OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4771

Decision Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4771
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 22 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

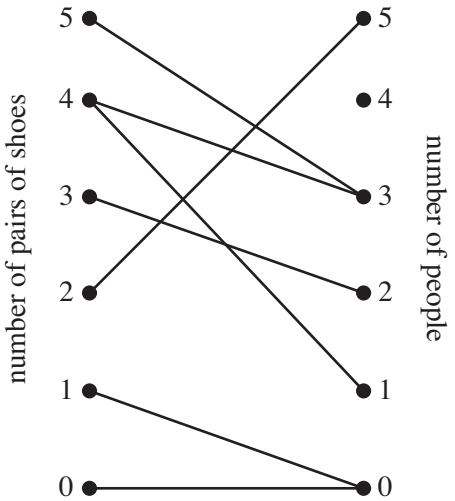
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

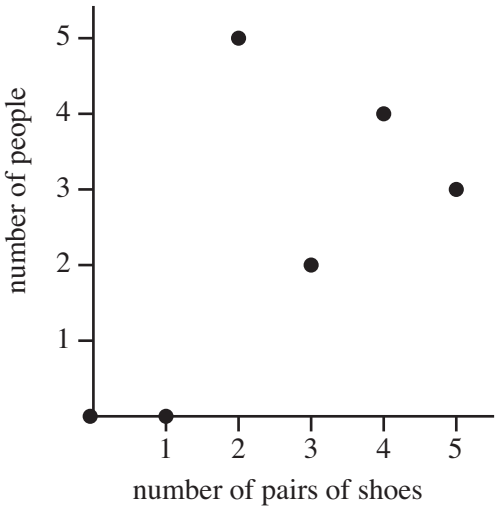
- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (24 marks)

1 Two students draw graphs to represent the numbers of pairs of shoes owned by members of their class. Andrew produces a bipartite graph, but gets it wrong. Barbara produces a completely correct frequency graph. Their graphs are shown below.



Andrew's graph



Barbara's graph

(i) Draw a correct bipartite graph. **[3]**

(ii) How many people are in the class? **[1]**

(iii) How many pairs of shoes in total are owned by members of the class? **[2]**

(iv) Which points on Barbara's graph may be deleted without losing any information? **[1]**

Charles produces the same frequency graph as Barbara, but joins consecutive points with straight lines.

(v) Criticise Charles's graph. **[1]**

2 The algorithm gives a method for drawing two straight lines, if certain conditions are met.

Start with the equations of the two straight lines

Line 1 is $ax + by = c, \quad a, b, c > 0$
Line 2 is $dx + ey = f, \quad d, e, f > 0$

Let X = minimum of $\frac{c}{a}$ and $\frac{f}{d}$

Let Y = minimum of $\frac{c}{b}$ and $\frac{f}{e}$

If $X = \frac{c}{a}$ then $X^* = \frac{c - bY}{a}$ and $Y^* = \frac{f - dX}{e}$

If $X = \frac{f}{d}$ then $X^* = \frac{f - eY}{d}$ and $Y^* = \frac{c - aX}{b}$

Draw an x-axis labelled from 0 to X, and a y-axis labelled from 0 to Y
Join $(0, Y)$ to (X, Y^*) with a straight line
Join (X^*, Y) to $(X, 0)$ with a straight line

(i) Apply the algorithm with $a = 1, b = 5, c = 25, d = 10, e = 2, f = 85$. **[7]**

(ii) Why might this algorithm be useful in an LP question? **[1]**

3 John has a standard die in his pocket (ie a cube with its six faces labelled from 1 to 6).

(i) Describe how John can use the die to obtain realisations of the random variable X, defined below.

x	1	2	3
Probability$(X = x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$

[3]

(ii) Describe how John can use the die to obtain realisations of the random variable Y, defined below.

y	1	2	3
Probability$(Y = y)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

[3]

(iii) John attempts to use the die to obtain a realisation of a uniformly distributed 2-digit random number. He throws the die 20 times. Each time he records one less than the number showing. He then adds together his 20 recorded numbers.

Criticise John's methodology. **[2]**

Section B (48 marks)

4 An eco-village is to be constructed consisting of large houses and standard houses.

Each large house has 4 bedrooms, needs a plot size of 200 m^2 and costs £60 000 to build.

Each standard house has 3 bedrooms, needs a plot size of 120 m^2 and costs £50 000 to build.

The area of land available for houses is 120 000 m^2. The project has been allocated a construction budget of £42.4 million.

The market will not sustain more than half as many large houses as standard houses. So, for instance, if there are 500 standard houses then there must be no more than 250 large houses.

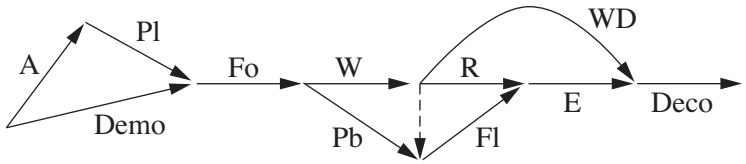
(i) Define two variables so that the three constraints can be formulated in terms of your variables. Formulate the three constraints in terms of your variables. **[5]**

(ii) Graph your three inequalities from part **(i)**, indicating the feasible region. **[4]**

(iii) Find the maximum number of bedrooms which can be provided, and the corresponding numbers of each type of house. **[2]**

(iv) Modify your solution if the construction budget is increased to £45 million. **[5]**

5 The activity network and table together show the tasks involved in constructing a house extension, their durations and precedences.



Activity	Description	Duration (days)
A	Architect produces plans	10
Pl	Obtain planning permission	14
Demo	Demolish existing structure	3
Fo	Excavate foundations	4
W	Build walls	3
Pb	Install plumbing	2
R	Construct roof	3
Fl	Lay floor	2
E	Fit electrics	2
WD	Install windows and doors	1
Deco	Decorate	5

(i) Show the immediate predecessors for each activity. **[2]**

(ii) Perform a forward pass and a backward pass to find the early time and the late time for each event. **[4]**

(iii) Give the critical activities, the project duration, and the total float for each activity. **[4]**

(iv) The activity network includes one dummy activity. Explain why this dummy activity is needed. **[2]**

Whilst the foundations are being dug the customer negotiates the installation of a decorative corbel. This will take one day. It must be done after the walls have been built, and before the roof is constructed. The windows and doors cannot be installed until it is completed. It will not have any effect on the construction of the floor.

(v) Redraw the activity network incorporating this extra activity. **[3]**

(vi) Find the revised critical activities and the revised project duration. **[1]**

6 The table shows the distances in miles, where direct rail connections are possible, between 11 cities in a country. The government is proposing to construct a high-speed rail network to connect the cities.

	P	S	F	Ln	Br	Nr	Bm	Ld	Nc	Lv	M
P	–	150	–	240	125	–	–	–	–	–	–
S	150	–	150	80	105	–	135	–	–	–	–
F	–	150	–	80	–	–	–	–	–	–	–
Ln	240	80	80	–	120	115	120	–	–	–	–
Br	125	105	–	120	–	230	90	–	–	–	–
Nr	–	–	–	115	230	–	160	175	255	–	–
Bm	–	135	–	120	90	160	–	120	–	–	90
Ld	–	–	–	–	–	175	120	–	210	100	90
Nc	–	–	–	–	–	255	–	210	–	175	–
Lv	–	–	–	–	–	–	–	100	175	–	35
M	–	–	–	–	–	–	90	90	–	35	–

(i) Use the tabular form of Prim's algorithm, starting at vertex P, to find a minimum connector for the network. Draw your minimum connector and give its total length. **[6]**

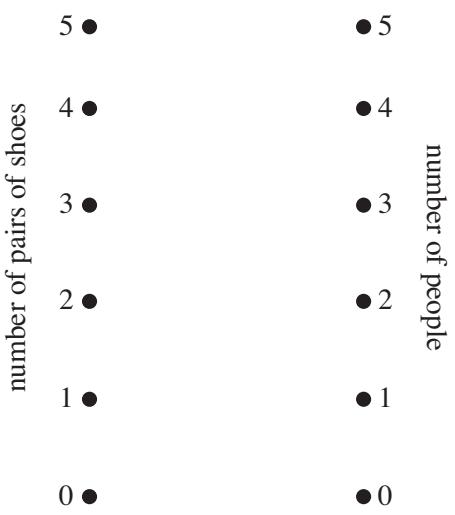
(ii) Give one advantage and two disadvantages of constructing a rail network using only the arcs of a minimum connector. **[3]**

(iii) Use Dijkstra's algorithm on the diagram in the Printed Answer Book, to find the shortest route and distance from P to Nr in the original network. **[6]**

(iv) Give the shortest distance from P to Nr using only arcs in your minimum connector. **[1]**

Section A (24 marks)

1 (i)



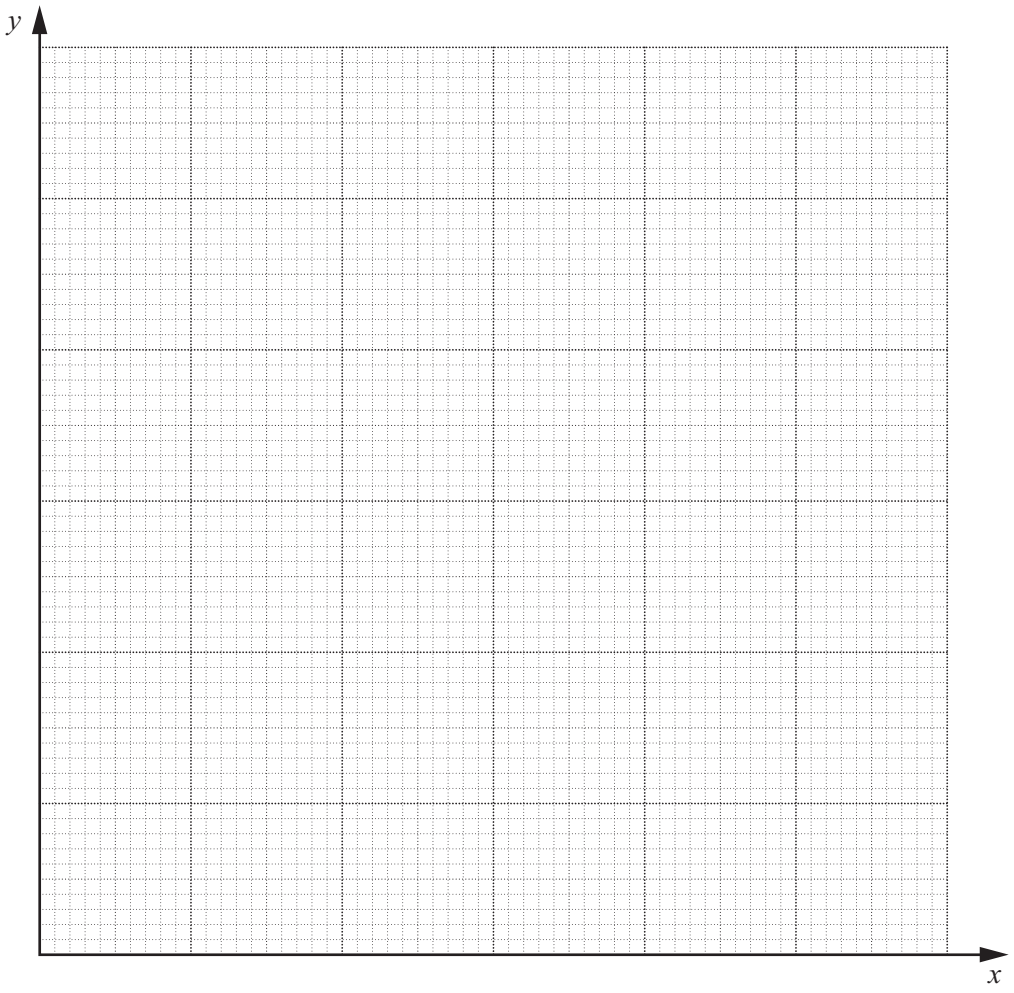
1 (ii)

1 (iii)

1 (iv)

1 (v)

2 (i)



2 (ii)

3 (i)

x	1	2	3
Probability$(X = x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$

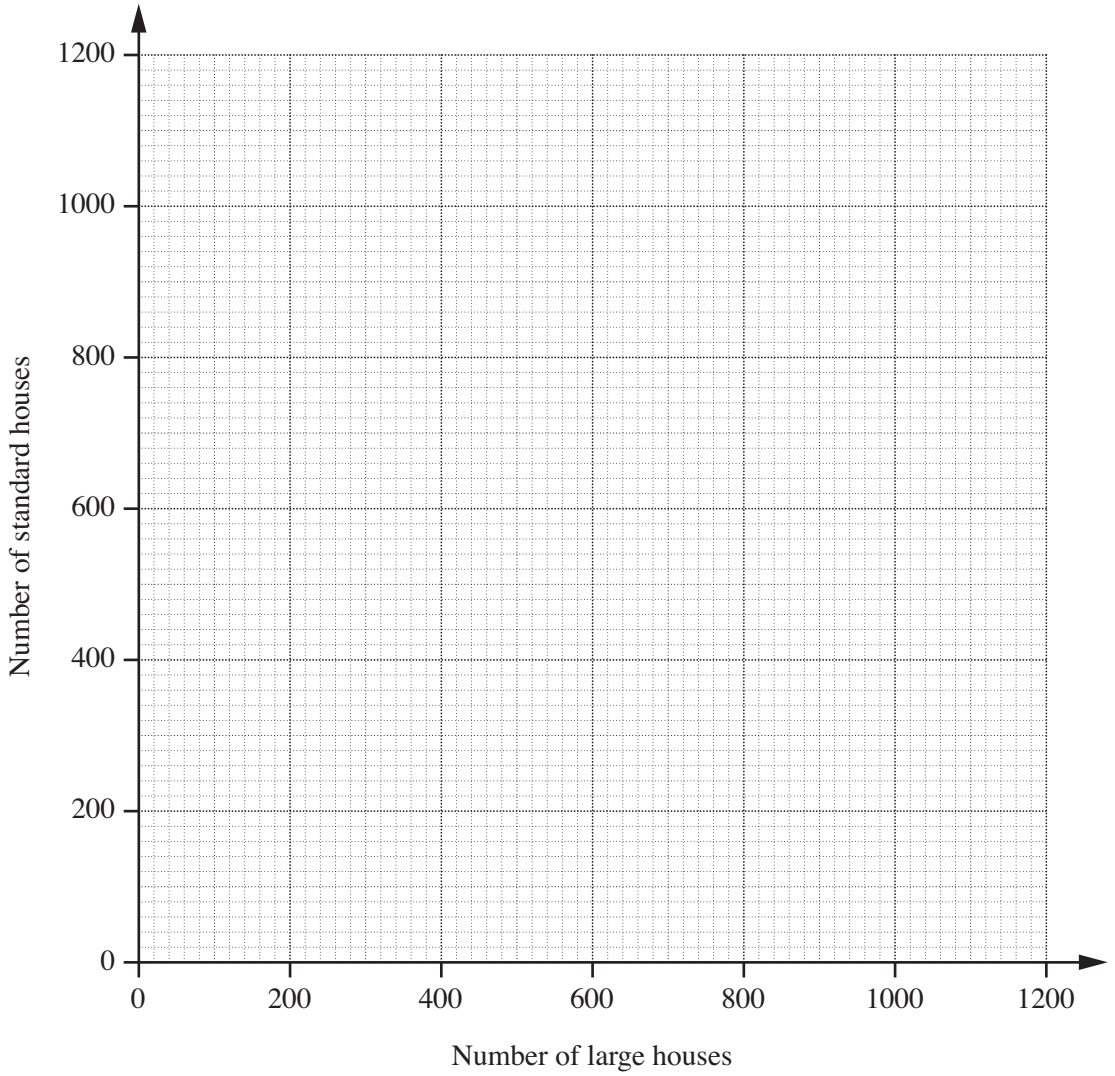
3 (ii)

y	1	2	3
Probability$(Y = y)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

Section B (48 marks)

4 (i)

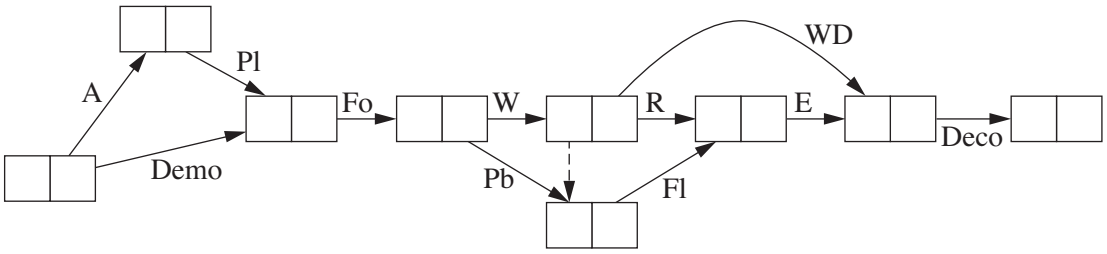
4 (ii)



5 (i)

Activity	Immediate predecessor(s)
A	
Pl	
Demo	
Fo	
W	
Pb	
R	
Fl	
E	
WD	
Deco	

5 (ii)



5 (iii)

task: A Pl Demo Fo W Pb R Fl E WD Deco

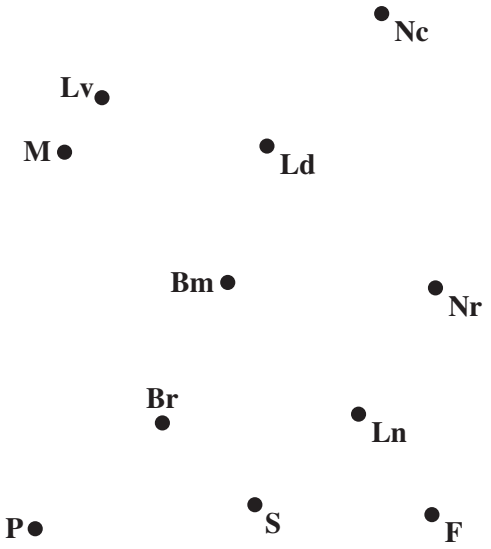
float:

5 (iv)

6 (i)

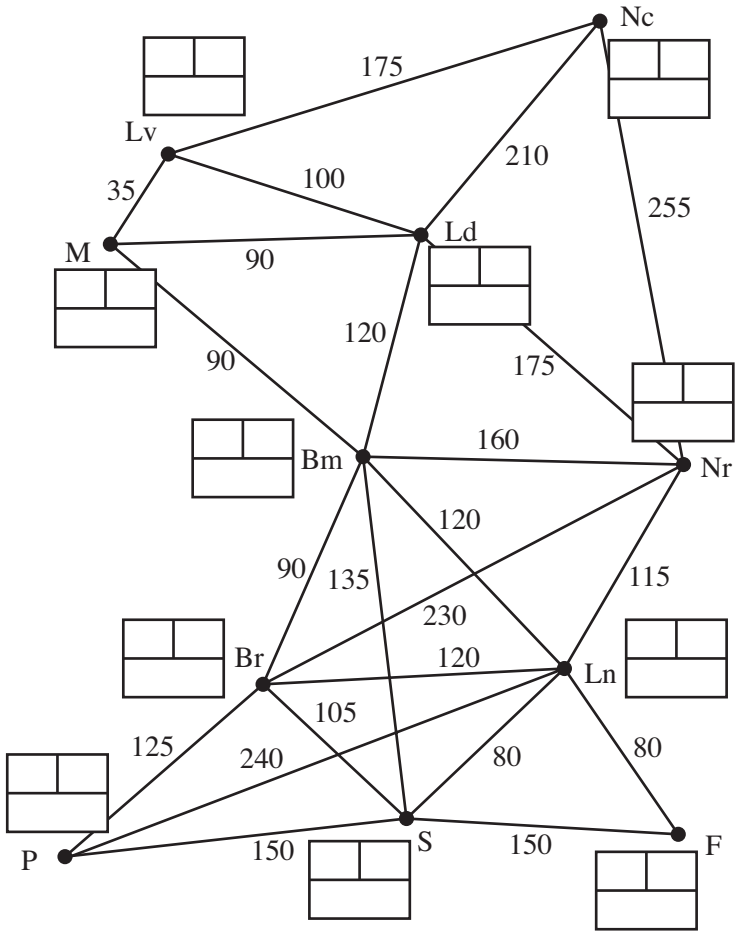
	P	S	F	Ln	Br	Nr	Bm	Ld	Nc	Lv	M
P	–	150	–	240	125	–	–	–	–	–	–
S	150	–	150	80	105	–	135	–	–	–	–
F	–	150	–	80	–	–	–	–	–	–	–
Ln	240	80	80	–	120	115	120	–	–	–	–
Br	125	105	–	120	–	230	90	–	–	–	–
Nr	–	–	–	115	230	–	160	175	255	–	–
Bm	–	135	–	120	90	160	–	120	–	–	90
Ld	–	–	–	–	–	175	120	–	210	100	90
Nc	–	–	–	–	–	255	–	210	–	175	–
Lv	–	–	–	–	–	–	–	100	175	–	35
M	–	–	–	–	–	–	90	90	–	35	–

Min connector



6 (ii)

6 (iii)

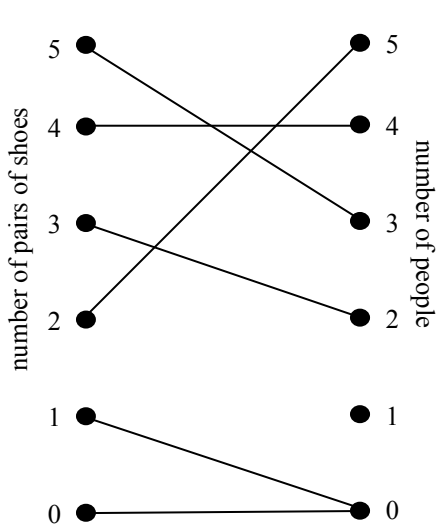


THERE IS A SPARE COPY OF THIS NETWORK ON PAGE 12.

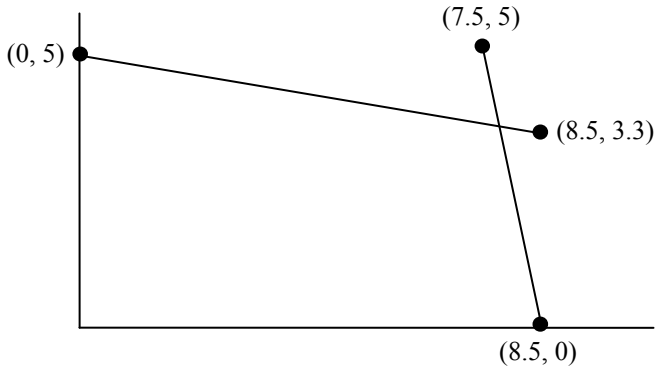
6 (iv)

4771, June 2011, Markscheme

1.

(i) 	B1 3 to 4 deleted B1 1 to 4 deleted B1 4 to 4 added	-1 for each arc in error
(ii) 14	B1	
(iii) 47	M1 A1 cao	Award method mark if answer correct, or if wrong but with a sum of products shown.
(iv) (0, 0) and (1, 0)	B1	Award only if correct **points** are specified in some way.
(v) Explanation should recognise that a line is a set of points − not appropriate in this context.	B1	e.g. "Intermediate points have no meaning." e.g. "Can't have one and a half pairs of shoes." (sic)

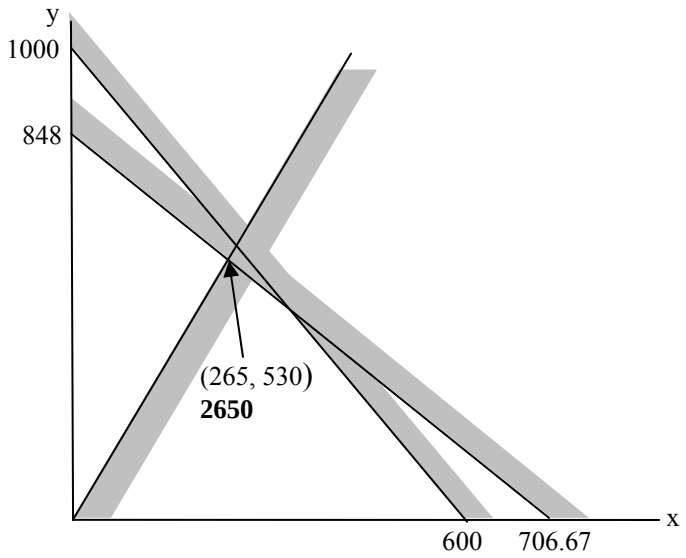
2.

(i) $X = \min(25, 8.5) = 8.5$ or equivalent	B1	cao	OK if only seen once or more on graph
$Y = \min(5, 42.5) = 5$ oe	B1	cao	OK if only seen once or more on graph
$X^* = (85–10)/10 = 7.5$ oe	B1	cao	OK if only seen on graph
$Y^* = (25–8.5)/5 = 3.3$ oe	B1	cao	OK if only seen on graph
	B1	allow ft	sensibly scaled for their X and Y e.g. disallow if either of the lines in the question could intersect both axes.
	B1	cao	lines - can extend to beyond segment
	B1	cao	condone minor errors in plotting (e.g. 8.5 plotted at 9)
(ii) Avoids tiny feasible regions.	B1		need comment on size of region

3.

(i) e.g. 1, 2, 3 → 1 4 → 2 5, 6 → 3	M1 A1 A1	function with domain {1,2,3,4,5,6} and range {1,2,3} (special cases are possible – if correct!) proportions 3:2:1 all OK
(ii) e.g. 1, 2 → 1 3 → 2 4 → 3 (5, 6 → reject and throw again)	M1 reject some A1 reject two A1 rest	(Special cases are possible – if correct! e.g. allow throwing die twice and allocating correct proportions of 36.)
(iii) non uniform allows 100	B1 B1	"101 values" OK no credit for, e.g. "3 is not a two-digit number"

4.

(i)	e.g. x = number of large houses y = number of standard houses	M1 A1	M1 for variables for large and for standard A1 for "number"
	land: 200x + 120y <= 120000 oe cash: 60x + 50y <= 42400 oe market: x <= 0.5y oe	B1 B1 B1	use "isw" for incorrect simplifications -1 once only for any " < "
(ii)		B1 line 1, allow ft B1 line 2, allow ft B1 line 3, allow ft B1 feasible region	for instance, if x <= 2y in part (i), then allow correct graph of x <= 0.5y or ft graph of x <= 2y plotting tolerance on axis intersection points – within correct small square must consider 3 lines ft if region includes y-axis interval from origin upwards allow any clear indication of feasible region ignore any indication(s) of boundary lines included or excluded
(iii)	intersection of y=2x and 6x+5y=4240, (265, 530) 2650	M1 correct point, cao A1	identification only - coordinates not required here their 4x+3y from (260-280, 520-540)
(iv)	their 60x + 50y <= 45000 or line from their (0, 900) to (750, 0)	B1 ft	can be implied from final M1 working
	Best point is at the intersection of the land constraint and the new cash constraint, and not on y=2x	M1 comparison of two (or more) points A1	not just ringing points their identified best point is not on y=2x or an axis
	(214, 643) 2785	M1 correct point, cao A1	identification, coordinates not required here bedrooms - their 4x+3y from (200-220, 620-660)

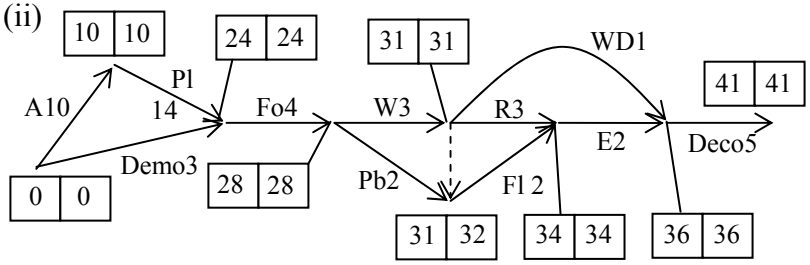
5.

(i)

Activity	Immediate predecessors
A	–
Pl	A
Demo	–
Fo	Pl; Demo
W	Fo
Pb	Fo
R	W
Fl	Pb; W
E	R; Fl
WD	W
Deco	WD; E

M1 Fl correct
A1 rest

(ii)



M1 at least one correct nontrivial join
A1 forward pass

M1 at least one correct nontrivial burst
A1 backward pass

excluding start node

(iii) critical activities: A; Pl; Fo; W; R; E; Deco

B1 cao

project duration = 41 days

B1 cao

act	A	Pl	Dm	Fo	W	Pb	R	Fl	E	WD	Dc
float	0	0	21	0	0	2	0	1	0	4	0

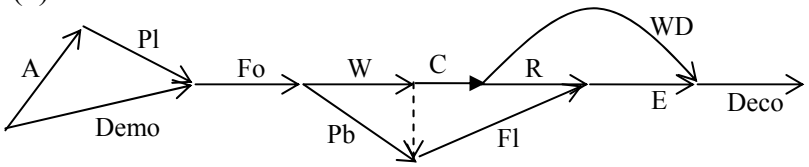
B1 A, Pl, Dm, Fo, W
B1 rest

cao
cao – most see zeros, dashes or empty spaces won't do

(iv) Fl has both W and Pb as immediate predecessors.
R and WD have only W as immediate predecessor.

B1
B1 one of R/WD

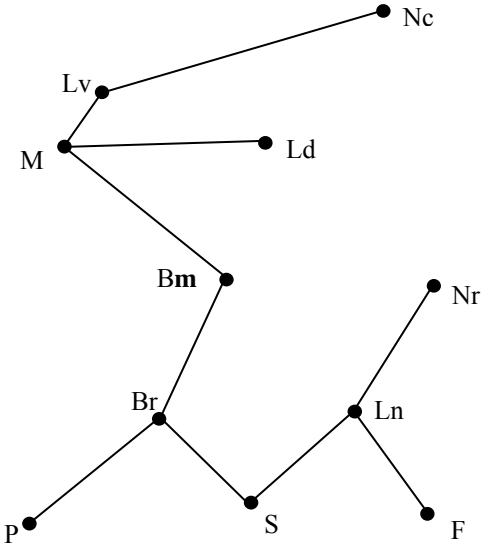
SC1 for a convincing but not specific answer, e.g. "A dummy is needed to cater for both joint and separate precedences".

(v)  	M1 C between W and R A1 Fl + dummy OK A1 WD OK	
(vi) new duration = 42 days critical activities: A; Pl; Fo; W; C; R; E; Deco	B1 cao	both needed

6.

(i)

	1	7	9	8	2	10	3	6	11	5	4
	P	S	F	Ln	Br	Nr	Bm	Ld	Nc	Lv	M
P	–	150	–	240	125	–	–	–	–	–	–
S	150	–	150	80	105	–	135	–	–	–	–
F	–	150	–	80	–	–	–	–	–	–	–
Ln	240	80	80	–	120	115	120	–	–	–	–
Br	125	105	–	120	–	230	90	–	–	–	–
Nr	–	–	–	115	230	–	160	175	255	–	–
Bm	–	135	–	120	90	160	–	120	–	–	90
Ld	–	–	–	–	–	175	120	–	210	100	90
Nc	–	–	–	–	–	255	–	210	–	175	–
Lv	–	–	–	–	–	–	–	100	175	–	35
M	–	–	–	–	–	–	90	90	–	35	–



Length = 985 miles

M1 tabular Prim — 125 in P column and 90 in Br column ringed, with both rows crossed

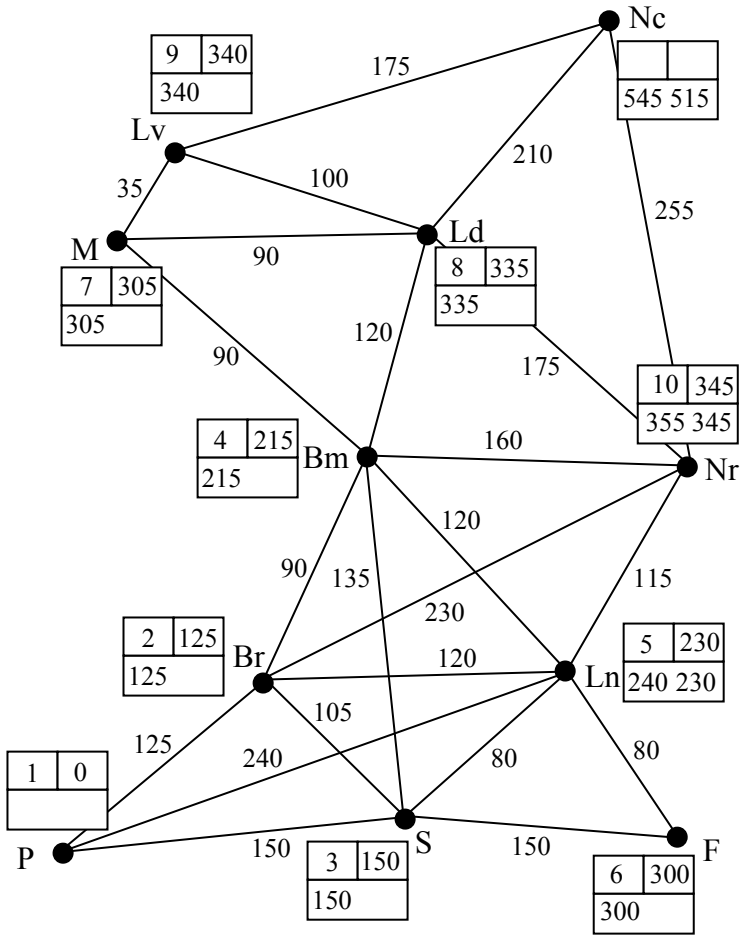
A2 choosings — all circles in correct place; -1 each error (watch for one error making two changes to a row)

A1 crossings — all rows crossed out except, possibly, Nc row.

accept convincing transpose

B1 cao

B1 cao

(ii)	Advantage: shortest length of track Disadvantage: tree, no redundancy ≡ fragility (breakdown et al) Disadvantage: some journeys are not shortest paths	B1 cao B1 B1	allow cost minimisation could say "no cycles" disallow comments relating to direct connectivity, or relating to more stops "longer journeys" or "takes longer" allowed allow "min connector arcs may be more expensive" oe don't allow two marks for the same point described differently. e.g. longer journeys/more time/more upkeep
(iii)		M1 Dijkstra A1 working values B1 labels B1 order of labelling	correct working values (no extras) at Ln and Nr, and working values only superseded at Ln and Nr (ignore Nc for this M) (need to check Nc here)
	Route: P S Ln Nr Distance: 345 miles	B1 cao B1 cao	
(iv)	Distance by min connector = 425 miles	B1 ft their mc	