describe the subsequent motion of the sledge. **[2]**

The explorer now pulls the rope, still at an angle of 34° to the horizontal, so that the tension in it is 155 N. The frictional force is now 95 N.

(iii) Calculate the acceleration of the sledge. **[3]**

In a new situation, there is no rope and the sledge slides down a uniformly rough slope inclined at 26° to the horizontal. The sledge starts from rest and reaches a speed of $5\,\mathrm{m\,s^{-1}}$ in 2 seconds.

(iv) Calculate the frictional force between the slope and the sledge. **[5]**

8 A toy boat moves in a horizontal plane with position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors east and north respectively. The origin of the position vectors is at O. The displacements x and y are in metres.

First consider only the motion of the boat parallel to the x-axis. For this motion

$$x = 8t - 2t^2.$$

The velocity of the boat in the x-direction is v_x m s^{-1}.

(i) Find an expression in terms of t for v_x and determine when the boat instantaneously has zero speed in the x-direction. **[3]**

Now consider only the motion of the boat parallel to the y-axis. For this motion

$$v_y = (t - 2)(3t - 2),$$

where v_y m s^{-1} is the velocity of the boat in the y-direction at time t seconds.

(ii) Given that $y = 3$ when $t = 1$, use integration to show that $y = t^3 - 4t^2 + 4t + 2$. **[4]**

The position vector of the boat is given in terms of t by $\mathbf{r} = (8t - 2t^2)\mathbf{i} + (t^3 - 4t^2 + 4t + 2)\mathbf{j}$.

(iii) Find the time(s) when the boat is due north of O and also the distance of the boat from O at any such times. **[4]**

(iv) Find the time(s) when the boat is instantaneously at rest. Find the distance of the boat from O at any such times. **[5]**

(v) Plot a graph of the path of the boat for $0 \leqslant t \leqslant 2$. **[3]**

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Q 1		Mark	Comment	Sub
(i)	6 m s^{-1} 4 m s^{-2}	B1 B1	Neglect units. Neglect units.	2
(ii)	$v(5) = 6 + 4\times5 = 26$ $s(5) = 6\times5 + 0.5\times4\times25 = 80$ so 80 m	B1 M1 A1	Or equiv. FT (i) and **their** $v(5)$ where necessary. cao	3
(iii)	distance is 80 + $26\times(15-5) + 0.5\times3\times(15-5)^2$ = 490 m	M1 M1 A1	Their 80 + attempt at distance with $a = 3$ Appropriate *uvast* . Allow $t = 15$. FT **their** v(5). cao	3
		8		

Q 2		Mark	Comment	Sub
(i)	When $t = 2$, velocity is $6 + 4\times2 = 14$	M1 A1	Recognising that areas under graph represent changes in velocity in (i) or (ii) or equivalent *uvast.*	2
(ii)	Require velocity of – 6 so must inc by – 20 $-8\times(t-2) = -20$ so $t = 4.5$	M1 F1	FT ±(6 + **their** 14) used in any attempt at area/ *uvast* FT **their** 14 [Award SC2 for 4.5 WW and SC1 for 2.5 WW]	2
		4		

Q 3		Mark	Comment	Sub
(i)	$\mathbf{F} + \begin{pmatrix}-4\\8\end{pmatrix} = 6\begin{pmatrix}2\\3\end{pmatrix}$ $\mathbf{F} = \begin{pmatrix}16\\10\end{pmatrix}$	M1 B1 B1 A1	N2L. $F = ma$. All forces present Addition to get resultant. May be implied. For $\mathbf{F} \pm \begin{pmatrix}-4\\8\end{pmatrix} = 6\begin{pmatrix}2\\3\end{pmatrix}$. SC4 for $\mathbf{F} = \begin{pmatrix}16\\10\end{pmatrix}$ WW. If magnitude is given, final mark is lost unless vector answer is clearly intended.	4
(ii)	$\arctan\left(\frac{16}{10}\right)$ 57.994… so 58.0° (3 s. f.)	M1 A1	Accept equivalent and FT **their F** only. Do not accept wrong angle. Accept 360 - $\arctan\left(\frac{16}{10}\right)$ cao. Accept 302º (3 s.f.)	2
		6		

Q4		Mark	Comment	Sub
	either We need $3.675 = 9.8t - 4.9t^2$ Solving $4t^2 - 8t + 3 = 0$ gives $t = 0.5$ or $t = 1.5$ **or** Time to greatest height $0 = 35 \times 0.28 - 9.8t$ so $t = 1$ Time to drop is 0.5 total is 1.5 s **then** Horiz distance is $35 \times 0.96t$ So distance is $35 \times 0.96 \times 1.5 = 50.4$ m	*M1 M1* A1 F1 M1 A1 A1 A1 B1 F1	Equating given expression or **their** attempt at y to ± 3.675 . If **they** attempt y, allow sign errors, $g = 9.81$ etc. and $u = 35$. Dependent. Any method of solution of a 3 term quadratic. cao. Accept only the larger root given Both roots shown and larger chosen provided both +ve. Dependent on 1st M1. [Award M1 M1 A1 for 1.5 seen WW] Complete method for total time from motion in separate parts. Allow sign errors, $g = 9.81$ etc. Allow $u = 35$ initially only. Time for 1st part Time for 2nd part cao Use of $x = u\cos\alpha t$. May be implied. FT **their** quoted t provided it is positive.	6
		6		

Q5		Mark	Comment	Sub
(i)	For the parcel ↑ N2L $55 - 5g = 5a$ $a = 1.2$ so 1.2 m s^{-2}	M1 A1 A1	Applying N2L to the parcel. Correct mass. Allow $F = mga$. Condone missing force but do not allow spurious forces. Allow only sign error(s). Allow –1.2 only if sign convention is clear.	3
(ii)	$R - 80g = 80 \times 1.2$ or $R - 75g - 55 = 75 \times 1.2$ $R = 880$ so 880 N	M1 A1	N2L. Must have correct mass. Allow only sign errors. FT **their** a cao [NB beware spurious methods giving 880 N]	2
		5		

Q6		Mark	Comment	Sub
	Method 1			
	$\uparrow\ v_A = 29.4 - 9.8T \qquad \downarrow\ v_B = 9.8T$	M1	Either attempted. Allow sign errors and $g = 9.81$ etc	
		A1	Both correct	
	For same speed $29.4 - 9.8T = 9.8T$	M1	Attempt to equate. Accept sign errors and $T = 1.5$ substituted in both.	
	so $T = 1.5$	E1	If 2 subs there must be a statement about equality	
	and $V = 14.7$	F1	FT T or V, whichever is found second	
	$H = 29.4 \times 1.5 - 0.5 \times 9.8 \times 1.5^2$ $+ 0.5 \times 9.8 \times 1.5^2$	M1	Sum of the distance travelled by each attempted	
	$= 44.1$	A1	cao	
	Method 2			
	$V^2 = 29.4^2 - 2 \times 9.8 \times x = 2 \times 9.8 \times (H - x)$	M1	Attempts at V^2 for each particle equated. Allow sign errors, 9.81 etc Allow h_1, h_2 without $h_1 = H - h_2$	
		B1	Both correct. Require $h_1 = H - h_2$ but not an equation.	
	$29.4^2 = 19.6H$ so $H = 44.1$	A1	cao	
	Relative velocity is 29.4 so	M1	Any method that leads to T or V	
	$T = \dfrac{44.1}{29.4}$	E1		
	Using $v = u + at$	M1	Any method leading to the other variable	
	$V = 0 + 9.8 \times 1.5 = 14.7$	F1		
			Other approaches possible. If 'clever' ways seen, reward according to weighting above.	
				7
		7		

Q7		Mark	Comment	Sub
(i)	Diagram	B1 B1	Weight, friction and 121 N present with arrows. All forces present with suitable labels. Accept W, mg, $100g$ and 980. No extra forces.	
	Resolve $\rightarrow$ $121\cos 34 - F = 0$ $F = 100.313\ldots$ so 100 N (3 s. f.)	M1 E1	Resolving horiz. Accept $s \leftrightarrow c$. Some evidence required for the *show*, e.g. at least 4 figures. Accept $\pm$.	
	Resolve $\uparrow$ $R + 121\sin 34 - 980 = 0$ $R = 912.337\ldots$ so 912 N (3 s. f.)	M1 B1 A1	Resolve vert. Accept $s \leftrightarrow c$ and sign errors. All correct	7
(ii)	It will continue to move at a constant speed of $0.5\ \text{m s}^{-1}$.	E1 E1	Accept no reference to direction Accept no reference to direction [Do not isw: conflicting statements get zero]	2
(iii)	Using N2L horizontally $155\cos 34 - 95 = 100a$ $a = 0.335008\ldots$ so $0.335\ \text{m s}^{-2}$ (3 s. f.)	M1 A1 A1	Use of N2L. Allow $F = mga$, F omitted and 155 not resolved. Use of $F = ma$ with resistance and T resolved. Allow $s \leftrightarrow c$ and signs as the **only** errors.	3
(iv)	$a = 5 \div 2 = 2.5$ N2L down the slope $100g\sin 26 - F = 100 \times 2.5$ $F = 179.603\ldots$ so 180 N (3 s. f.)	M1 A1 M1 B1 A1	Attempt to find a from information $F = ma$ using **their** "new" a. All forces present. No extras. Require attempt at wt cpt. Allow $s \leftrightarrow c$ and sign errors. Weight term resolved correctly, seen in an equn or on a diagram. cao. Accept – 180 N if consistent with direction of F on their diagram	5
		17		

Q8		Mark	Comment	Sub
(i)	$v_x = 8 - 4t$ $v_x = 0 \Leftrightarrow t = 2$ so at $t = 2$	M1 A1 F1	**either** Differentiating **or** Finding 'u' and 'a' from x and use of $v = u + at$ FT **their** $v_x = 0$	3
(ii)	$y = \int (3t^2 - 8t + 4)\, dt$ $= t^3 - 4t^2 + 4t + c$ $y = 3$ when $t = 1$ so $3 = 1 - 4 + 4 + c$ so $c = 3 - 1 = 2$ and $y = t^3 - 4t^2 + 4t + 2$	M1 A1 M1 E1	Integrating v_y with at least one correct integrated term. All correct. Accept no arbitrary constant. Clear evidence Clearly shown and stated	4
(iii)	We need $x = 0$ so $8t - 2t^2 = 0$ so $t = 0$ or $t = 4$ $t = 0$ gives $y = 2$ so 2 m $t = 4$ gives $y = 4^3 - 4^3 + 16 + 2 = 18$ so 18 m	M1 A1 A1 A1	May be implied. Must have both Condone 2**j** Condone 18**j**	4
(iv)	We need $v_x = v_y = 0$ From above, $v_x = 0$ only when $t = 2$ so evaluate $v_y(2)$ $v_y(2) = 0$ [$(t - 2)$ is a factor] so yes only at $t = 2$ At $t = 2$, the position is (8, 2) Distance is $\sqrt{8^2 + 2^2} = \sqrt{68}$ m (8.25 3 s.f.)	M1 M1 A1 B1 B1	**either** Recognises $v_x = 0$ when $t = 2$ **or** Finds time(s) when $v_y = 0$ **or** States or implies $v_x = v_y = 0$ Considers $v_x = 0$ **and** $v_y = 0$ with **their** time(s) $t = 2$ recognised as only value (accept as evidence only $t = 2$ used below). For the last 2 marks, no credit lost for reference to $t = \frac{2}{3}$. May be implied FT from **their** position. Accept one position followed through correctly.	5
(v)	$t = 0$, 1 give (0, 2) and (6, 3)	B1 B1 B1	At least one value $0 \le t < 2$ correctly calc. This need not be plotted Must be x-y curve. Accept sketch. Ignore curve outside interval for t. Accept unlabelled axes. Condone use of line segments. At least three correct points **used** in x-y graph or sketch. General shape correct. Do not condone use of line segments.	3
		19		