OCR
Oxford Cambridge and RSA

Wednesday 3 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4772/01   Decision Mathematics 2

QUESTION PAPER

Duration: 1 hour 30 minutes
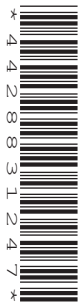
INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

*   The Question Paper will be found inside the Printed Answer Book.
*   Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
*   **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
*   Use black ink. HB pencil may be used for graphs and diagrams only.
*   Read each question carefully. Make sure you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   You are permitted to use a graphical calculator in this paper.
*   Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

*   The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
*   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
*   The total number of marks for this paper is **72**.
*   The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

*   Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A furniture manufacturer is planning a production run. He will be making wardrobes, drawer units and desks. All can be manufactured from the same wood.

He has available $200\,m^2$ of wood for the production run. Allowing for wastage, a wardrobe requires $5\,m^2$, a drawer unit requires $3\,m^2$, and a desk requires $2\,m^2$.

He has 200 hours available for the production run. A wardrobe requires 4.5 hours, a drawer unit requires 5.2 hours, and a desk requires 3.8 hours.

The completed furniture will have to be stored at the factory for a short while before being shipped. The factory has $50\,m^3$ of storage space available. A wardrobe needs $1\,m^3$, a drawer unit needs $0.75\,m^3$, and a desk needs $0.5\,m^3$.

The manufacturer needs to know what he should produce to maximise his income. He sells the wardrobes at £80 each, the drawer units at £65 each and the desks at £50 each.

(i) Formulate the manufacturer's problem as an LP. **[6]**

(ii) Use the Simplex algorithm to solve the LP problem. **[6[**

(iii) Interpret the results. **[3]**

(iv) An extra $25\,m^2$ of wood is found and is to be used. The new optimal solution is to make 44 wardrobes, no drawer units and no desks. However, this leaves some of each resource (wood, hours and space) left over. Explain how this can be possible. **[1]**

2   **(i)** Given that x and y are propositions, draw a 4-line truth table for $x \Rightarrow y$, allowing x and y to take all combinations of truth values.

   If x is false and $x \Rightarrow y$ is true, what can be deduced about the truth value of y?   **[2]**

A story has it that, in a lecture on logic, the philosopher Bertrand Russell (1872–1970) mentioned that a false proposition implies any proposition.

A student challenged this, saying "In that case, given that $1 = 0$, prove that you are the Pope."

Russell immediately replied, "Add 1 to both sides of the equation: then we have $2 = 1$. The set containing just me and the Pope has 2 members. But $2 = 1$, so the set has only 1 member; therefore, I am the Pope."

Russell's string of statements is an example of a deductive sequence. Let a represent "$1 = 0$", b represent "$2 = 1$", c represent "Russell and the Pope are 2" and d represent "Russell and the Pope are 1". Then Russell's deductive sequence can be written as $(a \wedge (a \Rightarrow b) \wedge c) \Rightarrow d$.

   (ii) Assuming that a is false, b is false, $a \Rightarrow b$ is true, c is true, and that d can take either truth value, draw a 2-line truth table for $(a \wedge (a \Rightarrow b) \wedge c) \Rightarrow d$.   **[2]**

   (iii) What does the table tell you about d with respect to the false proposition a?   **[2]**

   (iv) Explain why Russell introduced propositions b and c into his argument.   **[1]**

   (v) Russell could correctly have started a deductive sequence:

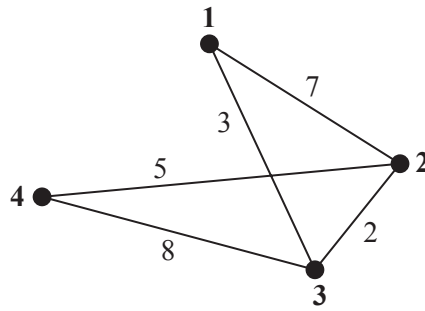 $a \wedge [a \Rightarrow ((0.5 = -0.5) \Rightarrow (0.25 = 0.25))]$.

   Had he have done so could he correctly have continued it to end at d?

   Justify your answer.   **[2]**

   (vi) Draw a combinatorial circuit to represent $(a \wedge (a \Rightarrow b) \wedge c) \Rightarrow d$.   **[7]**

3 Floyd's algorithm is applied to the incomplete network on 4 nodes drawn below. The weights on the arcs represent journey times.

The final matrices are shown below.

final time matrix

	1	2	3	4
1	6	5	3	10
2	5	4	2	5
3	3	2	4	7
4	10	5	7	10

final route matrix

	1	2	3	4
1	3	3	3	3
2	3	3	3	4
3	1	2	2	2
4	2	2	2	2

  (i) Draw the complete network of shortest times. **[2]**

 (ii) Explain how to use the final route matrix to find the quickest route from node **4** to node **1** in the original incomplete network. Give this quickest route. **[3]**

A new node, node **5**, is added to the original incomplete network. The new journey times are shown in the table.

	1	2	3	4
5	4	–	–	2

(iii) Draw the complete 5-node network of shortest times. (You are not required to use an algorithm to find the shortest times.) **[3]**

 (iv) Write down the final time matrix and the final route matrix for the complete 5-node network. (You do not need to apply Floyd's algorithm.) **[3]**

  (v) (*A*) Apply the nearest neighbour algorithm to the complete 5-node network of shortest times, starting at node **1**. Give the time for the cycle you produce. **[2]**

        (*B*) Starting at node **3**, a possible cycle in the complete 5-node network of shortest times is **3 2 1 5 4 3**. Give the actual cycle to which this corresponds in the incomplete 5-node network of journey times. **[1]**

 (vi) By deleting node **5** and its arcs from the complete 5-node network of shortest times, and then using Kruskal's algorithm, produce a lower bound for the solution to the associated practical travelling salesperson problem. Show clearly your use of Kruskal's algorithm. **[3]**

(vii) In the incomplete 5-node network of journey times, find a quickest route starting at node **5** and using each of the 7 arcs at least once. Give the time of your route. **[3]**

4   Helen has a meeting to go to in London. She has to travel from her home in G on the south coast to KC in London. She can drive from G to the west to A to catch a train, or she can drive to the east to W to catch a train on a different line. From both A and W she can travel to mainline terminuses V or LB in London. She will then travel by tube either from V to KC, or from LB to KC.

The times for the various steps of her journey are shown in the tables. Both train services and tube services are subject to occasional delays, and these are modelled in the tables.

Driving times	to A	to W
From G	20 min	15 min

Train journey	To V			To LB		
	normal time	probability of delay	delay	normal time	probability of delay	delay
From A	1 hr 40 min	0.05	10 min	1 hr 45 min	0.05	10 min
From W	1 hr 30 min	0.10	20 min	1 hr 35 min	0.10	20 min

Tube journey	To KC		
	normal time	probability of delay	delay
From V	7 min	0.20	2 min
From LB	9 min	0.10	2 min

Helen wants to choose the route which will give the shortest expected journey time.

(i) Draw a decision tree to model Helen's decisions and the possible outcomes. **[8]**

(ii) Calculate Helen's shortest expected journey time and give the route which corresponds to that shortest expected journey time. **[8]**

As she gets into her car, Helen hears a travel bulletin on the radio warning of a broken escalator at V. This means that routes through V will take Helen 10 minutes longer.

(iii) Find the value of the radio information, explaining your calculation. **[3]**

(iv) Why might the shortest expected journey time not be the best criterion for choosing a route for Helen? **[1]**
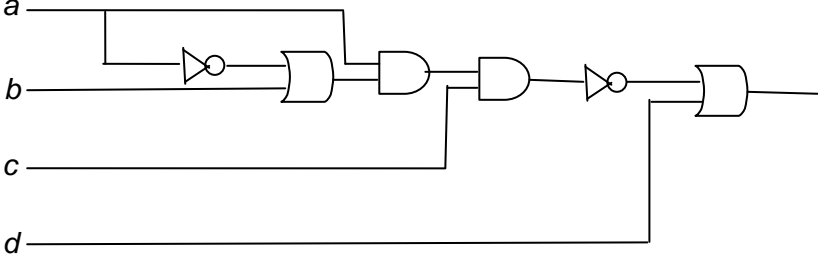
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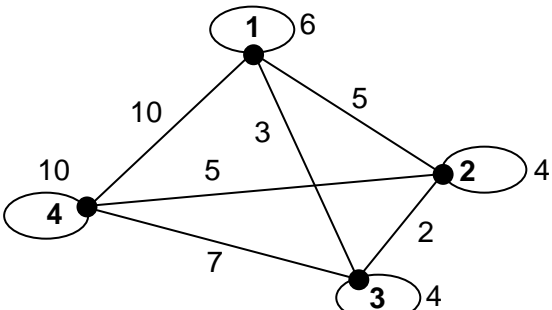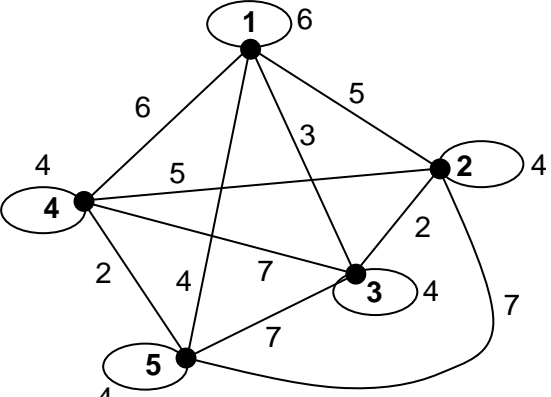
3 (iii)

3 (iv)

final time matrix

	1	2	3	4	5
1					
2					
3					
4					
5					
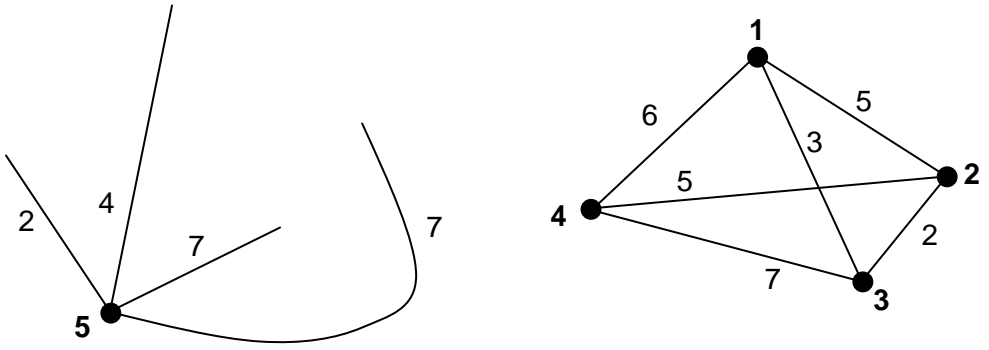
final route matrix

	1	2	3	4	5
1					
2					
3					
4					
5					

Turn over
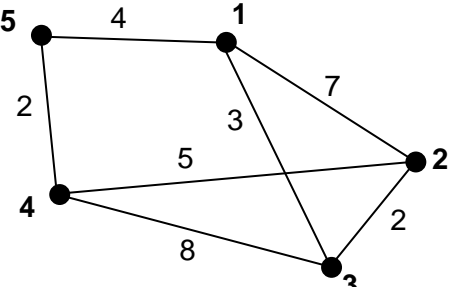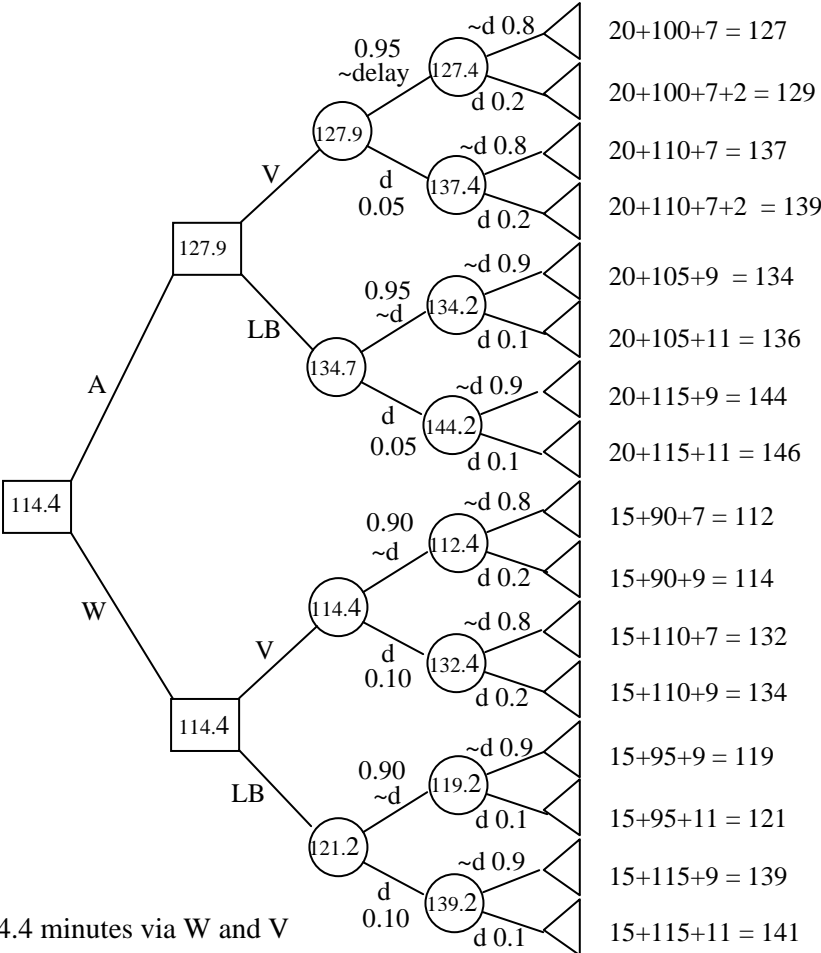
Question			Answer	Marks	Guidance
1	**(i)**		Let w be the number of wardrobes made. Let u be the number of drawer units made. Let d be the number of desks made.	B1 B1	definition "number of"

			Max 80w+65u+50d
			st 5w+3u+2d < 200
			        4.5w+5.2u+3.8d < 200
			        w+0.75u+0.5d < 50
				B1
				B1
				B1
				B1
					SC …
					-2 for consistent ">"

1	**(ii)**		<table><tr><td>M</td><td>w</td><td>u</td><td>d</td><td>s1</td><td>s2</td><td>s3</td><td>RHS</td></tr><tr><td>1</td><td>-80</td><td>-65</td><td>-50</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>5</td><td>3</td><td>2</td><td>1</td><td>0</td><td>0</td><td>200</td></tr><tr><td>0</td><td>4.5</td><td>5.2</td><td>3.8</td><td>0</td><td>1</td><td>0</td><td>200</td></tr><tr><td>0</td><td>1</td><td>0.75</td><td>0.5</td><td>0</td><td>0</td><td>1</td><td>50</td></tr><tr><td>1</td><td>0</td><td>-17</td><td>-18</td><td>16</td><td>0</td><td>0</td><td>3200</td></tr><tr><td>0</td><td>1</td><td>0.6</td><td>0.4</td><td>0.2</td><td>0</td><td>0</td><td>40</td></tr><tr><td>0</td><td>0</td><td>2.5</td><td>2</td><td>-0.9</td><td>1</td><td>0</td><td>20</td></tr><tr><td>0</td><td>0</td><td>0.15</td><td>0.1</td><td>-0.2</td><td>0</td><td>1</td><td>10</td></tr><tr><td>1</td><td>0</td><td>5.5</td><td>0</td><td>7.9</td><td>9</td><td>0</td><td>3380</td></tr><tr><td>0</td><td>1</td><td>0.1</td><td>0</td><td>0.38</td><td>-0.2</td><td>0</td><td>36</td></tr><tr><td>0</td><td>0</td><td>1.25</td><td>1</td><td>-0.45</td><td>0.5</td><td>0</td><td>10</td></tr><tr><td>0</td><td>0</td><td>0.025</td><td>0</td><td>-0.155</td><td>-0.05</td><td>1</td><td>9</td></tr></table> w=36, u=0, d=10 and M=3380 with s1=s2=0 and s3=9 (not needed)	M1 A1√ M1 A1√ M1 A1√	initial tableau obj + 3 constraints initial tableau OK pivot pivot
1	**(iii)**		Make 36 wardrobes and 10 desks. Income = £3380 9m^3 of storage space spare	B1√ B1√ B1√	
1	**(iv)**		An integer solution is required, and that may mean that no resource is exhausted.	B1	

	Question		Answer	Marks	Guidance
2	**(i)**		<table><tr><td>x</td><td>y</td><td>$x \Rightarrow y$</td></tr><tr><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>0</td><td>1</td></tr></table> y can either be true or false.	B1 B1	cao
2	**(ii)**		<table><tr><td>a</td><td>b</td><td>c</td><td>$a \Rightarrow b$</td><td>$a \land (a \Rightarrow b)$</td><td>$a \land (a \Rightarrow b) \land c$</td><td>$d$</td><td>$(a \land (a \Rightarrow b) \land c) \Rightarrow d$</td></tr><tr><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td></tr></table>	B1 B1	SC … -1 for shortened form with order not explicit
2	**(iii)**		That one can build a (correct) deductive sequence from a to d, whether d is true or false.	B1 B1	deduction d true or false
2	**(iv)**		So that he could establish a (correct) deductive sequence from "1=0" to "Russell is the Pope".	B1	
2	**(v)**		No, because from a true statement only truth can correctly be deduced, and Russell was not the Pope.	M1 A1	true to false needed so "no"
2	**(vi)**			B1 M1 A1 M1 A1 M1 A1	using *abcd* as inputs implying *b* implying *d* using an "and" both correct

	Question		Answer	Marks	Guidance
3	**(i)**			B1 B1	complete network, including loops all 10 times correct
3	**(ii)**		Look in row 4, column 1 to find first vertex en route ... **2** Now look in row 2 column 1 ... **3** Now row 3 column 1 ... **1** So route is **4 → 2 → 3 → 1**	M1 A1 B1	explanation of one correct step SC … -1 for reverse
3	**(iii)**			M1 A1 A1	complete inc. loops 5-5, 5-2 and 5-3 1-4 and 4-4 SC … -1 only here if no loops if already penalised in (i)

Question			Answer	Marks	Guidance
3	**(iv)**		Time Matrix Route Matrix 	B1 B1 B1√	cao for times 3 correct changes to old routes all new routes correct
3	**(v)**	**(A)**	Route (fastest times) = **1 3 2 4 5 1** Time = 3+2+5+2+4 = 16	B1 B1	
		(B)	**3 2 1 5 4 3** has actual route **3 2 (3) 1 5 4 (2) 3**	B1	
3	**(vi)**		 Kruskal ... **3**-**2** then **3**-**1** then **2**-**4** ... length = 10 So lower bound = 10 + 2 + 4 = 16	M1 A1 B1√	Kruskal

Time Matrix

	1	**2**	**3**	**4**	**5**
1	6	5	3	6	4
2	5	4	2	5	7
3	3	2	4	7	7
4	6	5	7	4	2
5	4	7	7	2	4

Route Matrix

	1	**2**	**3**	**4**	**5**
1	3	3	3	5	5
2	3	3	3	4	4
3	1	2	2	2	1
4	5	2	2	5	5
5	1	4	1	4	4

Question	Answer	Marks	Guidance
3 **(vii)**	costs of pairings ... (**1-2**, **3-4**) = 5+7 = 12 (**1-3**, **2-4**) = 3+5 = 8 (**1-4**, **2-3**) = 6+2 = 8 So quickest time is 31+8 = 39. There are several routes, e.g. **5 1 3 1 2 4 2 3 4 5**	B1 B1 B1	correct costs 39 **1-3** repeated (or **1-4**) **2-4** repeated (or **3-2**)

Question	Answer	Marks	Guidance
4 **(i)(ii)**		M1	A v W first
		A1	drawn correctly
		M1	V v LB second
		A1	drawn correctly
		M1	train delay v ~ delay third
		A1	drawn correctly
		M1	tube delay v ~ delay third
		A1	drawn correctly
		M1	16 end values
		A1	cao
		M1	8 2nd level chance computations
		A1	cao
		M1	4 1st level chance computations
		A1	cao
		M1	3 correct decision values √
		A1	

Tree diagram:

- 114.4 (root)
  - **A** → 127.9
    - **V** 127.9
      - 0.95 ~delay → 127.4
        - ~d 0.8 → 20+100+7 = 127
        - d 0.2 → 20+100+7+2 = 129
      - d 0.05 → 137.4
        - ~d 0.8 → 20+110+7 = 137
        - d 0.2 → 20+110+7+2 = 139
    - **LB** 134.7
      - 0.95 ~d → 134.2
        - ~d 0.9 → 20+105+9 = 134
        - d 0.1 → 20+105+11 = 136
      - d 0.05 → 144.2
        - ~d 0.9 → 20+115+9 = 144
        - d 0.1 → 20+115+11 = 146
  - **W** → 114.4
    - **V** 114.4
      - 0.90 ~d → 112.4
        - ~d 0.8 → 15+90+7 = 112
        - d 0.2 → 15+90+9 = 114
      - d 0.10 → 132.4
        - ~d 0.8 → 15+110+7 = 132
        - d 0.2 → 15+110+9 = 134
    - **LB** 121.2
      - 0.90 ~d → 119.2
        - ~d 0.9 → 15+95+9 = 119
        - d 0.1 → 15+95+11 = 121
      - d 0.10 → 139.2
        - ~d 0.9 → 15+115+9 = 139
        - d 0.1 → 15+115+11 = 141

114.4 minutes via W and V

Question			Answer	Marks	Guidance
4	**(iii)**		3.2 mins (Expected time via W and V + 10 – Expected time via W and LB)	M1 M1 A1√	(114.4+10) −121.2 3.2
4	**(iv)**		The timetable ... when a train leaves and when it arrives … or cost	B1	