OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4772

Decision Mathematics 2

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4772
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 23 June 2011
Morning

Duration: 1 hour 30 minutes

OCE/18045

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

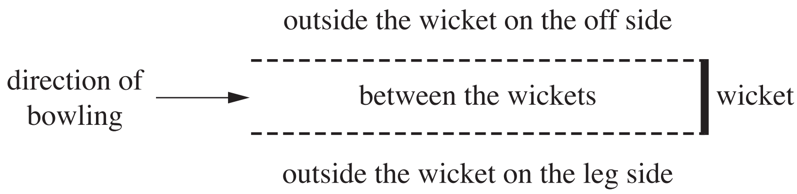
Turn over

1 **(a)** Heard in Parliament: “Will the minister not now discontinue her proposal to ban the protest?”

The minister replied “Yes I will.”

To what had the minister committed herself logically, and why might that not have been her intention? **[4]**

(b) In a cricket tournament an umpire might be required to decide whether or not a batsman is out ‘lbw’, ie ‘leg before wicket’. The lbw law for the tournament refers to parts of the cricket pitch as shown in the diagram (assuming a right-handed batsman):



The umpire has to make a number of judgements:

A Would the ball have hit the wicket?

B Did the ball hit the batsman, or part of his equipment other than the bat, without hitting the bat?

C Did the ball hit the batsman, or part of his equipment other than the bat, before hitting the bat?

D Was the part of the batsman or his equipment which was hit by the ball, between the wickets when it was hit?

E Was the part of the batsman or his equipment which was hit by the ball, outside of the wicket on the off side when it was hit?

F Was the batsman attempting to play a stroke?

The law can be interpreted as saying that the batsman is out lbw if $[(A \wedge B) \vee (A \wedge C)] \wedge [D \vee (E \wedge {\sim}F)]$.

The tournament’s umpiring manual, in attempting to simplify the law, states that the batsman is out lbw if $A \wedge (B \vee C) \wedge (D \vee E) \wedge (D \vee {\sim}F)$.

For an lbw decision this requires 4 conditions each to be true.

(i) Use the rules of Boolean algebra to show that the manual’s rule is logically equivalent to the law as stated above, naming the rules used at each step. **[7]**

A trainee umpire, using the manual, considers each condition in turn and judges that the following are true: A; B; E; D.

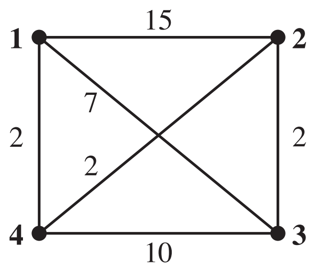
(ii) What is her decision and why? **[2]**

(iii) What is odd about her judgement, and does this make the logic invalid? **[3]**

2 A government has just created a new ministry, the Ministry of Administrative Affairs. The ministry is to have four departments:

the Administration
the Bureaucracy
the Certification Service
the Duplication Section.

Each of these departments is to be established in a separate office on one of four existing sites. The diagram shows the direct journey times in minutes between these four sites.



(i) Use Floyd's algorithm to find the shortest journey times between the four office sites. **[8]**

(ii) Draw a network showing your shortest times. **[1]**

(iii) Use appropriate algorithms to find upper and lower bounds for the optimum solution to the Travelling Salesperson Problem in the original network, briefly explaining the steps taken. **[4]**

(iv) A van is to be organised to deliver bundles of paperwork between the departments. Why might the optimum solution to the TSP be useful in planning this, and why might it not be? **[2]**

(v) Journeys to locations **2** and **3**, from locations **1** and **4**, are subject to a congestion charge which is equivalent in costing terms to 15 minutes of journey time. What sort of network would be needed to model this? **[1]**

3 Magnus has been researching career possibilities. He has just completed his GCSEs, and could leave school and get a good job. He estimates, discounted at today's values and given a 49 year working life, that there is a 50% chance of such a job giving him lifetime earnings of £1.5m, a 30% chance of £1.75m, and a 20% chance of £2m.

Alternatively Magnus can stay on at school and take A levels. He estimates that, if he does so, there is a 75% chance that he will achieve good results. If he does not achieve good results then he will still be able to take the same job as earlier, but he will have lost two years of his lifetime earnings. This will give a 50% chance of lifetime earnings of £1.42m, a 30% chance of £1.67m and a 20% chance of £1.92m.

If Magnus achieves good A level results then he could take a better job, which should give him discounted lifetime earnings of £1.6m with 50% probability or £2m with 50% probability. Alternatively he could go to university. This would cost Magnus another 3 years of lifetime earnings and would not guarantee him a well-paid career, since graduates sometimes choose to follow less well-paid vocations. His research shows him that graduates can expect discounted lifetime earnings of £1m with 20% probability, £1.5m with 30% probability, £2m with 30% probability, and £3m with 20% probability.

(i) Draw up a decision tree showing Magnus's options. **[7]**

(ii) Using the EMV criterion, find Magnus's best course of action, and give its value. **[7]**

Magnus has read that money isn't everything, and that one way to reflect this is to use a utility function and then compare expected utilities. He decides to investigate the outcome of using a function in which utility is defined to be the square root of value.

(iii) Using the expected utility criterion, find Magnus's best course of action, and give its utility. **[4]**

(iv) The possibility of high earnings (£3m) swings Magnus's decision towards a university education. Find what value instead of £3m would make him indifferent to choosing a university education under the EMV criterion. (Do not change the probabilities.) **[2]**

4 A small alpine hotel is planned. Permission has been obtained for no more than 60 beds, and these can be accommodated in rooms containing one, two or four beds.

The total floor areas needed are $15\,\text{m}^2$ for a one-bed room, $25\,\text{m}^2$ for a two-bed room and $40\,\text{m}^2$ for a four-bed room. The total floor area of the bedrooms must not exceed $700\,\text{m}^2$.

Marginal profit contributions per annum, in thousands of euros, are estimated to be 5 for a one-bed room, 9 for a two-bed room and 15 for a four-bed room.

(i) Formulate a linear programming problem to find the mix of rooms which will maximise the profit contribution within the two constraints. **[3]**

(ii) Use the simplex algorithm to solve the problem, and interpret your solution. **[8]**

It is decided that, for marketing reasons, at least 5 one-bed rooms must be provided.

(iii) Solve this modified problem using either the two-stage simplex method or the big-M method. You may wish to adapt your final tableau from part **(ii)** to produce an initial tableau, but you are not required to do so. **[5]**

(iv) The simplex solution to the revised problem is to provide 5 one-bed rooms, 15 two-bed rooms and 6.25 four-bed rooms, giving a profit contribution of €253 750. Interpret this solution in terms of the real world problem. **[1]**

(v) Compare the following solution to your answer to part **(iv)**: 8 one-bed rooms, 12 two-bed rooms and 7 four-bed rooms. Explain your findings. **[3]**

2(i)

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

THERE ARE SPARE COPIES OF THESE TABLES ON PAGE 12

2(ii)

4(i)

4(ii)

THERE IS A SPARE COPY OF THIS GRID ON PAGE 11

4772, June 2011, Markscheme

1.

(a) To not discontinue, i.e. to continue. "Will the minister not …" is a form of words in which the negation is not intended.	B1 to continue B1 double negation B1 understanding ~~ B1 language confusion	
(b)(i) $[(A\wedge B) \vee (A\wedge C)] \wedge [D \vee (E\wedge \sim F)]$ $\Leftrightarrow[(A\vee(A\wedge C)) \wedge (B\vee(A\wedge C))] \wedge [(D\vee E) \wedge (D\vee \sim F)]$ $\Leftrightarrow(A\vee(A\wedge C)) \wedge (B\vee(A\wedge C)) \wedge [(D\vee E) \wedge (D\vee \sim F)]$ $\Leftrightarrow A \wedge [(B\vee A) \wedge (B\vee C)] \wedge (D\vee E) \wedge (D\vee \sim F)$ $\Leftrightarrow A \wedge (B\vee A) \wedge (B\vee C) \wedge (D\vee E) \wedge (D\vee \sim F)$ $\Leftrightarrow A \wedge (B\vee C) \wedge (D\vee E) \wedge (D\vee \sim F)$	M1 uses distributive rule A1 distributive rule A1 "distributive rule" M1 uses associative rule A1 associative rule A1 "associative rule" B1 rest correct	 ft ft SC if refers to "absorption" then +1
or $A \wedge (B\vee C) \wedge (D\vee E) \wedge (D\vee \sim F)$ $\Leftrightarrow[A \wedge (B\vee C)] \wedge (D\vee E) \wedge (D\vee \sim F)$ $\Leftrightarrow[A \wedge (B\vee C)] \wedge [(D\vee E) \wedge (D\vee \sim F)]$ $\Leftrightarrow[(A\wedge B) \vee (A\wedge C)] \wedge [(D\vee E) \wedge (D\vee \sim F)]$ $\Leftrightarrow[(A\wedge B) \vee (A\wedge C)] \wedge [D \vee (E\wedge \sim F)]$	M1 uses distributive rule A1 distributive rule A1 "distributive rule" M1 uses associative rule A1 associative rule A1 "associative rule" B1 rest correct	 ft ft
(ii) Out, LBW! Either first square bracket and second square bracket, or all 4 conditions are satisfied	B1 "out" B1 using either test	
(iii) Can't have D and E both true at the same time. Logic still valid. Logic not concerned with consistency of input, only whether out or not.	B1 B1 B1	

2.

(i)

	1	**2**	**3**	**4**
1	∞	15	7	2
2	15	∞	2	2
3	7	2	∞	10
4	2	2	10	∞

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	2	3	4
3	1	2	3	4
4	1	2	3	4

B1 time matrix
B1 route matrix

	1	**2**	**3**	**4**
1	∞	15	7	2
2	15	30	2	2
3	7	2	14	9
4	2	2	9	4

	1	**2**	**3**	**4**
1	1	2	3	4
2	1	1	3	4
3	1	2	1	1
4	1	2	1	1

M1 replacing an ∞ by a correct value
A1

	1	**2**	**3**	**4**
1	30	15	7	2
2	15	30	2	2
3	7	2	4	4
4	2	2	4	4

	1	**2**	**3**	**4**
1	2	2	3	4
2	1	1	3	4
3	1	2	2	2
4	1	2	2	1

A1 ft

	1	**2**	**3**	**4**
1	14	9	7	2
2	9	4	2	2
3	7	2	4	4
4	2	2	4	4

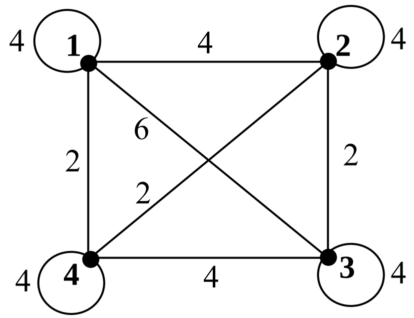
	1	**2**	**3**	**4**
1	3	3	3	4
2	3	3	3	4
3	1	2	2	2
4	1	2	2	1

A1 ft

	1	**2**	**3**	**4**
1	4	4	6	2
2	4	4	2	2
3	6	2	4	4
4	2	2	4	4

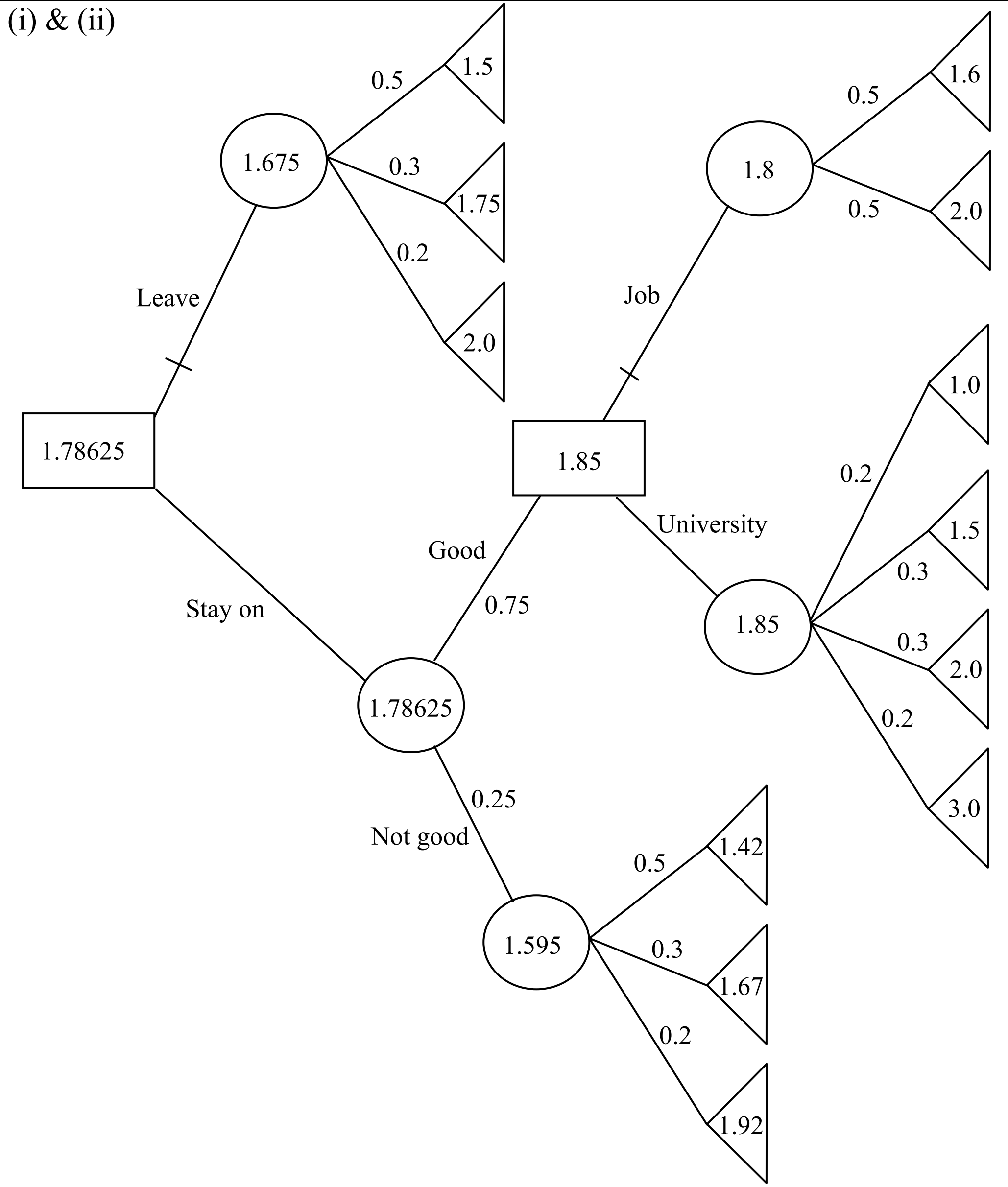
	1	**2**	**3**	**4**
1	4	4	4	4
2	4	3	3	4
3	2	2	2	2
4	1	2	2	1

A1 entries other than row 3 col 1 of route matrix ... ft
B1 row 3 col 1 of route matrix ... cao

(ii) 	B1 ft	
(iii) Upper – nearest neighbour – e.g. 2+2+2+6 = 12	M1 nearest neighbour A1	mention of nearest neighbour or a nearest neighbour computation allow 2+2+2+7=13 etc for working in original network
Lower – e.g. “delete” **1**, and compute (2+2)+2+4 = 10	M1 delete a vertex A1 rest of computation	needs to be consistent with above
(iv) e.g. if the requirement is for part loads, and deliver to one department en route to another, then might save time. e.g. if the requirement is for part whole loads then might not be relevant.	B1 B1	answer should be valid and refer to the specific situation of the DAA
(v) A directed network.	B1	

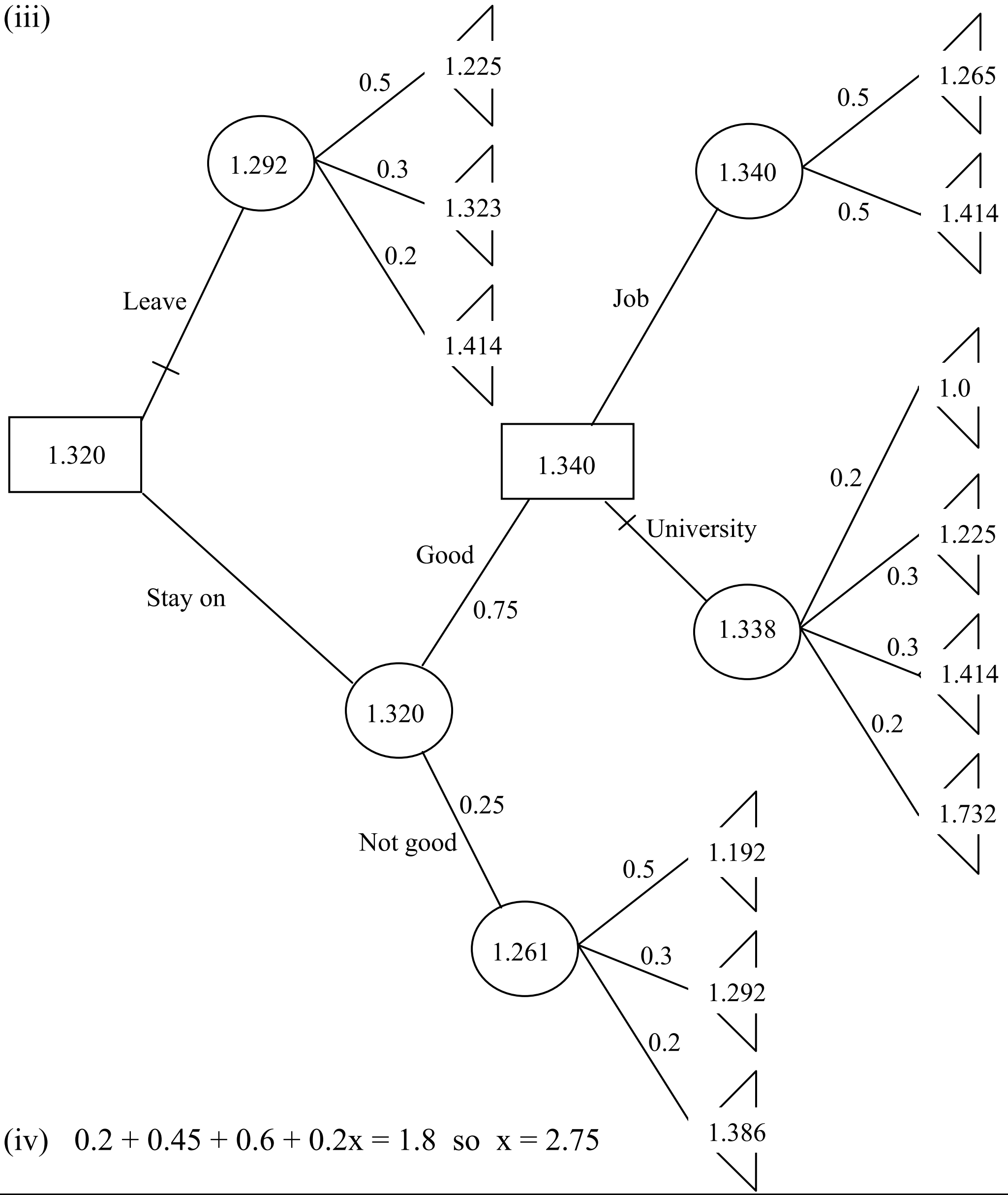
3.

(i) & (ii)



B1	stay on/leave	decision node
B1	leave	chance node with 3 branches
B1	good A/not good	chance node
B1	not good	chance node with 3 branches
B1	job/uni	decision node
B1	job	chance node with 2 branches
B1	uni	chance node with 4 branches
B1	leave computation	cao
B1	job computation	cao
B1	uni computation	cao
B1	good comp	ft
B1	not good comp	cao
B1	good/not good	ft
B1	stay on for 1.78625	cao

3 (cont)

<table>
<tr><td>(iii)
</td><td>M1 utilities of outcomes
A1 cao

M1 computing backwards
A1 ft</td><td></td></tr>
<tr><td>(iv) $0.2 + 0.45 + 0.6 + 0.2x = 1.8$ so $x = 2.75$</td><td>M1 A1 cao</td><td>equation with 0.2 x or division by 0.2</td></tr>
</table>

4.

<table>
<tr><td>(i) Definition of variables
Max 5x + 9y + 15z
st x + 2y + 4z ≤ 60
15x + 25y + 40z ≤ 700</td><td>B1
B1 objective

B1 constraints</td><td>needs to say “number of”</td></tr>
<tr><td>(ii)

P	x	y	z	s1	s2	RHS
1	–5	–9	–15	0	0	0
0	1	2	4	1	0	60
0	15	25	40	0	1	700
1	–5/4	–3/2	0	15/4	0	225
0	1/4	1/2	1	1/4	0	15
0	5	5	0	–10	1	100
1	1/4	0	0	3/4	3/10	255
0	–1/4	0	1	5/4	–1/10	5
0	–1	1	0	–2	1/5	20

Identification of basic variables (y and z)
+ values (inc objective)</td><td>M1 initial tableau
A1 ft

M1 first iteration
A1 ft

M1 second iteration
A1 ft

B1 ft
B1 ft</td><td>two slack variables

identifying correct pivot

identifying correct pivot</td></tr>
</table>

(iii)

A	P	x	y	z	s1	s2	s3	a	RHS
1	0	1	0	0	0	0	–1	0	5
0	1	1/4	0	0	3/4	3/10	0	0	255
0	0	–1/4	0	1	5/4	–1/10	0	0	5
0	0	1	1	0	–2	1/5	0	0	20
0	0	1	0	0	0	0	–1	1	5
1	0	0	0	0	0	0	0	–1	0
0	1	0	0	0	3/4	3/10	1/4	–1/4	253.75
0	0	0	0	1	5/4	–1/10	–1/4	1/4	6.25
0	0	0	1	0	–2	1/5	1	–1	15
0	0	1	0	0	0	0	–1	1	5

B1 $\geq$ row
B1 new objective

M1 pivot
A1 objectives cao
A1 constraints cao for basic variables

If from scratch, then M1 for first pivot, A1 for final objective row(s) and A1 for final constraint rows.

or

or (same scheme)

P	x	y	z	s1	s2	s3	RHS
1	–M+1/4	0	0	3/4	3/10	M	255–5M
0	–1/4	0	1	5/4	–1/10	0	5
0	1	1	0	–2	1/5	0	20
0	1	0	0	0	0	–1	5
1	0	0	0	3/4	3/10	1/4	253.75
0	0	0	1	5/4	–1/10	–1/4	6.25
0	0	1	0	–2	1/5	1	15
0	1	0	0	0	0	–1	5

B1 $\geq$ row
B1 new objective

M1 pivot
A1 objective cao
A1 constraints cao for basic variables

(iv) 5, 15 and 6 at £250000 — B1

(v) 8, 12 and 7 is feasible and gives £253000 — B1 B1
IP solution need not be "near" to LP solution — B1