OCR
Oxford Cambridge and RSA

Wednesday 14 June 2017 – Morning

AS GCE MATHEMATICS (MEI)

4771/01 Decision Mathematics 1

QUESTION PAPER

A 52
B 46
C 41
D 36
E 31

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4771/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

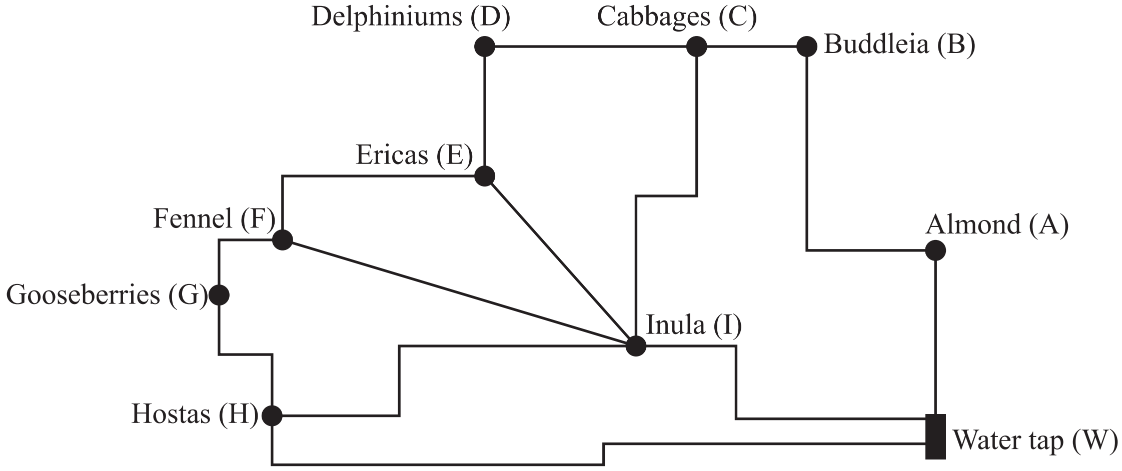
INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 Pippa is planning an irrigation system for a new garden. The graph shows the water tap, the plants and trees which need to be watered, and possible routes for the irrigation pipes.



The lengths of possible pipe runs (m) are given in the table.

	W	A	B	C	D	E	F	G	H	I
W		10							40	20
A	10		17							
B		17		7						
C			7		12					18
D				12		8				
E					8		14			11
F						14		5		15
G							5		6	
H	40							6		18
I	20			18		11	15		18	

(i) Starting at W, use Prim's algorithm in tabular form to find the least length of pipe that is needed for the system. Give the length of pipe and draw the connections that are used in this solution. **[5]**

In fact, Pippa decides to connect the Inula directly to the water tap and then to use all of the direct connections from the Inula. The Almond, Buddleia, Delphiniums and Gooseberries she will subsequently connect in optimally.

(ii) Draw Pippa's connections. Give the minimum length of pipe she will need for this solution, and give a reason why she might choose it. **[3]**

2 The following is an algorithm for 'shop subtraction'. It applies to two whole numbers, each between 0 and 999.

```
10  Let P be the smaller number
20  Let M be the larger number
30  Let C be 0
40  If P + C + 100 > M then goto 70
50  Let C = C + 100
60  Goto 40
70  If P + C + 10 > M then goto 100
80  Let C = C + 10
90  Goto 70
100 If P + C + 1 > M then goto 130
110 Let C = C + 1
120 Goto 100
130 Print 'The answer is' C
```

(i) Apply the algorithm to the numbers 112 and 250. Show the steps as you apply them and give the answer. **[4]**

(ii) Show how to modify the algorithm so that it can be applied to numbers between 0 and 9999. **[3]**

(iii) What is the connection between the algorithm and giving change for a purchase made in a shop? **[1]**

3 $K_{2,4}$ is the complete bipartite graph on sets of 2 and 4 elements, i.e. all possible ways of joining 2 elements in one set to 4 elements in another set.

(i) Draw $K_{2,4}$ with no lines crossing. **[1]**

(ii) Explain how $K_{2,n}$ can be drawn without any lines crossing for any positive integer n. **[1]**

(iii) The smallest value of n for which $K_{3,n}$ cannot be drawn without any lines crossing is $n = 3$. Start from a drawing of $K_{2,3}$ and explain why it is not possible to construct $K_{3,3}$ from this without having any lines crossing. **[3]**

It is claimed that the minimum number of line crossings needed to draw $K_{3,n}$ is given by $\frac{(n-1)^2}{4}$ if n is odd and $\frac{n(n-2)}{4}$ if n is even.

(iv) Draw $K_{3,5}$ with the number of line crossings given by the formula. **[2]**

(v) Explain the consequences of parts (i) to (iv) for circuit board design. **[1]**

Section B (48 marks)

4 A nursery has $9000\,\text{m}^2$ of land available for growing deciduous trees and evergreen trees from saplings (young trees). Each deciduous tree needs $8\,\text{m}^2$ of space, and each evergreen tree needs $6\,\text{m}^2$ of space.

The costs of purchasing saplings, labour, fertilisers, etc. are £16 for each deciduous tree and £16 for each evergreen tree, and £20 000 is available to invest.

The nursery can obtain at most 800 deciduous saplings for planting, and at most 1000 evergreen saplings.

When the saplings have grown into trees, the nursery sells the deciduous trees for £25 each, and the evergreen trees for £22 each.

(i) Calculate the profit which will be made from each deciduous tree and from each evergreen tree. **[1]**

The nursery manager wants to find how many trees of each type should be grown so as to maximise the profit.

(ii) Formulate the manager's problem as a linear program, ignoring the fact that the numbers of trees must be integers. **[5]**

(iii) Draw the feasible region for your problem in part (ii), and hence show that the solution to the LP is 800 deciduous trees and $433\frac{1}{3}$ evergreen trees. Give the maximum profit. **[7]**

(iv) How much extra profit would be made for an extra $100\,\text{m}^2$ of land, still allowing for non-integer solutions?

How much extra profit (compared to your answer to part (iii)) would be made for an extra $1000\,\text{m}^2$ of land, still allowing for non-integer solutions? **[2]**

(v) Saplings have to be purchased in bundles of 50 at a time. For the original problem, with $9000\,\text{m}^2$ of land available, find the optimal number of each type of sapling to purchase. **[1]**

5 John wants to generate integers between 0 and 15, each being equally likely. He has a coin available. He constructs a table with headings as shown, and with as many rows as he needs.

8	**4**	**2**	**1**

…

To complete the first row he throws the coin four times, recording the results as a 1 for 'heads' or a 0 for 'tails' in the four columns of the first row, starting from the right and moving left. For example, if he gets two heads, then a tail and then another head, this will be recorded in the table as 1, 0, 1, 1 (remembering that the order is from the right). He then multiplies the entries in the row by the column headings to get a score. In the example the score is $8 + 2 + 1 = 11$.

So the example has the following result and score:

8	**4**	**2**	**1**	
1	0	1	1	11

…

(i) Explain why this method generates the scores 0 to 15, each with the same probability. **[3]**

John wants to repeat this experiment 64 times. His last 8 coin throws are as follows:

H T T T T H T H

(ii) Compute John's final two scores. **[2]**

John is trying to simulate a fairground game in which 16 jars are arranged in a square formation, and a ball is repeatedly thrown until it lands in one of them.

00	01	02	03
04	05	06	07
08	09	10	11
12	13	14	15

(The jars have been numbered to help in the rest of this question.)

When John produces a score, that score corresponds to the number of a jar.

(iii) Explain why John's simulation will not be a good model of reality. **[1]**

John decides instead to use two-digit random numbers to simulate the game. He decides that he will allocate a probability of $\frac{1}{36}$ for a ball to land in each of the corner jars (jars 00, 03, 12 and 15), a probability of $\frac{2}{36}$ for each of the other eight edge jars (jars 01, 02, 04, 07, 08, 11, 13 and 14), and a probability of $\frac{4}{36}$ for each of the other four jars.

(iv) Give a rule for John to use, i.e. specify which jar should be represented by each two-digit random number. **[8]**

(v) Use your rule together with the string of two-digit random numbers in the answer book to simulate two repetitions of the game. **[2]**

6 Pippa owns a garden construction company. She is preparing a quotation for constructing a garden on a $1000\,\text{m}^2$ site. Her quotation will need to include an estimate of how long the construction will take.

The tasks, together with limitations on their starting times and durations, are as follows:

A Produce a detailed survey of the site. This must be done before anything else, except that clearing the site can be started at the same time. The surveying will take 1 day.

B Clear the site. This will take 3 days.

C Put forward a plan, debate it and adjust it. This can't be done until the survey is complete. It involves meeting with the client, and will take 3 days.

D Build walls and other fixed features. This cannot be started until the site is cleared and the plan agreed. It will take 3 days.

E Have a specialist contractor install the pond. This cannot be started until the site is cleared and the plan agreed. It will take 5 days.

F Plant the trees. This cannot be started until the walls and other fixed features are completed. It will take 3 days.

G Plant the plants. This cannot be started until the walls and other fixed features are completed. It will take 2 days.

H Install the irrigation. This cannot be started until the trees are planted and the plants are planted. It will take 2 days.

(i) Draw an activity-on-arc precedence network for this project. **[4]**

(ii) Complete a forward pass and a backward pass to determine the minimum completion time and the critical activities, using the durations given above. **[6]**

Activities D and E will be contracted to specialist companies. Pippa plans to complete activities A, B, C, F, G and H herself, but she can employ her friend, Afzal, to do or to help with any of these tasks except for A and C. Each of B, F, G and H can be done by two people, with the total time taken by the two of them being the same as the activity duration.

(iii) The project is to be completed in the shortest time. Produce a schedule to achieve this whilst using as little of Afzal's time as possible. Show who does what and when. Give the minimum completion time and give the total time for which Afzal must be employed. **[5]**

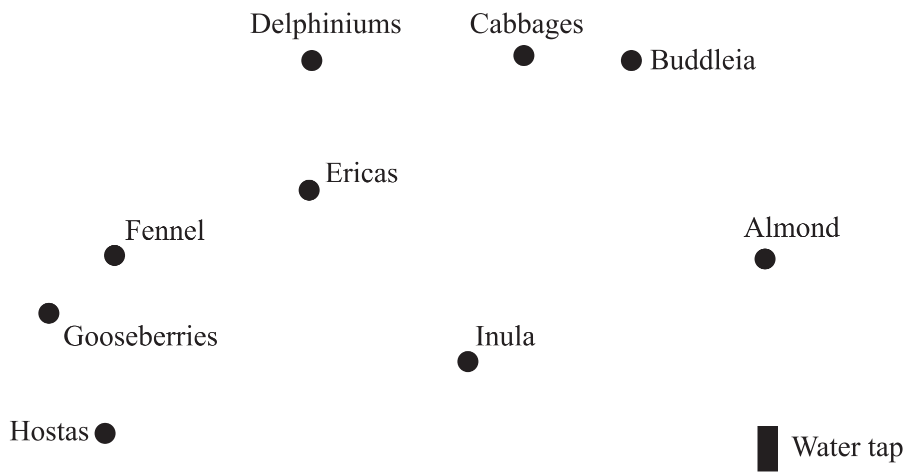
(iv) How long will it take to complete the project if Pippa does not use Afzal at all? **[1]**

END OF QUESTION PAPER

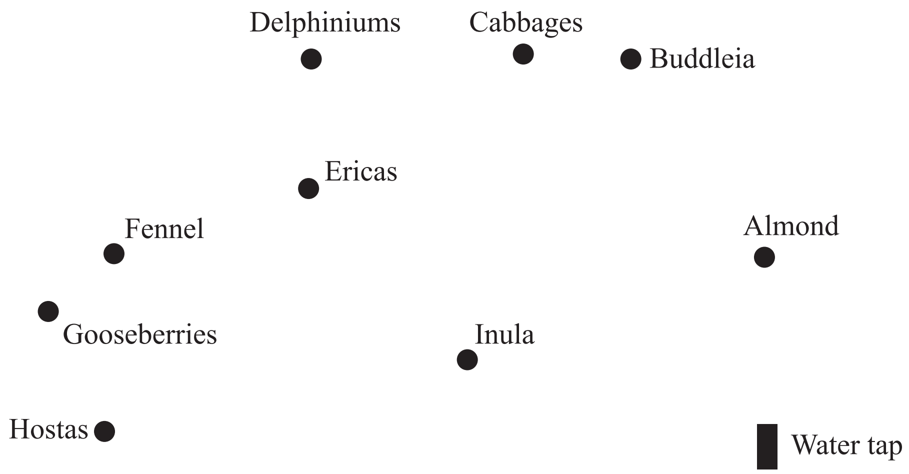
1 (i)

	W	A	B	C	D	E	F	G	H	I
W		10							40	20
A	10		17							
B		17		7						
C			7		12					18
D				12		8				
E					8		14			11
F						14		5		15
G							5		6	
H	40							6		18
I	20			18		11	15		18	

Length of pipe used =



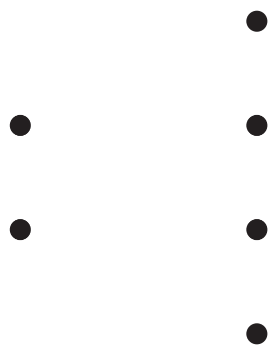
1 (ii)



Minimum length of pipe:

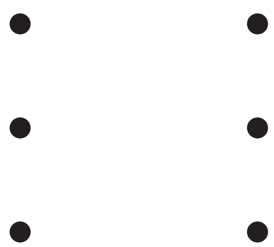
Reason:

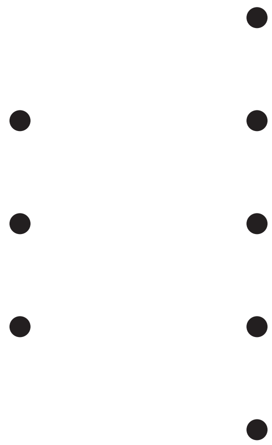
3 (i)



3 (ii)

3 (iii)



3 (iv)	
3 (v)	

4 (i)

4 (ii)

4 (iii) A spare copy of this graph can be found on page 11.

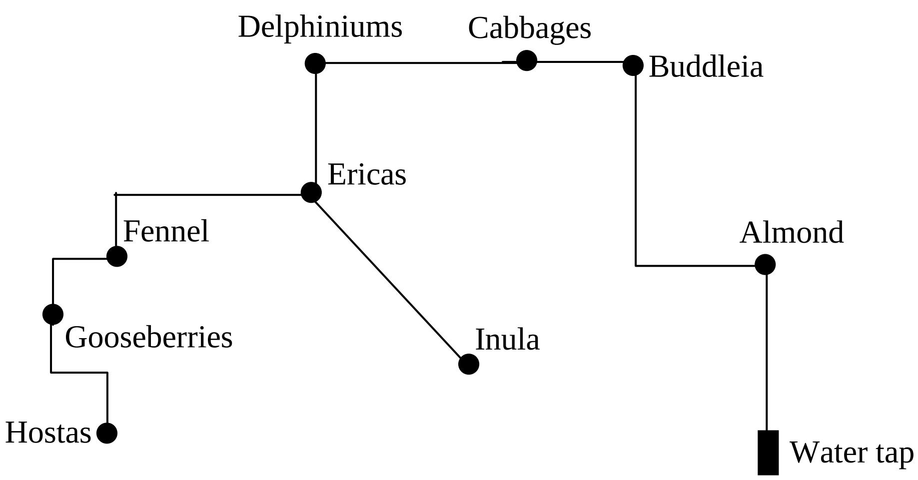
(answer space continued on next page)

5 (v)	Two-digit random numbers: 01 99 52 67 23 62 85

6 (i) & (ii)	
6 (iii)	
6 (iv)	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>1</td><td>(i)</td><td></td><td>
<table>
<tr><th></th><th>W1</th><th>A2</th><th>B3</th><th>C4</th><th>D5</th><th>E6</th><th>F8</th><th>G9</th><th>H</th><th>I7</th></tr>
<tr><td>W</td><td></td><td>10</td><td></td><td></td><td></td><td></td><td></td><td></td><td>40</td><td>20</td></tr>
<tr><td>A</td><td>10</td><td></td><td>17</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>B</td><td></td><td>17</td><td></td><td>7</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>C</td><td></td><td></td><td>7</td><td></td><td>12</td><td></td><td></td><td></td><td></td><td>18</td></tr>
<tr><td>D</td><td></td><td></td><td></td><td>12</td><td></td><td>8</td><td></td><td></td><td></td><td></td></tr>
<tr><td>E</td><td></td><td></td><td></td><td></td><td>8</td><td></td><td>14</td><td></td><td></td><td>11</td></tr>
<tr><td>F</td><td></td><td></td><td></td><td></td><td></td><td>14</td><td></td><td>5</td><td></td><td>15</td></tr>
<tr><td>G</td><td></td><td></td><td></td><td></td><td></td><td></td><td>5</td><td></td><td>6</td><td></td></tr>
<tr><td>H</td><td>40</td><td></td><td></td><td></td><td></td><td></td><td></td><td>6</td><td></td><td>18</td></tr>
<tr><td>I</td><td>20</td><td></td><td></td><td>18</td><td></td><td>11</td><td>15</td><td></td><td>18</td><td></td></tr>
</table>

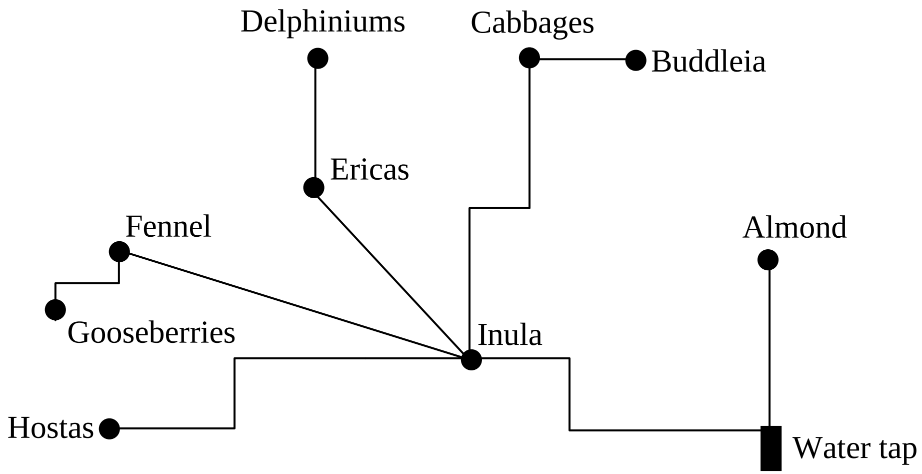
Length of pipe used = 90m



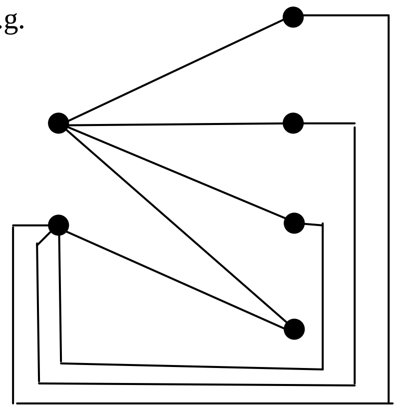
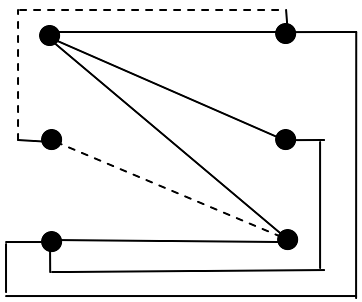
</td><td>B1
B1
B1

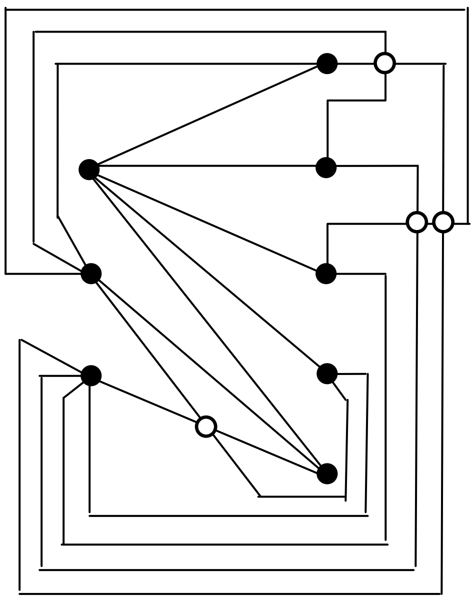
B1cao

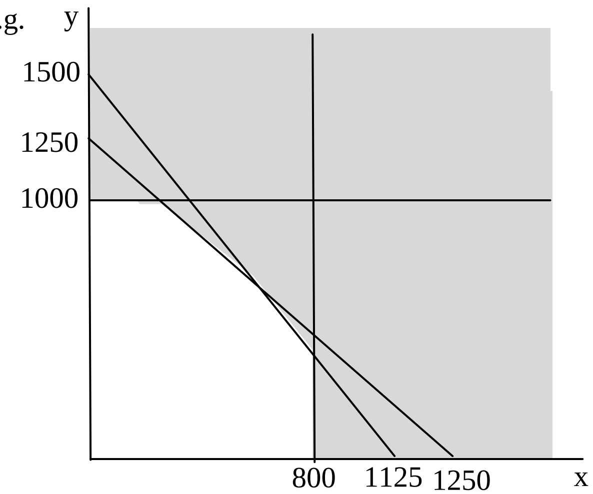
B1cao</td><td>indicating selections
deleting in rows
numbering columns</td></tr>
</table>

	(ii)			M1	3 out of 4 connections for A, B, D and G correct
			112m Shorter runs, or less exposure to risk.	A1 B1	cao

Question			Answer	Marks	Guidance
2	(i)		P 112 M 250 C (0) 100 110 120 130 131 132 133 134 135 136 137 138 The answer is 138	 B1 M1 A1 B1	 correct to statement 100 (i.e. 130)
	(ii)		e.g. add 34 If P + C + 1000 > M then goto 40 35 Let C = C + 1000 36 Goto 34	 B1 B1 B1	 (ignore “34” and “40”) (ignore “35”) logic all OK
	(iii)		e.g. P = price, M = money tendered, C = change	B1	No need to consider note denominations instead of powers of 10.

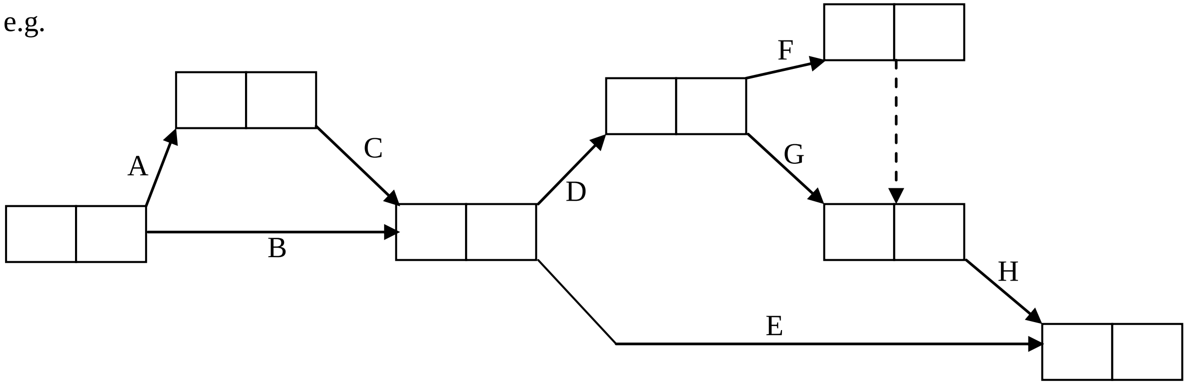
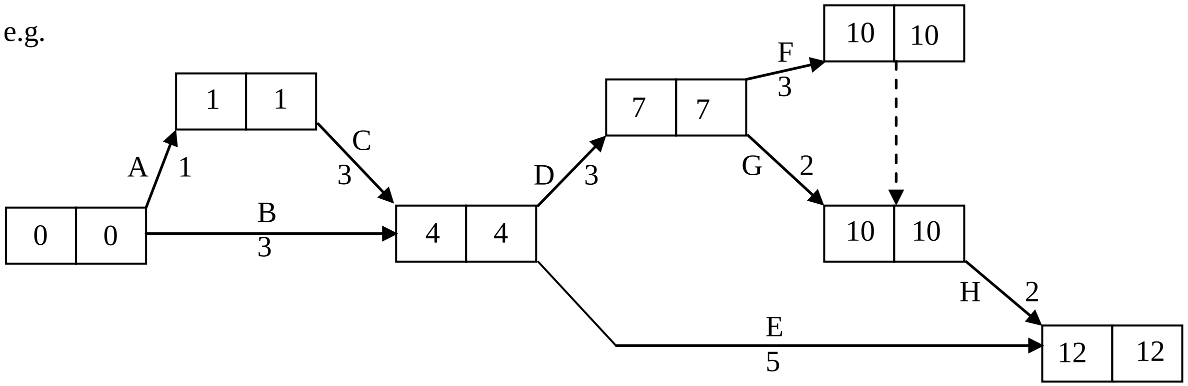
Question			Answer	Marks	Guidance
3	(i)		e.g. 	B1	
	(ii)		e.g. as per the above, with top left connected directly and bottom left connected around the back.	B1	
	(iii)		e.g. 	B1	$K_{2,3}$ seen
			e.g. (Dotted connections not needed.) The middle left cannot access the middle right.	M1 E1	choice of just two points that cannot be connected on the candidate's graph. dependent on the M1

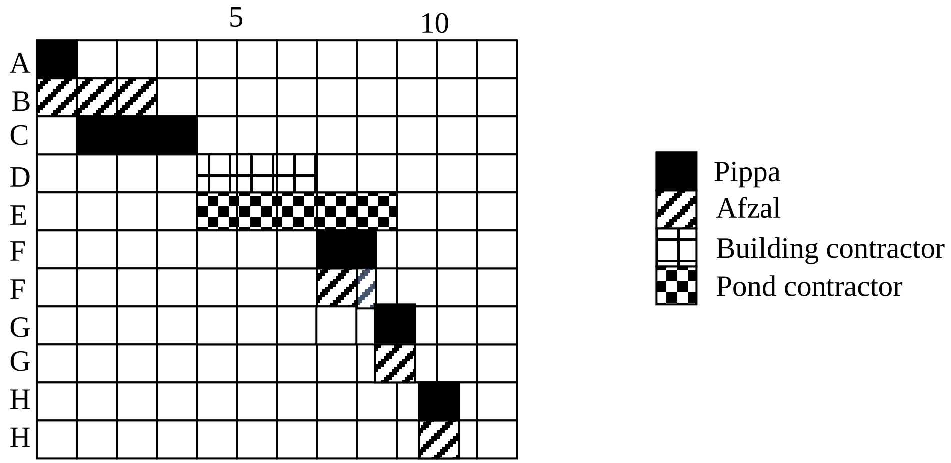
	(iv)		(5-1)×(5-1)/4 = 4 crossings e.g. 	B1 B1	can be implied
	(v)		e.g. They inform about how many layers will be needed.	B1	

Question			Answer	Marks	Guidance
4	(i)		£9 and £6 respectively	B1	
	(ii)		Let x be the number of deciduous trees and y the number of evergreens. Max $9x+6y$ st $8x+6y<9000$ $16x+16y<20000$ $x<800$ $y<1000$	B1 B1 B1 B1 B1	
	(iii)		e.g.  $(800,\ 433\frac{1}{3}) \rightarrow 9800$ $\big((800, 0) \rightarrow 7200\big)$ $(750, 500) \rightarrow 9750$ Profit is £9800	B1 B1 B1 B1 B1 B1 B1	labelling and scaling axes line for space constraint line for finance constraint lines for availability constraints feasible region indicated (with 6 or 5 lines correct) for profit at $(800,\ 433\frac{1}{3})$ and (750, 500) or gradient method with gradient -1.5 9800 indicated
	(iv)		£100 (at (800, 450)) £100 (also at (800, 450))	B1 B1	
	(v)		(750, 500) or 15 and 10 bundles (giving £9750 - but this not required)	B1	

Question			Answer	Marks	Guidance
5	(i)		stating 0000 gives a score of 0 stating 1111 gives a score of 15 all equally likely	B1 B1 B1	or 16 (B1) distinct numbers generated (B1)
	(ii)		1 10	B1 B1	penultimate last SC1 … 8, 5
	(iii)		The ball will not have an equal probability of landing in each jar	B1	
	(iv)		e.g. 00, 01 → 00 02, 03 → 03 04, 05 → 12 06, 07 → 15 08 – 11 → 01 12 – 15 → 02 16 – 19 → 04 20 – 23 → 07 24 – 27 → 08 28 – 31 → 11 32 – 35 → 13 36 – 39 → 14 40 – 47 → 05 48 – 55 → 06 56 – 63 → 09 64 – 71 → 10 72 – 99 → reject and repeat e.g. corner 00 00-01 2 edge 01 02-05 4 edge 02 06-09 4 corner 03 10-11 2 edge 04 12-15 4 inside 05 16-23 8 inside 06 24-31 8 edge 07 32-35 4 edge 08 36-39 4 inside 09 40-47 8 inside 10 48-55 8 edge 11 56-59 4 corner 12 60-61 2 edge 13 62-65 4 edge 14 66-69 4 corner 15 70-71 2 reject 72-99 28	M1 A1 M1 A1 M1 A1 M1 A1	reject some efficient – numbers stated rule for corner jars rule for edge jars rule for inside jars

	(v)		e.g. Using the above rule(s), the first ball lands in jar 00 (00) and the second in jar 06 (10).	B1 B1	√ subject to last 3 M marks

Question			Answer	Marks	Guidance
6	(i)		e.g. 	M1 A1 A1 A1	activity-on-arc A, B, C D, E Rest
6	(ii)		e.g.  minimum completion time – 12 days critical activities – A, C, D, F, H.	M1 A1 M1 A1 B1 B1	forward pass backward pass

6	**(iii)**		e.g. 	B1 B1 B1	A, B, C D, E F, G, H
			Minimum completion time = 10.5 days Afzal needs to be employed for 6.5 days.	B1 B1	
6	**(iv)**		17 days	B1	