OCR
Oxford Cambridge and RSA

Wednesday 3 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

5089069931

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **24** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Option 1: Vectors

1 The point A has coordinates (2, 5, 4) and the line BC has equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 25 \\ 43 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix}.$$

You are given that AB = AC = 15.

(i) Show that the coordinates of one of the points B and C are (4, 10, 18). Find the coordinates of the other point. These points are B and C respectively. **[6]**

(ii) Find the equation of the plane ABC in cartesian form. **[4]**

(iii) Show that the plane containing the line BC and perpendicular to the plane ABC has equation $5y - 3z + 4 = 0$. **[4]**

The point D has coordinates (1, 1, 3).

(iv) Show that $|\overrightarrow{BC} \times \overrightarrow{AD}| = \sqrt{7667}$ and hence find the shortest distance between the lines BC and AD. **[7]**

(v) Find the volume of the tetrahedron ABCD. **[3]**

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Option 2: Multi-variable calculus

2 A surface has equation $z = 3x^2 - 12xy + 2y^3 + 60$.

(i) Show that the point A $(8, 4, -4)$ is a stationary point on the surface. Find the coordinates of the other stationary point, B, on this surface. **[5]**

(ii) A point P with coordinates $(8+h, 4+k, p)$ lies on the surface.

(*A*) Show that $p = -4 + 3(h-2k)^2 + 2k^2(6+k)$. **[3]**

(*B*) Deduce that the stationary point A is a local minimum. **[3]**

(*C*) By considering sections of the surface near to B in each of the planes $x = 0$ and $y = 0$, investigate the nature of the stationary point B. **[4]**

(iii) The point Q with coordinates (1, 1, 53) lies on the surface.

Show that the equation of the tangent plane at Q is

$$6x + 6y + z = 65.$$ **[4]**

(iv) The tangent plane at the point R has equation $6x + 6y + z = \lambda$ where $\lambda \neq 65$.

Find the coordinates of R. **[5]**

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Option 3: Differential geometry

3 Fig. 3 shows an ellipse with parametric equations $x = a\cos\theta$, $y = b\sin\theta$, for $0 \leqslant \theta \leqslant 2\pi$, where $0 < b \leqslant a$.
The curve meets the positive x-axis at A and the positive y-axis at B.

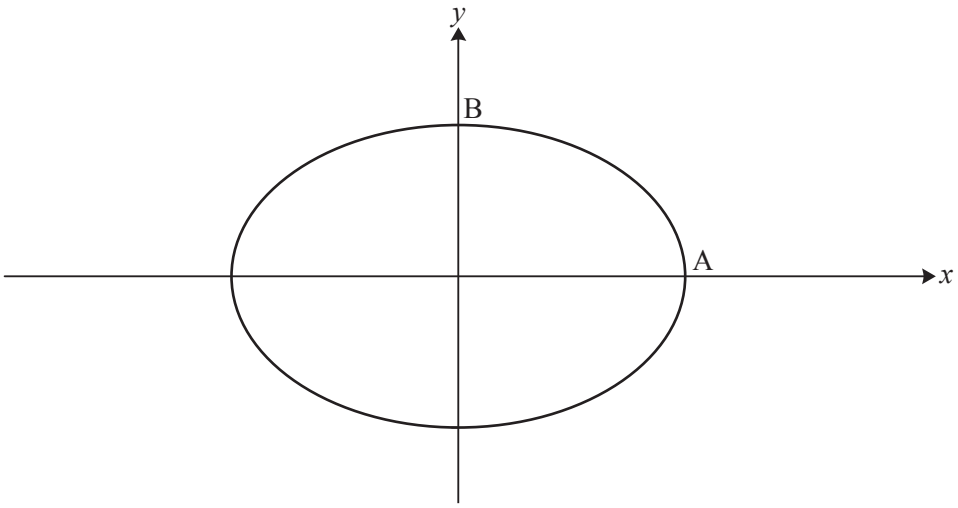

Fig. 3

(i) Show that the radius of curvature at A is $\frac{b^2}{a}$ and find the corresponding centre of curvature. **[7]**

(ii) Write down the radius of curvature and the centre of curvature at B. **[2]**

(iii) Find the relationship between a and b if the radius of curvature at B is equal to the radius of curvature at A. What does this mean geometrically? **[1]**

(iv) Show that the arc length from A to B can be expressed as

$$b\int_0^{\frac{\pi}{2}} \sqrt{1+\lambda^2\sin^2\theta}\, d\theta,$$

where λ^2 is to be determined in terms of a and b.

Evaluate this integral in the case $a = b$ and comment on your answer. **[7]**

(v) Find the cartesian equation of the evolute of the ellipse. **[7]**

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Option 4: Groups

4 M is the set of all 2×2 matrices m(a,b) where a and b are rational numbers and

$$\mathrm{m}(a,b)=\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix},\ a\neq 0.$$

(i) Show that under matrix multiplication M is a group. You may assume associativity of matrix multiplication. **[7]**

(ii) Determine whether the group is commutative. **[3]**

The set N_k consists of all 2×2 matrices m(k,b) where k is a fixed positive integer and b can take any integer value.

(iii) Prove that N_k is closed under matrix multiplication if and only if $k = 1$. **[4]**

Now consider the set P consisting of the matrices m(1,0), m(1,1), m(1,2) and m(1,3). The elements of P are combined using matrix multiplication but with arithmetic carried out modulo 4.

(iv) Show that $(\mathrm{m}(1,1))^2=\mathrm{m}(1,2)$. **[2]**

(v) Construct the group combination table for P. **[4]**

The group R consists of the set $\{e, a, b, c\}$ combined under the operation *. The identity element is e, and elements a, b and c are such that

$$a*a = b*b = c*c \qquad \text{and} \qquad a*c = c*a = b.$$

(vi) Determine whether R is isomorphic to P. **[4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 An inspector has three factories, A, B, C, to check. He spends each day in one of the factories. He chooses the factory to visit on a particular day according to the following rules.

- If he is in A one day, then the next day he will never choose A but he is equally likely to choose B or C.
- If he is in B one day, then the next day he is equally likely to choose A, B or C.
- If he is in C one day, then the next day he will never choose A but he is equally likely to choose B or C.

(i) Write down the transition matrix, **P**. **[2]**

(ii) On Day 1 the inspector chooses A.

(*A*) Find the probability that he will choose A on Day 4. **[3]**

(*B*) Find the probability that the factory he chooses on Day 7 is the same factory that he chose on Day 2. **[4]**

(iii) Find the equilibrium probabilities and explain what they mean. **[4]**

The inspector is not satisfied with the number of times he visits A so he changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 0.8, 0.1, 0.1, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

(iv) Write down the new transition matrix, **Q**, and find the new equilibrium probabilities. **[3]**

(v) On a particular day, the inspector visits factory A. Find the expected number of consecutive further days on which he will visit factory A. **[3]**

Still not satisfied, the inspector changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 1, 0, 0, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

The new transition matrix is **R**.

(vi) On Day 15 he visits C. Find the first subsequent day for which the probability that he visits B is less than 0.1. **[3]**

(vii) Show that in this situation there is an absorbing state, explaining what this means. **[2]**

END OF QUESTION PAPER

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1	**(i)**	Any point is $\begin{pmatrix}8\\25\\43\end{pmatrix}+\lambda\begin{pmatrix}4\\15\\25\end{pmatrix}$ $=\left((8+4\lambda),(25+15\lambda),(43+25\lambda)\right)$ $\text{AB}=\left((8+4\lambda),(25+15\lambda),(43+25\lambda)\right)-(2,5,4)$ $=\left((6+4\lambda),(20+15\lambda),(39+25\lambda)\right)$ Distance AB $=\sqrt{(6+4\lambda)^2+(20+15\lambda)^2+(39+25\lambda)^2}=15$ $\Rightarrow 866\lambda^2+2598\lambda+1732=0$ $\Rightarrow \lambda^2+3\lambda+2=0$ $\Rightarrow \lambda=-1,-2$ $\Rightarrow$ B $(4,10,18)$, C $(0,-5,-7)$	**M1** **A1** **M1** **A1** **B1** **A1**	Finding AB in terms of λ For B For C	B1 can also be given for verifying AB = 15 and showing B is on line
			6		
	(ii)	$\text{AC}=[2,10,11]$ $\mathbf{n}=\begin{pmatrix}2\\10\\11\end{pmatrix}\times\begin{pmatrix}4\\15\\25\end{pmatrix}=\begin{pmatrix}85\\-6\\-10\end{pmatrix}$ $\Rightarrow 85x-6y-10z=c$ Sub one value to give c $\Rightarrow 85x-6y-10z=100$	**B1** **M1** **A1** **A1**	Or any vector in plane other than BC Suitable vector product or other method for finding normal	
			4		

	(iii)	This plane contains the line BC and **n** $\mathbf{n'} = \begin{pmatrix} 85 \\ -6 \\ -10 \end{pmatrix} \times \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix} = \begin{pmatrix} 0 \\ -2165 \\ 1299 \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \\ 3 \end{pmatrix}$ $\Rightarrow 5y - 3z = c$ Sub one value to give c $\Rightarrow 5y - 3z + 4 = 0$	**M1** **A1** **M1** **A1**	Appropriate vector product **oe** Allow uncancelled vector Must be seen!	
			4		
	(iv)	$\mathbf{BC} = [-4, -15, -25] \quad \mathbf{AD} = [-1, -4, -1]$ $\mathbf{BC} \times \mathbf{AD} = \begin{bmatrix} 4 & 15 & 25 \\ 1 & 4 & 1 \end{bmatrix} = [-85, 21, 1]$ $\lvert\mathbf{BC} \times \mathbf{AD}\rvert = \sqrt{85^2 + 21^2 + 1} = \sqrt{7667}$ $\mathbf{AC} = [2, 10, 11]$ $\text{Distance} = \left\lvert \frac{(\mathbf{AC}).(\mathbf{BC} \times \mathbf{AD})}{\sqrt{7667}} \right\rvert$ $= \left\lvert \frac{-170 + 210 + 11}{\sqrt{7667}} \right\rvert = \frac{51}{\sqrt{7667}}$	**B1** **M1** **M1** **A1** **B1** **M1** **A1**	 Finding vector product Finding magnitude, pythagoras must be seen N.b. Answer given Other vectors possible Accept 0.582.....	
			7		

	(v)	A (2, 5, 4), B(4, 10, 18), C(0,-5, -7), D(1, 1, 3) $\mathbf{AB} = [2,5,14]$ $\mathbf{AC} = [-2,-10,-11]$ $\mathbf{AD} = [-1,-4,-1]$ Volume $= \left\lvert \frac{1}{6}(\mathbf{AB} \bullet (\mathbf{CA} \times \mathbf{DA}) \right\rvert$ $\mathbf{AC} \times \mathbf{AD} = \begin{bmatrix} -2 & -10 & -11 \\ -1 & -4 & -1 \end{bmatrix} = [-34,9,-2]$ $= \left\lvert \frac{1}{6}((2,5,14) \bullet (-34,9,-2)) \right\rvert = \left\lvert \frac{1}{6}(-68+45-28) \right\rvert$ $= \frac{51}{6} = \frac{17}{2}$	**M1** **A1** **A1**	Formula for volume Vector product	
			3		

2	**(i)**	$z = 3x^2 - 12xy + 2y^3 + 60$ $\frac{\partial z}{\partial x} = 6x - 12y = 0 \Rightarrow x = 2y$ $\frac{\partial z}{\partial y} = -12x + 6y^2 = 0 \Rightarrow y^2 = 2x = 4y$ $\Rightarrow y = 0$ or 4. $\Rightarrow x = 0$ or 8. $\Rightarrow z = 60$ or -4 Stationary points at A$(8, 4, -4)$, and B$(0, 0, 60)$	**M1** **A1** **M1** **B1** **A1**	For finding both partials and setting = 0 Both Solving simultaneously to get x or y Also by verification	
			5		
	(ii)(*A*)	$z = 3x^2 - 12xy + 2y^3 + 60$ Substitute $x = 8 + h,\ y = 4 + k$ $\Rightarrow z_p = 3(8+h)^2 - 12(8+h)(4+k) + 2(4+k)^3 + 60$ $= -4 + 3(h^2 - 4hk + 4k^2) + 12k^2 + 2k^3$ $= -4 + 3(h - 2k)^2 + 2k^2(k + 6)$	**M1** **M1** **A1**	For substitution and expansion For splitting $24k^2$	
			3		
	(ii)(*B*)	For all values of h and k, $(h - 2k)^2 > 0$ and $2k^2(k + 6) > 0$ providing $k > -6$ So $z > -4$ for all values of x and y close to A So is local minimum.	**M1** **M1** **A1**	Or for small k	
			3		

	(ii)(*C*)	When $x = 0,\ z = 2y^3 + 60$ and so either side of (0, 0, 60) the value of z will be greater or less than 60. When $y = 0,\ z = 3x^2 + 60$ and so either side of (0, 0, 60) the value of z will always be greater than 60. So B is a saddle point.	**M1** **A1** **A1** **A1**	For obtaining functions For pt of inflexion - can be by sketch For minimum - can be by sketch	
			4		
	(iii)	At (1, 1, 53), $\frac{\partial z}{\partial x} = 6x - 12y = -6$ $\frac{\partial z}{\partial y} = -12x + 6y^2 = -6$ Equation of tangent plane is: $z - 53 = \frac{\partial z}{\partial x}(x-1) + \frac{\partial z}{\partial y}(y-1)$ $z - 53 = -6x - 6y + 12$ $\Rightarrow 6x + 6y + z = 65$	**M1** **A1** **M1** **A1**	Finding values of derivatives Eqn of plane ag	Or $-6x - 6y - z = c$
			4		
	(iv)	$\frac{\partial z}{\partial x} = -6$ and $\frac{\partial z}{\partial y} = -6$ $\Rightarrow x = 2y - 1$ and $y^2 = 2x - 1$ $\Rightarrow y^2 - 4y + 3 = 0$ $\Rightarrow y = 1, 3$ $\Rightarrow$ Coordinates of R $(5, 3, 9)$	**M1** **A1** **M1** **A1** **A1**	Put partial derivatives = –6 Both correct Solve simultaneously Both values	
			5		

3	**(i)**	$x = a\cos\theta,\ x' = -a\sin\theta,\ x'' = -a\cos\theta$ $y = b\sin\theta,\ y' = b\cos\theta,\ y'' = -b\sin\theta$	**B1**	derivatives	
		$r = \dfrac{\left((x')^2 + (y')^2\right)^{3/2}}{x'y'' - x''y'} = \dfrac{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)^{3/2}}{-a\sin\theta.-b\sin\theta - -a\cos\theta.b\cos\theta}$	**M1**	Apply formula (or for κ)	
		$= \dfrac{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)^{3/2}}{ab}$	**M1**	Set $\theta = 0$	
		At A, $\theta = 0 \Rightarrow r = \dfrac{\left(a^2\sin^2 0 + b^2\cos^2 0\right)^{3/2}}{ab}$	**A1**	unsimplified	
		$= \dfrac{\left(b^2\right)^{3/2}}{ab} = \dfrac{b^2}{a}$	**A1**	ag	
		The centre is on the x-axis r less than a	**M1**		
		So centre of curvature is at $\left(a - \dfrac{b^2}{a}, 0\right)$ i.e. $\left(\dfrac{a^2 - b^2}{a}, 0\right)$	**A1**		
			7		
	(ii)	Radius = $\dfrac{a^2}{b}$	**B1**		
		Centre is at $\left(0, b - \dfrac{a^2}{b}\right)$ i.e. $\left(0, \dfrac{b^2 - a^2}{b}\right)$	**B1**		
			2		
	(iii)	$\dfrac{a^2}{b} = \dfrac{b^2}{a} \Rightarrow a = b$ The ellipse is a circle OR the centre of curvature for both points (and all points) is at (0, 0)	**B1**		
			1		

	(iv)	$s = \int_0^{\pi/2} \sqrt{\left(\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2\right)}\, d\theta$	**M1**	Applying formula	
		$= \int_0^{\pi/2} \sqrt{\left((a\sin\theta)^2 + (b\cos\theta)^2\right)}\, d\theta$			
		$= \int_0^{\pi/2} \sqrt{\left(a^2\sin^2\theta + b^2\cos^2\theta\right)}\, d\theta$	**A1**		
		$= \int_0^{\pi/2} \sqrt{\left(b^2\sin^2\theta + b^2\cos^2\theta + (a^2 - b^2)\sin^2\theta\right)}\, d\theta$	**M1**	Eliminate $\cos\theta$	
		$= \int_0^{\pi/2} \sqrt{\left(b^2 + (a^2 - b^2)\sin^2\theta\right)}\, d\theta$	**A1**		
		$= b\int_0^{\pi/2} \sqrt{\left(1 + \frac{(a^2 - b^2)}{b^2}\sin^2\theta\right)}\, d\theta$			
		$\Rightarrow \lambda^2 = \frac{(a^2 - b^2)}{b^2}$	**A1**		
		When $a = b$, $\lambda = 0$			
		$\Rightarrow s = b\int_0^{\pi/2} d\theta = \frac{1}{2}b\pi$	**A1**		
		This is a quarter of the circumference of a circle	**A1**	Or arc length of part of circle	
			7		

	(v)	Centre of curvature is at $\left(a\cos\theta-\rho\sin\psi, b\sin\theta+\rho\cos\psi\right)$ where $\rho=\dfrac{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{3/2}}{ab}$ giving $x=a\cos\theta-\dfrac{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{3/2}}{ab}.\dfrac{b\cos\theta}{\left(a^2\sin^2\theta+b^2\cos^2\theta\right)^{1/2}}$ $\Rightarrow ax=\cos\theta\left(a^2-\left(a^2\sin^2\theta+b^2\cos^2\theta\right)\right)$ $=\cos\theta\left(a^2-b^2\right)\cos^2\theta=\cos^3\theta\left(a^2-b^2\right)$ Similarly $by=\sin^3\theta\left(b^2-a^2\right)$ $\Rightarrow\left(\dfrac{ax}{a^2-b^2}\right)^{2/3}+\left(\dfrac{by}{a^2-b^2}\right)^{2/3}=1$ **oe**	**M1** **M1** **A1** **A1** **A1** **M1** **A1**	Parametric equations Deal with ψ i.e. $(ax)^{2/3}+(by)^{2/3}=\left(a^2-b^2\right)^{2/3}$	Or find equation of normal Or partial diffn of normal Eqn for x or y
			7		

4	**(i)**	The set is closed. i.e. $\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}\begin{pmatrix} c & d \\ 0 & \frac{1}{c} \end{pmatrix} = \begin{pmatrix} ac & ad+\frac{b}{c} \\ 0 & \frac{1}{ac} \end{pmatrix}$ Identity is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or m(1,0) Each element has an inverse Inverse of $\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}$ is $\begin{pmatrix} \frac{1}{a} & -b \\ 0 & a \end{pmatrix}$ Which is in M	**M1** **A1** **A1** **B1** **M1** **A1** **A1**	Attempt to demonstrate closure Two general and different matrices Correct product therefore shows closure Attempt to demonstrate inverse A general matrix and its inverse	
			7		
	(ii)	e.g. product $\begin{pmatrix} c & d \\ 0 & \frac{1}{c} \end{pmatrix}\times\begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix} = \begin{pmatrix} ac & cb+\frac{d}{a} \\ 0 & \frac{1}{ac} \end{pmatrix}$ OR one numeric example. e.g. $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & .5 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 0 & .5 \end{pmatrix}$ but $\begin{pmatrix} 2 & 3 \\ 0 & .5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ 0 & .5 \end{pmatrix}$ So No	**M1** **A1** **A1**	Demonstration by multiplying 2 matrices each way round (one way might be quoted from (i)) 2 correct (and different) products Dep on previous 2 marks	
			3		

	(iii)	e.g. $\begin{pmatrix} k & b \\ 0 & \frac{1}{k} \end{pmatrix}\begin{pmatrix} k & c \\ 0 & \frac{1}{k} \end{pmatrix}=\begin{pmatrix} k^2 & ..... \\ 0 & \frac{1}{k^2} \end{pmatrix}$ This is only in the set N_k if $k^2 = k$ Given $k \neq 0 \Rightarrow k = 1$ only	**M1** **A1** **A1** **A1**	Multiplying 2 matrices in N (Allow matrices the same) Sight of k^2	
			4		
	(iv)	$m(1,1)^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	**M1** **A1**		
			2		
	(v)	$\begin{matrix} & m(1,0) & m(1,1) & m(1,2) & m(1,3) \\ m(1,0) & m(1,0) & m(1,1) & m(1,2) & m(1,3) \\ m(1,1) & m(1,1) & m(1,2) & m(1,3) & m(1,0) \\ m(1,2) & m(1,2) & m(1,3) & m(1,0) & m(1,1) \\ m(1,3) & m(1,3) & m(1,0) & m(1,1) & m(1,2) \end{matrix}$	**B1** **B3**	m(1,0) the identity −1 each error	
			4		
	(vi)	Group table for R OR Any argument that states that: *a*, *b*, *c* have order 2 **B1** reason for this **B1** but only one element of P has order 2 **B1** So No	**B3** **B1**	−1 each error Dependent on previous B3	
			4		

5	**(i)**	$\mathbf{P} = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{pmatrix} 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{2} \end{pmatrix} \end{array}$	**B2**	B1 for two out of three columns correct	
			2		
	(ii)(*A*)	$\mathbf{P}^3 p$ $\begin{pmatrix} 0.138\dot{8} & - & - \\ - & - & - \\ - & - & - \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ gives $0.138\dot{8}\left(=\frac{5}{36}\right)$	**M1** **A1** **A1**	Cube P Sight of matrix soi	Allow M1 for P^4
			3		
	(ii)(*B*)	$P = M^5 p$ $\mathbf{P}^5 = \begin{pmatrix} ... & - & - \\ - & 0.4285 & - \\ - & - & 0.4286 \end{pmatrix}$ $p = 0.5\times 0.4285 + 0.5\times 0.4286$ $= 0.4286$	**M1** **M1** **A1** **A1**	Using diagonal elements from P^5 Using probabilities from 2nd day Ft Cao	
			4		
	(iii)	0.143, 0.429, 0.429 $\left(=\frac{1}{7},\frac{3}{7},\frac{3}{7}\right)$ Over a long period these are the probabilities that on any day at random the inspector is at these factories	**M1** **B1** **A1** **B1**	Obtaining equations Sight of $x + y + z = 1$ soi	Or **M1** considering P^n where n is large.(10 or more) **A2** for probabilities, A1 one error
			4		

	(iv)	$\begin{matrix} & \text{A} & \text{B} & \text{C} \\ \text{A} \\ \mathbf{Q} = \text{B} \\ \text{C} \end{matrix}\begin{pmatrix} 0.8 & 1/3 & 0 \\ 0.1 & 1/3 & 1/2 \\ 0.1 & 1/3 & 1/2 \end{pmatrix}$ 0.455, 0.273, 0.273 $\left(=\frac{5}{11},\frac{3}{11},\frac{3}{11}\right)$	**B1** **M1** **A1**	Solve or consider Q^n for large n.	
			3		
	(v)	P(from A to A) = 0.8. So $\alpha = 0.8$ $\Rightarrow \frac{\alpha}{1-\alpha} = \frac{0.8}{0.2} = 4$	**B1** **M1** **A1**	For using $\frac{\alpha}{1-\alpha}$ or $\frac{1}{1-\alpha}$	
			3		
	(vi)	New transition matrix: $\begin{matrix} & \text{A} & \text{B} & \text{C} \\ \text{A} \\ \mathbf{R} = \text{B} \\ \text{C} \end{matrix}\begin{pmatrix} 1 & 1/3 & 0 \\ 0 & 1/3 & 1/2 \\ 0 & 1/3 & 1/2 \end{pmatrix}$ We need 3rd entry of 2nd row to be < 0.1 $\mathbf{R}^9 = \begin{pmatrix} ... & ... & ... \\ ... & ... & 0.1162 \\ ... & ... & ... \end{pmatrix}$, $\mathbf{R}^{10} = \begin{pmatrix} ... & ... & ... \\ ... & ... & 0.0969 \\ ... & ... & ... \end{pmatrix}$ So 10 days later (which is day 25).	 **M1** **A1** **A1**	Method by "trial" with new matrix. For sight of one value	
			3		
	(vii)	A is the absorbing state. If it goes to A then it stays there.	**B1** **B1**	**oe**	
			2		