## ADVANCED SUBSIDIARY GCE UNIT MATHEMATICS (MEI)

Mechanics 1
WEDNESDAY 10 JANUARY 2007

Additional materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \mathrm{~m} \mathrm{~s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g=9.8$.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- $\quad$ The total number of marks for this paper is 72 .


## ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.


## Section A (36 marks)

1 Fig. 1 is the velocity-time graph for the motion of a body. The velocity of the body is $v \mathrm{~m} \mathrm{~s}^{-1}$ at time $t$ seconds.


Fig. 1
The displacement of the body from $t=0$ to $t=100$ is 1400 m . Find the value of $V$.

2 A particle moves along a straight line containing a point O . Its displacement, $x \mathrm{~m}$, from O at time $t$ seconds is given by

$$
x=12 t-t^{3}, \text { where }-10 \leqslant t \leqslant 10
$$

Find the values of $x$ for which the velocity of the particle is zero.

3 A box of mass 5 kg is at rest on a rough horizontal floor.
(i) Find the value of the normal reaction of the floor on the box.

The box remains at rest on the floor when a force of 10 N is applied to it at an angle of $40^{\circ}$ to the upward vertical, as shown in Fig. 3.


Fig. 3
(ii) Draw a diagram showing all the forces acting on the box.
(iii) Calculate the new value of the normal reaction of the floor on the box and also the frictional force.

4 Fig. 4 shows forces of magnitudes 20 N and 16 N inclined at $60^{\circ}$.


Fig. 4
(i) Calculate the component of the resultant of these two forces in the direction of the 20 N force.
(ii) Calculate the magnitude of the resultant of these two forces.

These are the only forces acting on a particle of mass 2 kg .
(iii) Find the magnitude of the acceleration of the particle and the angle the acceleration makes with the 20 N force.

5 A block of mass 4 kg slides on a horizontal plane against a constant resistance of 14.8 N . A light, inextensible string is attached to the block and, after passing over a smooth pulley, is attached to a freely hanging sphere of mass 2 kg . The part of the string between the block and the pulley is horizontal. This situation is shown in Fig. 5.


Fig. 5
The tension in the string is $T \mathrm{~N}$ and the acceleration of the block and of the sphere is $a \mathrm{~m} \mathrm{~s}^{-2}$.
(i) Write down the equation of motion of the block and also the equation of motion of the sphere, each in terms of $T$ and $a$.
(ii) Find the values of $T$ and $a$.

6 The velocity of a model boat, $\mathrm{v} \mathrm{m} \mathrm{s}^{-1}$, is given by

$$
\mathbf{v}=\binom{-5}{10}+t\binom{6}{-8}
$$

where $t$ is the time in seconds and the vectors $\binom{1}{0}$ and $\binom{0}{1}$ are east and north respectively.
(i) Show that when $t=2.5$ the boat is travelling south-east (i.e. on a bearing of $135^{\circ}$ ). Calculate its speed at this time.

The boat is at a point O when $t=0$.
(ii) Calculate the bearing of the boat from O when $t=2.5$.

Section B (36 marks)
7 A horizontal force of 24 N acts on a block of mass 12 kg on a horizontal plane. The block is initially at rest.

This situation is first modelled assuming the plane is smooth.
(i) Write down the acceleration of the block according to this model.

The situation is now modelled assuming a constant resistance to motion of 15 N .
(ii) Calculate the acceleration of the block according to this new model. How much less distance does the new model predict that the block will travel in the first 4 seconds?

The 24 N force is removed and the block slides down a slope at $5^{\circ}$ to the horizontal. The speed of the block at the top of the slope is $1.5 \mathrm{~m} \mathrm{~s}^{-1}$, as shown in Fig. 7. The answers to parts (iii) and (iv) should be found using the assumption that the resistance to the motion of the block is still a constant 15 N .


Fig. 7
(iii) Calculate the acceleration of the block in the direction of its motion.
(iv) For how much time does the block slide down the slope before coming to rest and how far does it slide in that time?

Measurements show that the block actually comes to rest in 3.5 seconds.
(v) Assuming that the error in the prediction is due only to the value of the resistance, calculate the true value of the resistance.

## [Question 8 is printed overleaf.]

## 8 In this question the value of $g$ should be taken as $10 \mathrm{~m} \mathrm{~s}^{-2}$.

As shown in Fig. 8, particles A and B are projected towards one another. Each particle has an initial speed of $10 \mathrm{~m} \mathrm{~s}^{-1}$ vertically and $20 \mathrm{~m} \mathrm{~s}^{-1}$ horizontally. Initially A and B are 70 m apart horizontally and B is 15 m higher than A . Both particles are projected over horizontal ground.


Fig. 8
(i) Show that, $t$ seconds after projection, the height in metres of each particle above its point of projection is $10 t-5 t^{2}$.
(ii) Calculate the horizontal range of A . Deduce that A hits the horizontal ground between the initial positions of A and B.
(iii) Calculate the horizontal distance travelled by B before reaching the ground.
(iv) Show that the paths of the particles cross but that the particles do not collide if they are projected at the same time.

In fact, particle $A$ is projected 2 seconds after particle $B$.
(v) Verify that the particles collide 0.75 seconds after A is projected.

| Q 1 |  | mark |  | sub |
| :---: | :---: | :---: | :---: | :---: |
|  | either <br> 70 V obtained So $70 V=1400$ and $V=20$ or $V=20$ | M1 <br> A1 <br> M1 <br> A1 <br> M1 <br> A1 <br> M1 <br> A1 | Attempt at area. If not trapezium method at least one <br> part area correct. Accept equivalent. <br> Or equivalent - need not be evaluated. <br> Equate their 70 V to 1400 . Must have attempt at complete areas or equations. <br> cao <br> Attempt to find areas in terms of ratios (at least one <br> correct) <br> Correct total ratio - need not be evaluated. <br> (Evidence <br> may be 800 or 400 or 200 seen). <br> Complete method. (Evidence may be 800/40 or 400/20 <br> or 200/10 seen). <br> cao <br> [ Award 3/4 for 20 seen WWW] |  |
|  |  |  |  | 4 |


| Q 2 |  | mark |  | sub |
| :---: | :---: | :---: | :---: | :---: |
|  | $(v=) 12-3 t^{2}$ $v=0 \Rightarrow 12-3 t^{2}=0$ <br> so $t^{2}=4$ and $t= \pm 2$ $x= \pm 16$ | M1 <br> A1 <br> M1 <br> A1 <br> A1 | Differentiating <br> Allow confusion of notation, including $x=$ Dep on $1^{\text {st }} \mathrm{M} 1$. Equating to zero. <br> Accept one answer only but no extra answers. FT only <br> if quadratic or higher degree. <br> cao. Must have both and no extra answers. |  |
|  |  |  |  | 5 |


| Q 3 |  | mark |  | sub |
| :---: | :---: | :---: | :---: | :---: |
| (i) | $R=m g$ so 49 N | B1 | Equating to weight. Accept $5 g$ (but not $m g$ ) | 1 |
| (ii) |  | B1 <br> B1 | All except $F$ correct (arrows and labels) (Accept $m g$, $W$ etc and no angle). Accept cpts instead of 10 N . No extra forces. F clearly marked and labelled | 2 |
| (iii) | $\uparrow \quad R+10 \cos 40-49=0$ $\begin{aligned} & R=41.339 \ldots \text { so } 41.3 \mathrm{~N}(3 \mathrm{~s} . \mathrm{f} .) \\ & F=10 \sin 40=6.4278 \ldots \text { so } 6.43 \mathrm{~N}(3 \mathrm{s.} \mathrm{f.}) \end{aligned}$ | M1 <br> B1 <br> A1 <br> B1 | Resolve vertically. All forces present and 10N resolved <br> Resolution correct and seen in an equation. <br> (Accept <br> $R= \pm 10 \cos 40$ as an equation) <br> Allow -ve if consistent with the diagram. | 4 |
|  |  |  |  | 7 |


| Q 4 |  | mark |  | sub |
| :---: | :---: | :---: | :---: | :---: |
| (i) | $\downarrow \quad 20+16 \cos 60=28$ | B1 |  | 1 |
| (ii) | either $\rightarrow 16 \sin 60$ $\text { Mag } \sqrt{28^{2}+192}=31.2409 \ldots$ $\text { so } 31.2 \text { N (3 s.f.) }$ <br> or <br> Cos rule $\begin{aligned} & \text { mag }^{2}=16^{2}+20^{2}-2 \times 16 \times 20 \times \cos 120 \\ & 31.2 \mathrm{~N}(3 \text { s. f.) } \end{aligned}$ | B1 <br> M1 <br> F1 <br> M1 <br> A1 <br> A1 | Any form. May be seen in (i). Accept any appropriate equivalent resolution. <br> Use of Pythag with 2 distinct cpts (but not 16 and $\pm 20)$ <br> Allow 34.788... only as FT <br> Must be used with $20 \mathrm{~N}, 16 \mathrm{~N}$ and $60^{\circ}$ or $120^{\circ}$ Correct substitution | 3 |
| (iii) | Magnitude of accn is $15.620 \ldots \mathrm{~m} \mathrm{~s}^{-2}$ so $15.6 \mathrm{~m} \mathrm{~s}^{-2}$ (3 s. f.) <br> angle with 20 N force is $\arctan \left(\frac{16 \sin 60}{28}\right)$ <br> so $26.3295 \ldots$ so $26.3^{\circ}$ (3 s. f.) | B1 <br> M1 <br> A1 | Award only for their $F \div 2$ <br> Or equiv. May use force or acceleration. Allow use <br> of sine or cosine rules. FT only $\mathrm{s} \leftrightarrow \mathrm{c}$ and sign errors. Accept reciprocal of the fraction. cao | 3 |
|  |  |  |  | 7 |
| Q 5 |  | mark |  | sub |
| (i) | $\begin{array}{ll} \text { sphere } & 19.6-T=2 a \\ \text { block } & T-14.8=4 a \end{array}$ | M1 <br> A1 <br> A1 | N2L. All forces attempted in one equation. <br> Allow <br> sign errors. No extra forces. Don't condone $F=$ mga. <br> Accept $2 g$ for 19.6 | 3 |
| (ii) | Solving $T=18 \quad a=0.8$ | M1 <br> A1 <br> F1 | Attempt to solve. Award only if two equations present both containing a and $T$. Either variable eliminated. <br> Either found cao <br> Other value. Allow wrong equation(s) and wrong working for $1^{\text {st }}$ value <br> [If combined equation used award: M1 as in (i) for <br> the equation with mass of 6 kg ; A1 for $a=0.8$; M1 as <br> in (i) for equation in $T$ and a for either sphere or block; A1 equation correct; F1 for $T$, FT their a; B1 Second equation in $T$ and a.] | 3 |
|  |  |  |  | 6 |


| Q 6 |  | mark |  | sub |
| :---: | :---: | :---: | :---: | :---: |
| (i) | $\begin{aligned} & t=2.5 \Rightarrow \mathbf{v}=\binom{-5}{10}+2.5\binom{6}{-8}=\binom{10}{-10} \\ & \text { speed is } \sqrt{10^{2}+10^{2}}=14.14 \ldots \\ & \text { so } 14.1 \mathrm{~m} \mathrm{~s}^{-1}(3 \mathrm{s.} . \mathrm{f} \text { ) } \end{aligned}$ | B1 <br> E1 <br> F1 | Need not be in vector form <br> Accept diag and/or correct derivation of just $\pm 45^{\circ}$ <br> FT their v | 3 |
| (ii) | $\mathbf{s}=2.5\binom{-5}{10}+\frac{1}{2} \times 2.5^{2} \times\binom{ 6}{-8}$ $=\binom{6.25}{0}$ <br> so $090^{\circ}$ | M1 <br> A1 <br> A1 <br> A1 | Consideration of $\mathbf{s}$ (const accn or integration) <br> Correct sub into uvast with $\mathbf{u}$ and $\mathbf{a}$. (If integration used it must be correct but allow no arb constant) <br> cao. CWO. | 4 |
|  |  |  |  | 7 |


| Q 7 |  | mark |  | sub |
| :---: | :---: | :---: | :---: | :---: |
| (i) | acceleration is $\frac{24}{12}$ so $2 \mathrm{~m} \mathrm{~s}^{-2}$ | B1 |  | 1 |
| (ii) | $\begin{aligned} & 24-15=12 a \\ & a=0.75 \mathrm{~m} \mathrm{~s}^{-2} \\ & 1^{\text {st }} \text { distance is } 0.5 \times 2 \times 16=16 \\ & 2^{\text {nd }} \text { distance is } 0.5 \times 0.75 \times 16=6 \\ & \text { Difference is } 10 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | Use of N2L. Both forces present. Must be $F=$ ma. No extra forces. <br> Appropriate uvast applied at least once. <br> Need not evaluate. Both found. May be implied. <br> FT (i) <br> cao | 5 |
| (iii) | $12 g \sin 5-15=12 a$ $\begin{aligned} & a=-0.39587 \ldots \\ & \text { so }-0.396 \mathrm{~m} \mathrm{~s}^{-2}(3 \mathrm{~s} . \text { f. }) \end{aligned}$ | M1 <br> M1 <br> A1 <br> A1 | Use of $F=$ ma, allow 15 N missing or weight not resolved. No extra forces. Allow use of $12 \sin 5$. Attempt at weight cpt. Allow $\sin \leftrightarrow \cos$. Accept seen on diagram. Accept the use of 12 instead of $12 g$. Weight cpt correct. Accept seen on diagram. Allow not used. <br> Correct direction must be made clear | 4 |
| (iv) | time $0=1.5+a t \Rightarrow t=3.789 \ldots$ <br> so 3.79 s (3 s. f.) <br> distance $s=0.5 \times(1.5+0) \times 3.789 \ldots(\text { or } \ldots)$ <br> giving $s=2.8418 \ldots$ so 2.84 m (3 s. f.) | M1 <br> A1 <br> M1 <br> A1 | Correct uvast . Use of 0, 1.5 and their a from (iii) or <br> their $s$ from (iv). Allow sign errors. Condone $u \leftrightarrow v$. <br> Correct uvast . Use of 0, 1.5 and their a from (iii) or their $t$ from (iv). Allow sign errors. Condone $u \leftrightarrow v$. <br> [The first A1 awarded for $t$ or $s$ has FT their $a$ if signs correct; the second awarded is cao] | 4 |
| (v) | accn is given by $0=1.5+3.5 a \Rightarrow a=-\frac{3}{7}=-0.42857 \ldots$ $\begin{aligned} & 12 g \sin 5-R=12 \times-0.42857 \ldots \\ & \text { so } R=15.39 \ldots \text { so } 15.4 \mathrm{~N} \text { (3 s. f.) } \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 | Use of $0,1.5$ and 3.5 in correct uvast. <br> Condone $u \leftrightarrow v$. <br> Allow $\pm$ <br> N2L. Must use their new accn. Allow only sign errors. <br> cao | 4 |
|  |  |  |  | 18 |


| Q 8 |  | mark |  | sub |
| :---: | :---: | :---: | :---: | :---: |
| (i) | Using $s=u t+0.5 a t^{2}$ with $u=10$ and $a$ $=-10$ | E1 | Must be clear evidence of derivation of -5 . Accept one calculation and no statement about the other. | 1 |
| (ii) | either <br> $s=0$ gives $10 t-5 t^{2}=0$ <br> so $5 t(2-t)=0$ <br> so $t=0$ or 2 . Clearly need $t=2$ <br> or <br> Time to highest point is given by $0=10-$ <br> 10t <br> Time of flight is $2 \times 1$ $=2 \mathrm{~s}$ <br> horizontal range is 40 m as $40<70$, hits the ground | B1 <br> M1 <br> A1 <br> M1 <br> M1 <br> A1 <br> B1 <br> E1 | Factorising <br> Award 3 marks for $t=2$ seen WWW <br> Dep on $1^{\text {st }} \mathrm{M} 1$. Doubling their $t$. <br> Properly obtained <br> FT $20 \times$ their $t$ <br> Must be clear. FT their range. | 5 |
| (iii) | need $10 t-5 t^{2}=-15$ <br> Solving $t^{2}-2 t-3=0$ <br> so $(t-3)(t+1)=0$ and $t=3$ <br> range is 60 m | M1 <br> M1 <br> A1 <br> M1 <br> A1 | [May divide flight into two parts] <br> Equate $s=-15$ or equivalent. Allow use of $\pm 15$. <br> Method leading to solution of a quadratic. <br> Equivalent form will do. <br> Obtaining $t=3$. Allow no reference to the other root. <br> [Award SC3 if $t=3$ seen WWW] <br> Range is $20 \times$ their $t$ (provided $t>0$ ) <br> cao. CWO. | 5 |
| (iv) | Using (ii) \& (iii), since $40+60>70$, paths cross <br> (For $0<t \leq 2$ ) both have same vertical motion so $B$ is always 15 m above $A$ | E1 <br> E1 | Must be convincing. Accept sketches. <br> Do not accept evaluation at one or more points alone. <br> That B is always above A must be clear. | 2 |
| (v) | Need $x$ components summing to 70 $20 \times 0.75+20 \times 2.75=15+55=70$ so true <br> Need $y$ components the same $\begin{aligned} & 10 \times 2.75-5 \times 2.75^{2}+15=4.6875 \\ & 10 \times 0.75-5 \times 0.75^{2}=4.6875 \end{aligned}$ | M1 <br> E1 <br> M1 <br> B1 <br> E1 | May be implied. <br> Or correct derivation of 0.75 s or 2.75 s <br> Attempt to use 0.75 and 2.75 in two vertical height equations (accept same one or wrong one) <br> 0.75 and 2.75 each substituted in the appropriate equn <br> Both values correct. <br> [Using cartesian equation: B1, B1 each equation: M1 <br> solving: A1 correct point of intersection: E1 Verify times] | 5 |
|  |  |  |  | 18 |

