Oxford Cambridge and RSA

Wednesday 8 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4772/01 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4772/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Martin is considering paying for a vaccination against a disease. If he catches the disease he would not be able to work and would lose £900 in income because he would have to stay at home recovering. The vaccination costs £20. The vaccination would reduce his risk of catching the disease during the year from 0.02 to 0.001.

(i) Draw a decision tree for Martin. **[3]**

(ii) Evaluate the EMV of Martin's loss at each node of your tree, and give the action that Martin should take to minimise the EMV of his loss. **[4]**

Martin can answer a medical questionnaire which will give an estimate of his susceptibility to the disease. If he is found to be susceptible, then his chance of catching the disease is 0.05. Vaccination will reduce that to 0.0025. If he is found not to be susceptible, then his chance of catching the disease is 0.01 and vaccination will reduce it to 0.0005. Historically, 25% of people are found to be susceptible.

(iii) What is the EMV of this questionnaire? **[6]**

Martin decides not to answer the questionnaire. He also decides that there is more than just his EMV to be considered in deciding whether or not to have the vaccination. The vaccination itself is likely to have side effects, but catching the disease would be very unpleasant. Martin estimates that he would find the effects of the disease 1000 times more unpleasant than the effects of the vaccination.

(iv) Analyse which course of action would minimise the unpleasantness for Martin. **[3]**

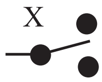
2 **(a)** Emelia: 'I won't go out for a walk if it's not dry or not warm.'
Gemma: 'It's warm. Let's go!'

Will what Gemma has said convince Emelia, and if not, why not? **[2]**

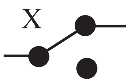
(b) If it is daytime and the car headlights are on, then it is raining.
If the dashboard lights are dimmed then the car headlights are on.
It is daytime.
It is not raining.

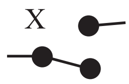
(i) What can you deduce? **[2]**

(ii) Prove your deduction. **[6]**

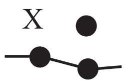
(c) In this part of the question the switch X is represented by 

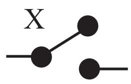
The switch can be wired into a circuit so that current flows when

the switch is up 

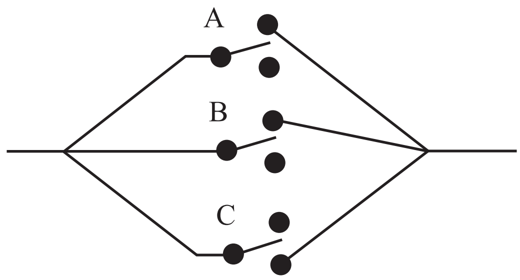
but does not flow when it is down 

Or the switch can be wired so that current flows when

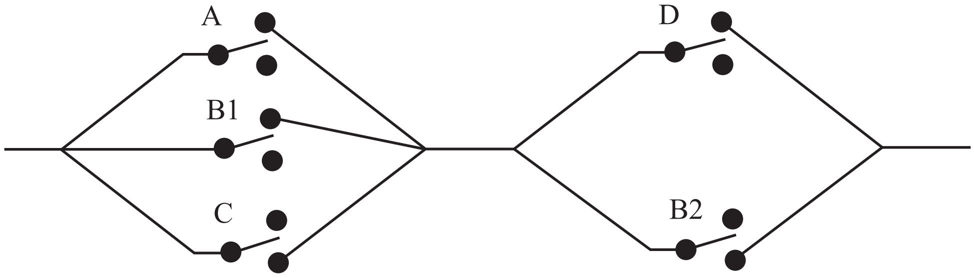
the switch is down 

but does not flow when it is up 

(i) Explain how the following circuit models $(A \wedge B) \Rightarrow C$. **[4]**



In the following circuit B1 and B2 represent 'ganged' switches. This means that the two switches are either both up or both down.



(ii) Given that A is down, C is up and current is flowing, what can you deduce? **[2]**

3 Neil is refurbishing a listed building. There are two types of paint that he can use for the inside walls. One costs £1.45 per m^2 and the other costs £0.95 per m^2. He must paint the lower half of each wall in the more expensive paint. He has $350\,m^2$ of wall to paint. He has a budget of £400 for wall paint.
The more expensive paint is easier to use, and so Neil wants to use as much of it as possible.

Initially, the following LP is constructed to help Neil with his purchasing of paint.

Let x be the number of m^2 of wall painted with the expensive paint.

Let y be the number of m^2 of wall painted with the less expensive paint.

Maximise $P = x + y$
subject to $1.45x + 0.95y \leqslant 400$
$y - x \leqslant 0$
$x \geqslant 0$
$y \geqslant 0$

(i) Explain the purpose of the inequality $y - x \leqslant 0$. **[1]**

(ii) The formulation does not include the inequality $x + y \geqslant 350$. State what this constraint models and why it has been omitted from the formulation. **[2]**

(iii) Use the simplex algorithm to solve the LP. Pivot first on the “1” in the y column. Interpret your solution. **[7]**

The solution shows that Neil needs to buy more paint. He negotiates an increase in his budget to £450.

(iv) Find the solution to the LP given by changing $1.45x + 0.95y \leqslant 400$ to $1.45x + 0.95y \leqslant 450$, and interpret your solution. **[2]**

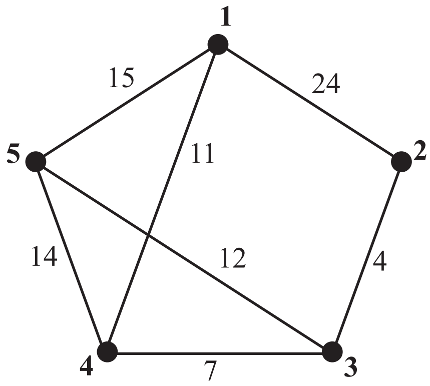
Neil realises that although he now has a solution, that solution is not the best for his requirements.

(v) Explain why the revised solution is not optimal for Neil. **[1]**

In order to move to an optimal solution Neil needs to change the objective of the LP and add another constraint to it.

(vi) Write down the new LP and the initial tableau for using two-stage simplex to solve it. Give a brief description of how to use two-stage simplex to solve it. **[7]**

4



(a) Solve the route inspection problem in the network above, showing the methodology you used to ensure that your solution is optimal. Show your route. **[4]**

(b) Floyd's algorithm is applied to the same network to find the complete network of shortest distances. After three iterations the distance and route matrices are as follows.

	1	**2**	**3**	**4**	**5**
1	48	24	28	11	15
2	24	8	4	11	16
3	28	4	8	7	12
4	11	11	7	14	14
5	15	16	12	14	24

	1	**2**	**3**	**4**	**5**
1	2	2	2	4	5
2	1	3	3	3	3
3	2	2	2	4	5
4	1	3	3	3	5
5	1	3	3	4	3

(i) Perform the fourth iteration of the algorithm, and show that there is no change to either matrix in the final iteration. **[4]**

(ii) Show how to use the matrices to give the shortest distance and the shortest route from vertex **1** to vertex **2**. **[3]**

(iii) Draw the complete network of shortest distances. **[2]**

(iv) Starting at vertex **1**, apply the nearest neighbour algorithm to the complete network of shortest distances to find a Hamilton cycle. Give the length of your cycle and interpret it in the original network. **[3]**

(v) By temporarily deleting vertex **1** and its connecting arcs from the complete network of shortest distances, find a lower bound for the solution to the Travelling Salesperson's Problem in that network. Say what this implies in the original network. **[4]**

END OF QUESTION PAPER

4 (a)

4 (b) (i) **A spare copy of these tables can be found on page 12.**

	1	**2**	**3**	**4**	**5**
1	48	24	28	11	15
2	24	8	4	11	16
3	28	4	8	7	12
4	11	11	7	14	14
5	15	16	12	14	24

	1	**2**	**3**	**4**	**5**
1	2	2	2	4	5
2	1	3	3	3	3
3	2	2	2	4	5
4	1	3	3	3	5
5	1	3	3	4	3

	1	**2**	**3**	**4**	**5**
1					
2					
3					
4					
5					

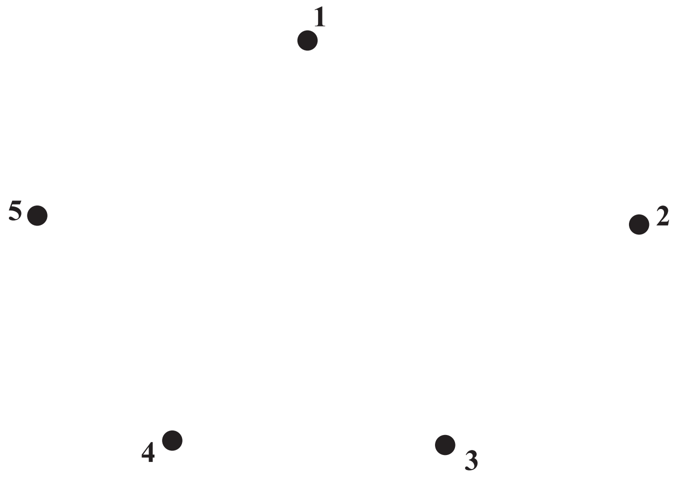
	1	**2**	**3**	**4**	**5**
1					
2					
3					
4					
5					

	1	**2**	**3**	**4**	**5**
1					
2					
3					
4					
5					

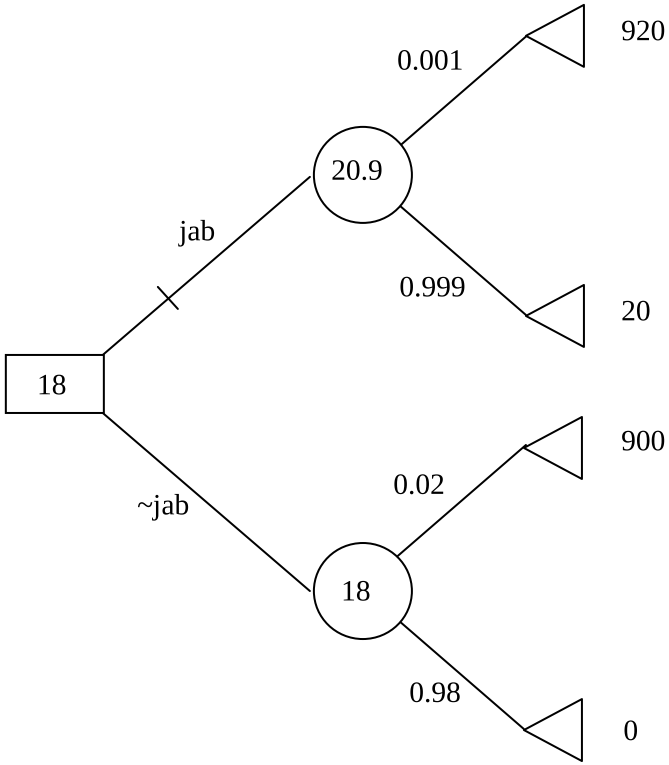
	1	**2**	**3**	**4**	**5**
1					
2					
3					
4					
5					

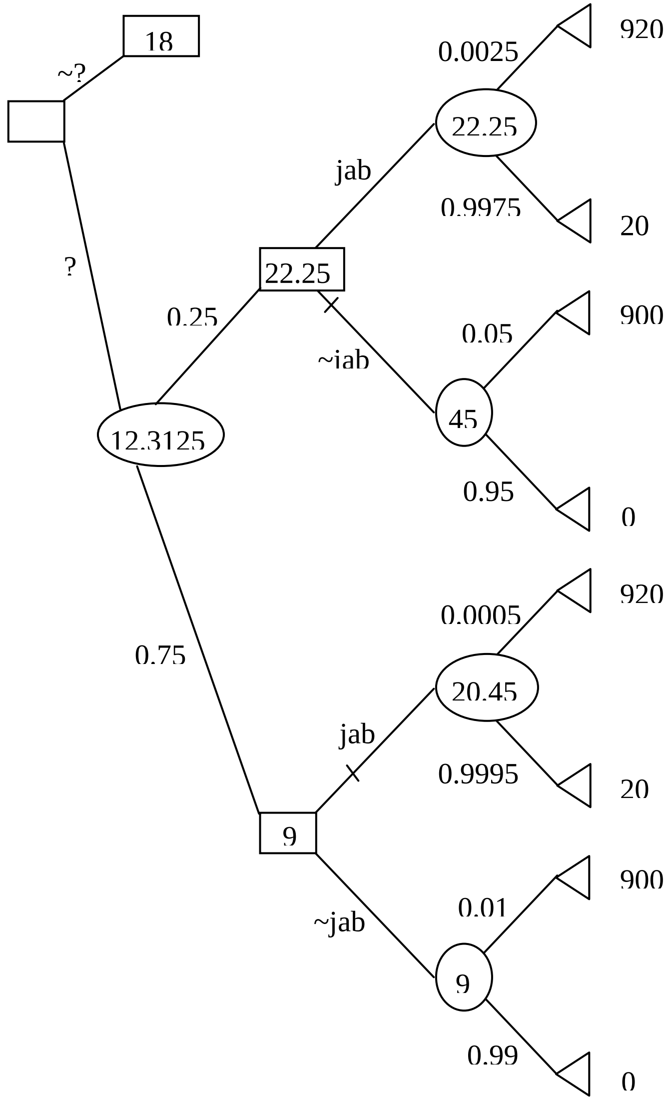
4 (b) (ii)

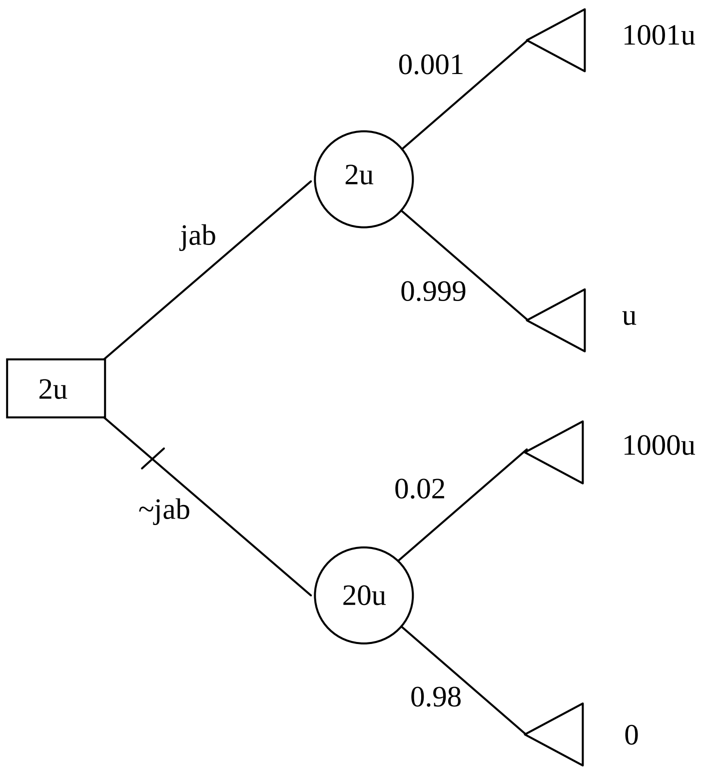
4 (b) (iii)



4 (b) (iv)

Question			Answer	Marks	Guidance
1	(i)& (ii)		 Do not have the vaccination.	M1 M1 A1 B1 B1√ B1√ B1√	decision node at first branch chance nodes at second branches all correct 920, 20, 900 and 0 20.9 and 18 18 at decision node action

Question		Answer	Marks	Guidance
	(iii)		B1 B1 M1 A1 A1 B1	? or ~? susceptible or not chance node decision structure on both branches susceptible branch, all calculations not susceptible branch, all calculations EMV = 18 – 12.3125 = 5.6875

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td></td><td>(iv)</td><td></td><td>
</td><td>B1

B1
B1</td><td>unpleasantness measure + final decision
jab computations
~jab computations</td></tr>
</table>

Question			Answer	Marks	Guidance
2	(a)		No It might be wet.	B1 B1	
	(b)	(i)	The car headlights are off. The dashboard lights are not dimmed.	B1 B1	
	(b)	(ii)	$(d \wedge l) \Rightarrow r$ so $\sim r \Rightarrow \sim d \vee \sim l$ so $\sim d \vee \sim l$ so $\sim l$ $dim \Rightarrow l$ so $\sim l \Rightarrow \sim dim$ so $\sim dim$ or 4 line truth table (since r=0 and d=1) for $((d \wedge l) \Rightarrow r) \wedge (dim \Rightarrow l)) \Rightarrow (\sim l \wedge \sim dim)$ SC One line of the above showing … $d \wedge l \Rightarrow r$ and then $dim \Rightarrow l$ 1 0 0 1 0 0 1 0	M1A1 A1 A1 B1 B1 M1 A2 A2 A1 (B3B1)	contrapositive modus ponens alternation contrapositive modus ponens (terminology not required) LHS (-1 each error) RHS (-1 each error) tautology
	(c)	(i)	Current flows through true path(s) "Up" signifies false and "down" signifies truth. eg So current only fails to flow when A and B are both true and C is false. or Circuit is $\sim A \vee \sim B \vee C$ $\Leftrightarrow \sim(A \wedge B) \vee C$ $\Leftrightarrow (A \wedge B) \Rightarrow C$	M1 M1 A1 A1 M1 A1 A1 A1	can be implied can be implied A and B true C false

Question			Answer	Marks	Guidance
	(c)	**(ii)**	B is up D is up (Alert candidates might try to present (c) as an answer to (b)(ii). This is not acceptable, though one might argue that a physical instantiation would be … a proof machine.)	B1 B1	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>3</td><td>(i)</td><td></td><td>“He must paint the lower half of each wall in the more expensive paint.”</td><td>B1</td><td></td></tr>
<tr><td></td><td>(ii)</td><td></td><td>“He has 350 m^2 of wall to paint.”
The Simplex algorithm deals with ≤ inequalities. Two-stage or Big-M needed for ≥.</td><td>B1
B1</td><td></td></tr>
<tr><td></td><td>(iii)</td><td></td><td>
<table>
<tr><th>P</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>RHS</th></tr>
<tr><td>1</td><td>-1</td><td>-1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1.45</td><td>0.95</td><td>1</td><td>0</td><td>400</td></tr>
<tr><td>0</td><td>-1</td><td>1</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>-2</td><td>0</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>0</td><td>2.4</td><td>0</td><td>1</td><td>-0.95</td><td>400</td></tr>
<tr><td>0</td><td>-1</td><td>1</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>0</td><td>0</td><td>5/6</td><td>5/24</td><td>333.33</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>5/12</td><td>-19/48</td><td>166.67</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>5/12</td><td>29/48</td><td>166.67</td></tr>
</table>
166.67 m^2 using expensive paint and 166.67 m^2 using less expensive paint.
Coverage 333.33 m^2.</td><td>M1
A1

M1
A1

M1
A1

B1</td><td>objective row

first pivot

second pivot

must be in terms of m^2 of coverage</td></tr>
<tr><td></td><td>(iv)</td><td></td><td>187.5
Coverage = 375m^2, or allow 187.5m^2 each.</td><td>B1
B1</td><td></td></tr>
<tr><td></td><td>(v)</td><td></td><td>The solution does not maximise the use of the more expensive paint.</td><td>B1</td><td></td></tr>
</table>

Question		Answer	Marks	Guidance
	(vi)	maximise $P = x$ subject to $1.45x + 0.95y < 450$ $y - x < 0$ $x + y > 350$ $x > 0$ $y > 0$	B1 B1	
		(tableau below)	B1 B1 B1 B1	surplus variable new objective artificial variable new constraint
		Minimise A to 0 and then proceed from the feasible solution (Solution is x=235 and y=115)	B1	

A	P	x	y	s_1	s_2	s_3	a	RHS
1	0	1	1	0	0	-1	0	350
0	1	-1	0	0	0	0	0	0
0	0	1.45	0.95	1	0	0	0	450
0	0	-1	1	0	1	0	0	0
0	0	1	1	0	0	-1	1	350

Question			Answer	Marks	Guidance
4	(a)		Odd vertices are **1**, **3**, **4** and **5**.	M1	
			Pairing and connecting ...		
			1 and **3**, **4** and **5** ... 18 + 14 = 32	M1	2 pairings
			1 and **4**, **3** and **5** ... 11 + 12 = 23	A1	all pairings
			1 and **5**, **3** and **4** ... 15 + 7 = 22		
			So min is 87 + 22 = 109, e.g. **1 5 1 2 3 4 3 5 4 1**	A1	length and a route.
4	(b)	(i)	(matrices below)	M1 A1 A1	use of the triple operation entries other than (2,1) “3” in (2,1)
				B1	final matrices the same

	1	**2**	**3**	**4**	**5**
1	48	24	28	11	15
2	24	8	4	11	16
3	28	4	8	7	12
4	11	11	7	14	14
5	15	16	12	14	24

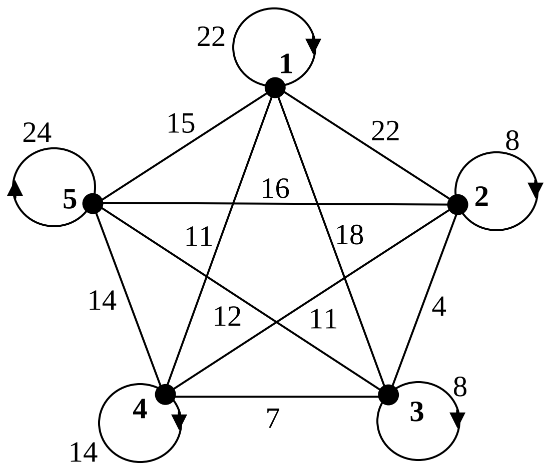
	1	**2**	**3**	**4**	**5**
1	2	2	2	4	5
2	1	3	3	3	3
3	2	2	2	4	5
4	1	3	3	3	5
5	1	3	3	4	3

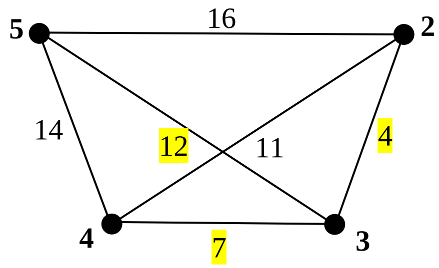
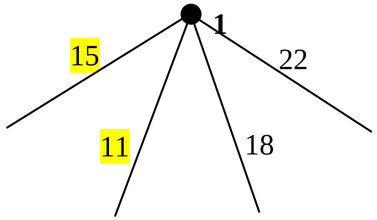
	1	**2**	**3**	**4**	**5**
1	22	22	18	11	15
2	22	8	4	11	16
3	18	4	8	7	12
4	11	11	7	14	14
5	15	16	12	14	24

	1	**2**	**3**	**4**	**5**
1	4	4	4	4	5
2	3	3	3	3	3
3	4	2	2	4	5
4	1	3	3	3	5
5	1	3	3	4	3

	1	**2**	**3**	**4**	**5**
1	22	22	18	11	15
2	22	8	4	11	16
3	18	4	8	7	12
4	11	11	7	14	14
5	15	16	12	14	24

	1	**2**	**3**	**4**	**5**
1	4	4	4	4	5
2	3	3	3	3	3
3	4	2	2	4	5
4	1	3	3	3	5
5	1	3	3	4	3

Question			Answer	Marks	Guidance
4	(b)	(ii)	Shortest distance is (1,2) entry in first matrix ... 22 Route ... (1,2) entry in second matrix is 4, so $1 \rightarrow 4$ (4,2) entry in second matrix is 3, so $1 \rightarrow 4 \rightarrow 3$ (3,2) entry in second matrix is 2, so $1 \rightarrow 4 \rightarrow 3 \rightarrow 2$	B1 M1 A1	2 to 1 OK backwards OK, if then corrected
4	(b)	(iii)	 	B1 B1	loops rest
4	(b)	(iv)	**1** 11 **4** 7 **3** 4 **2** 16 **5** 15 **1** ... total length 53 Represents **1 4 3 2 3 5 1**	B1B1 B1	

Question			Answer	Marks	Guidance
4	(b)	(v)	MST in reduced network has length 23.   26 to be added i, giving lower bound of 49 to TSP in network of shortest distances. So the best tour in the original network (revisiting allowed) must be between 49 and 53.	M1 M1 A1 B1	MST – 23 correct two arcs to add in lower bound

4772 Decision Mathematics 2

General Comments:

Candidates for this demanding paper present a wide variety of attainments. Some answers are very good indeed, with answers exceeding examiner expectations. Others struggled with the abstraction that is needed to apply maths to real problems.

Comments on Individual Questions:

Question No. 1

Decision Analysis represents a simple but powerful meta-tool, a methodology for structuring problem solving. Candidates were impressive in their mastery of it. Few put a foot wrong in the structuring of the problems, losing marks only in some details.

Parts (i) and (ii) presented very few difficulties. Some candidates worked in outcomes rather than losses, which was OK. Indeed, arguably it is better since it is consonant with applying a utility function. Less good was the practice seen on some scripts of incorporating the £20 cost of vaccination downstream from the relevant decision node. This is not a good idea. The total cost/benefit of each outcome needs to be evaluated at the end of the corresponding branch.

In part (iii) many candidates lost a mark by not starting from the decision to answer or not answer the questionnaire. The “not” branch does not need to repeat part (ii) - the outcome is sufficient. Very few candidates answered the question. They evaluated the cost corresponding to the “answer the questionnaire” branch, but they failed to then find, by subtraction, the value of the questionnaire.

A mark was reserved in part (iv) for establishing an “unpleasantness measure”, or at least recognising the need for it. Many failed to establish the outcomes of 1000u versus 0u, and 1001u versus 1u.

Question No. 2

This question, the logic question, was far more problematic. It was difficult to mark because many candidates who did understand what they were doing found it difficult to express that with sufficient clarity to distinguish themselves from candidates who were not scoring marks.

Part (a)(i) was surprising in the number of candidates it caught out. It seemed quite straightforward to note that Emelia needed Gemma to tell her about precipitation as well as about temperature, but many thought not. It is understandable that confusion can take hold in the stress of an examination, and tools might then help. Thus Emelia’s statement can be modelled as:

$(\sim d \vee \sim w) \Rightarrow \sim walk$, the contrapositive of which is:

$walk \Rightarrow (d \wedge w)$.

Some candidates pointed out that even if it is dry and warm, we don’t know if Emelia will walk, which is true – necessity versus sufficiency. But that comes after the need first for it to be dry as well as warm.

Most candidates collected both marks in part (b)(i), but proving their assertions in part (b)(ii) was a different matter. There were many ways to go about this. The most productive was to use the contrapositive, or the properties of the implication connective, which comes to the same thing.

Candidates reaching for their truth tables often failed to keep thinking, many doing no more that producing a truth table or truth tables for the given implication statement(s).

A full handle-turning, all-possibilities statement for a truth table would be:

$$\Big(\big((d \wedge h) \Rightarrow r\big) \wedge (\text{dim} \Rightarrow h) \wedge d \wedge \sim r\Big) \Rightarrow (\sim h \wedge \sim \text{dim})$$

This looks daunting, but it can be reduced to 4 lines and have the "$\wedge\ d \wedge \sim r$" dropped by restricting attention to d = 1 and r = 0. Even better to split it into two parts with two 2-line tables, as most did.

Part (c)(i) caused difficulties. Very few candidates were explicit in their identification of switches/up/down with statements/true/false, nor with current flowing or not flowing when the corresponding compound statement was true/false. Having said that there were some excellent solutions, arguably the best of which was to use Boolean algebra to show that $(A \wedge B) \Rightarrow C$ is equivalent to $\sim A \vee \sim B \vee C$, which is seen to be implemented in the circuit.

Question No. 3

This question was an attempt to demonstrate how LP is often used in practice – to explore options rather than as an all-encompassing model. Candidates seemed very comfortable with this.

Many candidates failed to score the first mark by offering answers such as "To have more expensive paint used than inexpensive paint". The correct answer was "He must paint the lower half of each wall in the more expensive paint".

In part (ii) most candidates scored the first mark, wall coverage. The preferred answer for the second mark was to keep it simple by avoiding two-stage Simplex. Here we departed from reality since this is dealt with easily in computing packages.

There was a very good level of skill shown in applying the algorithm in part (iii). Most candidates also scored the interpretation mark, which hung on them recognising that the numbers represented square metres of coverage. That also applied to part (iv).

At this point most candidates, probably tiring, lost sight of the objective. There was a collective assumption that Neil would wish to minimise cost, but that was never mentioned as an objective. It was stated that Neil wants to use as much of the more expensive paint as possible, within constraints on coverage and his budget … because it is easier to use. So most candidates missed this in (v) and did not model it in (vi). The mark scheme in part (vi) was kind to these candidates, and good drilling led to good marks for most.

Question No. 4

This question covered the usual mix of network algorithms but with an unusual balance, all applied to one given network.

Part (a) was the CPP, with 4 odd nodes. So 3 pairings were possible, and all needed to be seen. Most candidates were alert to what was needed. A few had no idea. Some lost marks on details. A surprising number of candidates tried to use nearest neighbour on this part.

Part (b)(i) was Floyd, and was done exceptionally well. Most also did well, but not as well, with the explanations in (b)(ii). Some gave answers rather than explanations, and some could only manage incoherent attempts at explanations

Part (b)(iii) was there to set up the remaining 2 parts. Part (b)(iv) was done very well. In part (b)(v) the minimum connector was often wrong, and the deduction was often incomplete..