

Friday 22 June 2012 – Afternoon**A2 GCE MATHEMATICS (MEI)**

4772 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4772
- MEI Examination Formulae and Tables (MF2)

Duration: 1 hour 30 minutes**Other materials required:**

- Scientific or graphical calculator

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 (a) When marking coursework, a teacher has to complete a form which includes the following:

In your opinion is this the original work of the pupil? (tick as appropriate)

I have no reason to believe that it is not

I cannot confirm that it is

- (i) The teacher suspects that a pupil has copied work from the internet. For each box, state whether the teacher should tick the box or not. [2]

- (ii) The teacher has no suspicions about the work of another pupil, and has no information about how the work was produced. Which boxes should she tick? [2]

- (iii) Explain why the teacher must always tick at least one box. [2]

- (b) Angus, the ski instructor, says that the class will have to have lunch in Italy tomorrow if it is foggy or if the top ski lift is not working. On the next morning Chloe, one of Angus's students, says that it is not foggy, so they can have lunch in Switzerland.

Produce a line of a truth table which shows that Chloe's deduction is incorrect. You may produce a complete truth table if you wish, but you must indicate a row which shows that Chloe's deduction is incorrect. [6]

- (c) You are given that the following two statements are true.

$$\begin{aligned}(X \vee \neg Y) &\Rightarrow Z \\ \neg Z\end{aligned}$$

Use Boolean algebra to show that Y is true.

[4]

- 2 Adrian is considering selling his house and renting a flat.

Adrian still owes £150 000 on his house. He has a mortgage for this, for which he has to pay £4800 annual interest. If he sells he will pay off the £150 000 and invest the remainder of the proceeds at an interest rate of 2.5% per annum. He will use the interest to help to pay his rent.

His estate agent estimates that there is a 30% chance that the house will sell for £225 000, a 50% chance that it will sell for £250 000, and a 20% chance that it will sell for £275 000.

A flat will cost him £7500 per annum to rent.

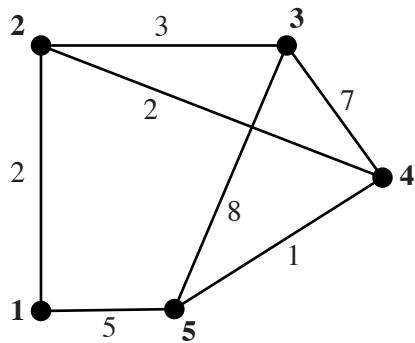
- (i) Draw a decision tree to help Adrian to decide whether to keep his house, or to sell it and rent a flat. Compare the EMVs of Adrian's annual outgoings, and ignore the costs of selling. [6]
- (ii) Would the analysis point to a different course of action if Adrian were to use a square root utility function, instead of EMVs? [3]

Adrian's circumstances change so that he has to decide now whether to sell or not in one year's time. Economic conditions might then be less favourable for the housing market, the same, or more favourable, these occurring with probabilities 0.3, 0.3 and 0.4 respectively. The possible selling prices and their probabilities are shown in the table.

Economic conditions and probabilities		Selling prices (£) and probabilities					
less favourable	0.3	200 000	0.2	225 000	0.3	250 000	0.5
unchanged	0.3	225 000	0.3	250 000	0.5	275 000	0.2
more favourable	0.4	250 000	0.3	300 000	0.5	350 000	0.2

- (iii) Draw a decision tree to help Adrian to decide what to do. Compare the EMVs of Adrian's annual outgoings. Assume that he will still owe £150 000 in one year's time, and that the cost of renting and interest rates do not change. [7]

- 3 The weights on the network represent distances.



- (i) The answer book shows the initial tables and the results of iterations 1, 2, 3 and 5 when Floyd's algorithm is applied to the network.
- (A) Complete the two tables for iteration 4. [6]
- (B) Use the final route table to give the shortest route from vertex 3 to vertex 5. [1]
- (C) Use the final distance table to produce a complete network with weights representing the shortest distances between vertices. [2]
- (ii) Using the complete network of shortest distances, find a lower bound for the solution to the Travelling Salesperson Problem by deleting vertex 5 and its arcs, and by finding the length of a minimum connector for the remainder. (You may find the minimum connector by inspection.) [3]
- (iii) Use the nearest neighbour algorithm, starting at vertex 1, to produce a Hamilton cycle in the complete network. Give the length of your cycle. [3]
- (iv) Interpret your Hamilton cycle in part (iii) in terms of the original network. [2]
- (v) Give a walk of minimum length which traverses every arc on the original network at least once, and which returns to the start. Give the length of your walk. [3]

- 4 A publisher is considering producing three books over the next week: a mathematics book, a novel and a biography. The mathematics book will sell at £10 and costs £4 to produce. The novel will sell at £5 and costs £2 to produce. The biography will sell at £12 and costs £5 to produce. The publisher wants to maximise profit, and is confident that all books will be sold.

There are constraints on production. Each copy of the mathematics book needs 2 minutes of printing time, 1 minute of packing time, and 300 cm^3 of temporary storage space.

Each copy of the novel needs 1.5 minutes of printing time, 0.5 minutes of packing time, and 200 cm^3 of temporary storage space.

Each copy of the biography needs 2.5 minutes of printing time, 1.5 minutes of packing time, and 400 cm^3 of temporary storage space.

There are 10 000 minutes of printing time available on several printing presses, 7500 minutes of packing time, and 2 m^3 of temporary storage space.

- (i) Explain how the following initial feasible tableau models this problem.

[5]

P	x	y	z	s_1	s_2	s_3	RHS
1	-6	-3	-7	0	0	0	0
0	2	1.5	2.5	1	0	0	10 000
0	1	0.5	1.5	0	1	0	7500
0	300	200	400	0	0	1	2 000 000

- (ii) Use the simplex algorithm to solve your LP, and interpret your solution.

[8]

- (iii) The optimal solution involves producing just one of the three books. By how much would the price of each of the other books have to be increased to make them worth producing?

[2]

There is a marketing requirement to provide at least 1000 copies of the novel.

- (iv) Show how to incorporate this constraint into the initial tableau ready for an application of the two-stage simplex method.

Briefly describe how to use the modified tableau to solve the problem. You are NOT required to perform the iterations.

[5]

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Question			Answer	Marks	Guidance
1	(a)	(i)	She should not tick the first box. She should tick the second box.	B1 B1 [2]	
1	(a)	(ii)	She should tick both boxes.	B1B1 [2]	
1	(a)	(iii)	eg To tick neither box would be contradictory, confirming that it is original, but having reason to believe that it is not.	M1 A1 [2]	looking at “neither” case, or equivalent.
1	(b)		eg I – lunch in Italy F – foggy T – top lift not working ((F \vee T) \Rightarrow I) \Leftrightarrow (\sim F \Rightarrow \sim I) (ignore \sim F \Rightarrow I if included) 0 1 1 1 1 0 10 0 01	M1 A1 A1 A1 A1 A1 [6]	identification of propositions Angus’s statement Chloe’s statement equivalence 0/1s for Angus and Chloe 0 for equivalence SC B1 for examining not foggy and lift not working
1	(c)		(X \vee \sim Y) \Rightarrow Z \sim Z \Rightarrow \sim (X \vee \sim Y) contrapositive \sim Z \Rightarrow \sim X \wedge Y De Morgan \sim Z given \sim X \wedge Y Y	M1A1 B1 B1 [4]	deducing Y from \sim X \wedge Y

Question		Answer	Marks	Guidance
2	(i)	<pre> graph LR D[4800] -- stay --> C1{4800} D -- sell --> C2((5062.5)) C2 -- "0.3" --> O1[5625] C2 -- "0.5" --> O2[5000] C2 -- "0.2" --> O3[4375] </pre>	B1 M1 A1 M1 A1 B1	decision node chance node 3-split rent costs (a correct value) -1 each error decision
2	(ii)	$0.3 \times \sqrt{5625} + 0.5 \times \sqrt{5000} + 0.2 \times \sqrt{4375} = 71.08$ $\sqrt{4800} = 69.28, \text{ so no change}$	[6] M1A1 A1 [3]	

Question	Answer	Marks	Guidance
(iii)	<pre> graph LR A[4700] -- stay --> B(()) A -- sell --> C(()) A -- same --> D(()) C -- less 0.3 --> E((5437.5)) C -- more 0.4 --> F((3875)) C -- same 0.3 --> G((5062.5)) E -- 0.2 --> H[6250] E -- 0.3 --> I[5625] E -- 0.5 --> J[5000] G -- 0.3 --> K[5625] G -- 0.5 --> L[5000] G -- 0.2 --> M[4375] F -- 0.3 --> N[5000] F -- 0.5 --> O[3750] F -- 0.2 --> P[2500] </pre> <p>The decision tree starts at node 4700. If 'stay', it leads to a chance node with outcome 4800. If 'sell', it branches into 'less' (0.3 probability) leading to node 5437.5, 'more' (0.4 probability) leading to node 3875, and 'same' (0.3 probability) leading to node 5062.5. Node 5437.5 further branches into three outcomes: 6250 (0.2 probability), 5625 (0.3 probability), and 5000 (0.5 probability). Node 3875 branches into three outcomes: 5000 (0.3 probability), 3750 (0.5 probability), and 2500 (0.2 probability). Node 5062.5 branches into three outcomes: 5625 (0.3 probability), 5000 (0.5 probability), and 4375 (0.2 probability).</p>	M1 A1 M1 A1 M1 A1 B1	new chance node 3-split “less” (a correct value) “more” (a correct value) 4700 (follow through)

Question			Answer	Marks	Guidance																																																																								
3	(i)	(a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>1</td><td>4</td><td>2</td><td>5</td><td>4</td><td>5</td><td></td><td>1</td><td>2</td><td>2</td><td>2</td><td>5</td></tr> <tr><td>2</td><td>2</td><td>4</td><td>3</td><td>2</td><td>3</td><td></td><td>2</td><td>1</td><td>1</td><td>3</td><td>4</td></tr> <tr><td>3</td><td>5</td><td>3</td><td>6</td><td>5</td><td>6</td><td></td><td>3</td><td>2</td><td>2</td><td>2</td><td>2</td></tr> <tr><td>4</td><td>4</td><td>2</td><td>5</td><td>4</td><td>1</td><td></td><td>4</td><td>2</td><td>2</td><td>2</td><td>5</td></tr> <tr><td>5</td><td>5</td><td>3</td><td>6</td><td>1</td><td>2</td><td></td><td>5</td><td>1</td><td>4</td><td>4</td><td>4</td></tr> </table>		1	2	3	4	5		1	2	3	4	5	1	4	2	5	4	5		1	2	2	2	5	2	2	4	3	2	3		2	1	1	3	4	3	5	3	6	5	6		3	2	2	2	2	4	4	2	5	4	1		4	2	2	2	5	5	5	3	6	1	2		5	1	4	4	4	M1 A2 M1 A2 [6]	(-1 each error) (-1 each error)
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3	(i)	(b)	$3 \rightarrow 2 \rightarrow 4 \rightarrow 5$	B1 [1]																																																																									
3	(i)	(c)		M1 A1 [2]	complete																																																																								

Question		Answer	Marks	Guidance
3	(ii)	<p>Lower bound = $(2 + 2 + 3) + (1 + 3) = 11$</p>	M1 A1 A1 [3]	delete vertex 5 plus arcs $(2 + 2 + 3)$ $1 + 3$
3	(iii)	$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 3 \rightarrow 1$ of total length 16	M1A1B1 [3]	M1 for $1 \rightarrow 2 \rightarrow 4 \rightarrow 5$
3	(iv)	$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow (4 \rightarrow 2) \rightarrow 3 \rightarrow (2) \rightarrow 1$	M1A1 [2]	SC 1 id seen elsewhere
3	(v)	eg $1 \rightarrow 2 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 5 \rightarrow 4 \rightarrow 5 \rightarrow 1$ Length = 32	M1A1 B1 [3]	2 \rightarrow 3 or 5 \rightarrow 4 repeated for M1
4	(i)	Let x be the number of maths books produced ... Line 1 $\Leftrightarrow \max 6x + 3y + 7z$ (10 - 4 = 6 etc.) Line 2 $\Leftrightarrow 2x + 1.5y + 2.5z \leq 10000$ (printing time) Line 3 $\Leftrightarrow x + 0.5y + 1.5z \leq 7500$ (packing time) Line 4 $\Leftrightarrow 300x + 200y + 400z \leq 2000000$ (storage space)	B1 B1 B1 B1 B1 [5]	variable defs. "number of" objective constraints

Question		Answer								Marks	Guidance	
4	(ii)											
		P	x	y	z	s1	s2	s3	RHS			
		1	-6	-3	-7	0	0	0	0	B1	pivot (pivot on x OK)	
		0	2	1.5	2.5	1	0	0	10000	M1	a correct row or column	
		0	1	0.5	1.5	0	1	0	7500	A1		
		0	300	200	400	0	0	1	2000000	B1	pivot	
		1	-0.4	1.2	0	2.8	0	0	28000	M1	a correct row or column	
		0	0.8	0.6	1	0.4	0	0	4000	A1		
		0	-0.2	-0.4	0	-0.6	1	0	1500	B1		
		0	-20	-40	0	-160	0	1	400000	M1	a correct row or column	
		1	0	1.5	0.5	3	0	0	30000	A1		
		0	1	0.75	1.25	0.5	0	0	5000	B1		
		0	0	-0.25	0.25	-0.5	1	0	2500			
		0	0	-25	25	-150	0	1	500000	[8]		
		Produce 5000 maths books – at a profit of £30000. (2500 packing minutes spare and 0.5 m ³ storage space spare.)										
4	(iii)	£1.50 and 50p respectively.								B1B1 [2]		

GCE Mathematics (MEI)											
			Max Mark	90% cp	a	b	c	d	e	u	
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper		Raw 100	72 18 18 100	66 16 16 90	60 15 15 80	53 13 13 70	47 11 11 60	41 9 9 50	34 8 8 40	0 0 0 0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework		Raw UMS	90 100	73 90	65 80	57 70	50 60	43 50	36 40	0 0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark		Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework		UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics		Raw UMS	90 100	73 90	65 80	57 70	50 60	43 50	36 40	0 0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics		Raw UMS	72 100	66 90	61 80	53 70	46 60	39 50	32 40	0 0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics		Raw UMS	72 100	61 90	54 80	47 70	40 60	34 50	28 40	0 0

4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw 18	72	68	63	57	51	45	39	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw 18	16	15	13	11	9	8	0	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw 100	18	16	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS 100	90	80	70	60	50	40	0	0
4762/01 (M2) MEI Mechanics 2	Raw 100	72	65	58	51	44	38	32	0
	UMS 100	90	80	70	60	50	40	0	0
4763/01 (M3) MEI Mechanics 3	Raw 100	72	67	63	56	50	44	38	0
	UMS 100	90	80	70	60	50	40	0	0
4764/01 (M4) MEI Mechanics 4	Raw 100	72	63	56	49	42	35	29	0
	UMS 100	90	80	70	60	50	40	0	0
4767/01 (S2) MEI Statistics 2	Raw 100	72	66	61	55	49	43	38	0
	UMS 100	90	80	70	60	50	40	0	0
4768/01 (S3) MEI Statistics 3	Raw 100	72	65	58	51	44	38	32	0
	UMS 100	90	80	70	60	50	40	0	0
4769/01 (S4) MEI Statistics 4	Raw 100	72	63	56	49	42	35	28	0
	UMS 100	90	80	70	60	50	40	0	0
4772/01 (D2) MEI Decision Mathematics 2	Raw 100	72	62	56	50	44	39	34	0
	UMS 100	90	80	70	60	50	40	0	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw 100	72	52	46	40	34	29	24	0
	UMS 100	90	80	70	60	50	40	0	0
4777/01 (NC) MEI Numerical Computation	Raw 100	72	63	55	47	39	32	25	0
	UMS 100	90	80	70	60	50	40	0	0