

**ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)**

Decision Mathematics 1

4771

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:

- Printed Answer Book 4771
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

- Scientific or graphical calculator

**Tuesday 22 June 2010
Afternoon**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

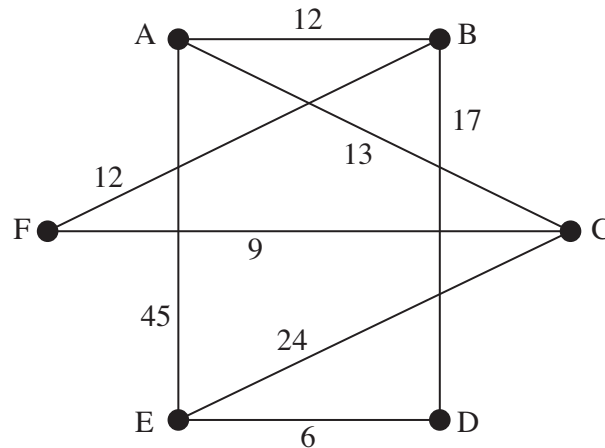
INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

Answer all questions on the Printed Answer Book provided.

Section A (24 marks)

- 1 (i) Use Dijkstra's algorithm to find the shortest distances and corresponding routes from A to each of the other vertices in the given network. [7]



- (ii) If the shortest distances and routes between every pair of vertices are required how many applications of Dijkstra will be needed? [1]

- 2 The following steps define an algorithm which acts on two numbers.

STEP 1 Write down the two numbers side by side on the same row.

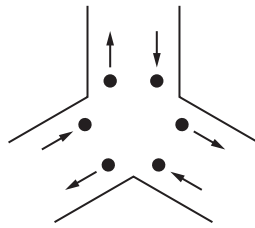
STEP 2 Beneath the left-hand number write down double that number. Beneath the right-hand number write down half of that number, ignoring any remainder.

STEP 3 Repeat STEP 2 until the right-hand number is 1.

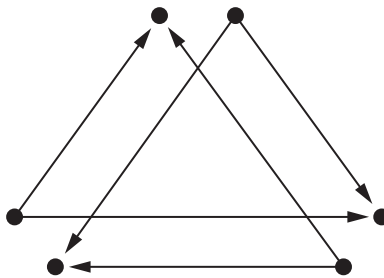
STEP 4 Delete those rows where the right-hand number is even. Add up the remaining left-hand numbers. This is the result.

- (i) Apply the algorithm to the left-hand number 3 and the right-hand number 8. [3]
- (ii) Apply the algorithm to the left-hand number 26 and the right-hand number 42. [4]
- (iii) Use your results from parts (i) and (ii), together with any other examples you may choose, to write down what the algorithm achieves. [1]

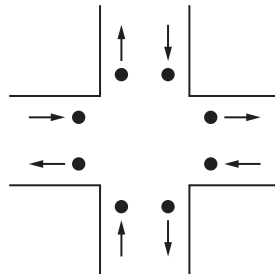
- 3 Traffic flows in and out of a junction of three roads as shown in the diagram.



Assuming that no traffic leaves the junction by the same road as it entered, then the digraph shows traffic flows across the junction.



- (i) Redraw the digraph to show that it is planar. [3]
- (ii) Draw a digraph to show the traffic flow across the junction of 4 roads, assuming that no traffic leaves the junction by the same road as it entered.



[4]

(Note that the resulting digraph is also planar, but you are not required to show this.)

- (iii) The digraphs showing flows across the junctions omit an important aspect in their modelling of the road junctions. What is it that they omit? [1]

Section B (48 marks)

- 4** A wall 4 metres long and 3 metres high is to be tiled. Two sizes of tile are available, 10 cm by 10 cm and 30 cm by 30 cm.

- (i) If x is the number of boxes of **ten** small tiles used, and y is the number of large tiles used, explain why $10x + 9y \geq 1200$. [3]

There are only 100 of the large tiles available.

The tiler advises that the area tiled with the small tiles should not exceed the area tiled with the large tiles.

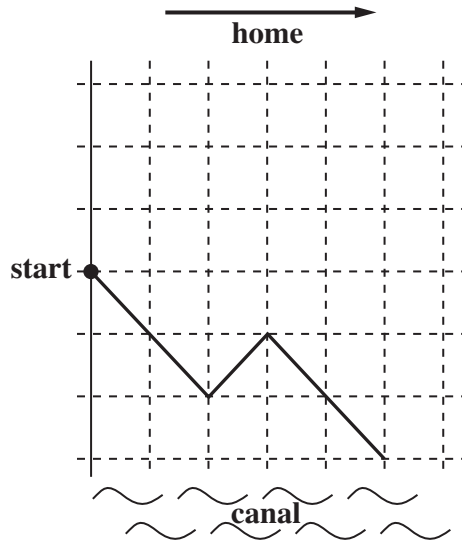
- (ii) Express these two constraints in terms of x and y . [3]

The smaller tiles cost 15p each and the larger tiles cost £2 each.

- (iii) Given that the objective is to minimise the cost of tiling the wall, state the objective function. Use the graphical approach to solve the problem. [8]

- (iv) Give **two** other factors which would have to be taken into account in deciding how many of each tile to purchase. [2]

- 5 The diagram shows the progress of a drunkard towards his home on one particular night. For every step which he takes towards his home, he staggers one step diagonally to his left or one step diagonally to his right, randomly and with equal probability. There is a canal three steps to the right of his starting point, and no constraint to the left. On this particular occasion he falls into the canal after 5 steps.



- (i) Explain how you would simulate the drunkard's walk, making efficient use of one-digit random numbers. [3]
- (ii) Using the random digits in the Printed Answer Book simulate the drunkard's walk and show his progress on the grid. Stop your simulation either when he falls into the canal or when he has staggered 6 steps, whichever happens first. [2]
- (iii) How could you estimate the probability of him falling into the canal within 6 steps? [2]

On another occasion the drunkard sets off carrying a briefcase in his right hand. This changes the probabilities of him staggering to the right to $\frac{2}{3}$, and to the left to $\frac{1}{3}$.

- (iv) Explain how you would now simulate this situation. [3]
- (v) Simulate the drunkard's walk (with briefcase) 10 times, and hence estimate the probability of him falling into the canal within 6 steps. (In your simulations you are not required to show his progress on a grid. You only need to record his steps to the right or left.) [6]

Question 6 is printed overleaf.

- 6 The table shows the tasks that have to be completed in building a stadium for a sporting event, their durations and their precedences. The stadium has to be ready within two years.

Task	Duration (months)	Immediate predecessors
A	4	–
B	2	–
C	7	–
D	12	A
E	5	A
F	7	A, B
G	6	D, J
H	3	C
I	12	E, F, H
J	7	E, F, H
K	12	C

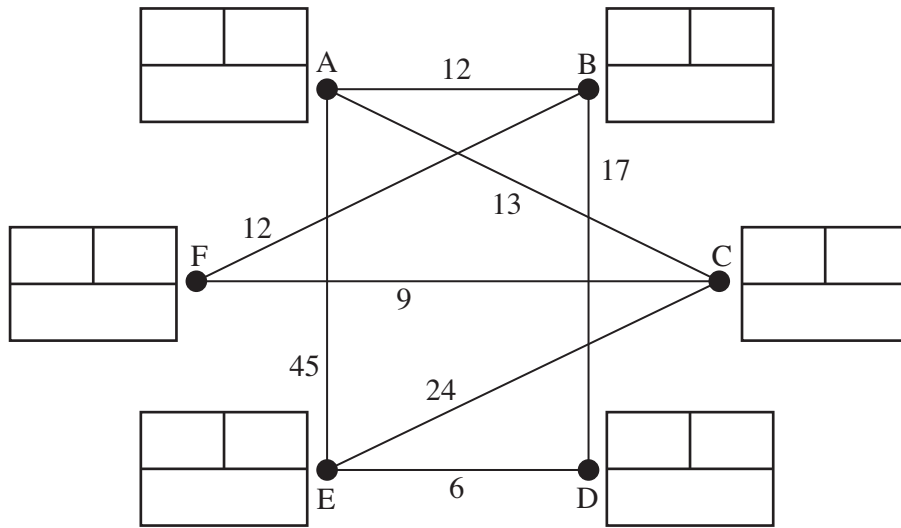
- (i) Draw an activity on arc network for these activities. [5]
- (ii) Mark on your diagram the early time and the late time for each event. Give the project duration and the critical activities. [6]

In the later stages of planning the project it is discovered that task J will actually take 9 months to complete. However, other tasks can have their durations shortened by employing extra resources. The costs of “crashing” tasks (i.e. the costs of employing extra resources to complete them more quickly) are given in the table.

Tasks which can be completed more quickly by employing extra resources	Number of months which can be saved	Cost per month of employing extra resources (£m)
A	2	3
D	1	1
C	3	3
F	2	2
G	2	4

- (iii) Find the cheapest way of completing the project within two years. [3]
- (iv) If the delay in completing task J is not discovered until it is started, how can the project be completed in time, and how much extra will it cost? [2]

1 (i)



Distance

Route

A to B

A to C

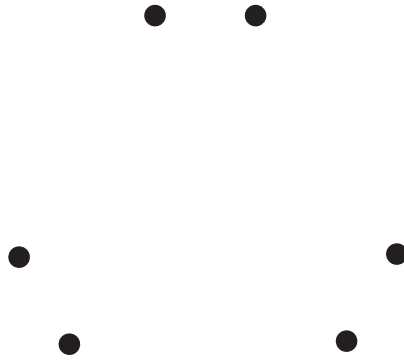
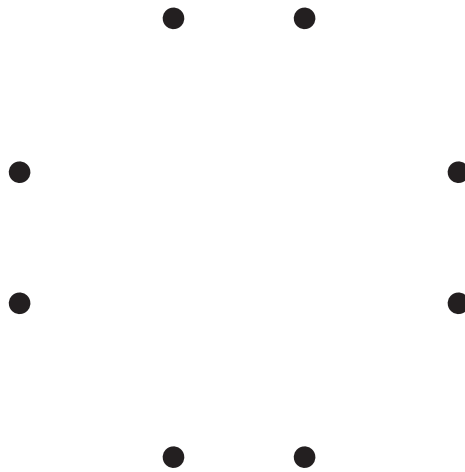
A to D

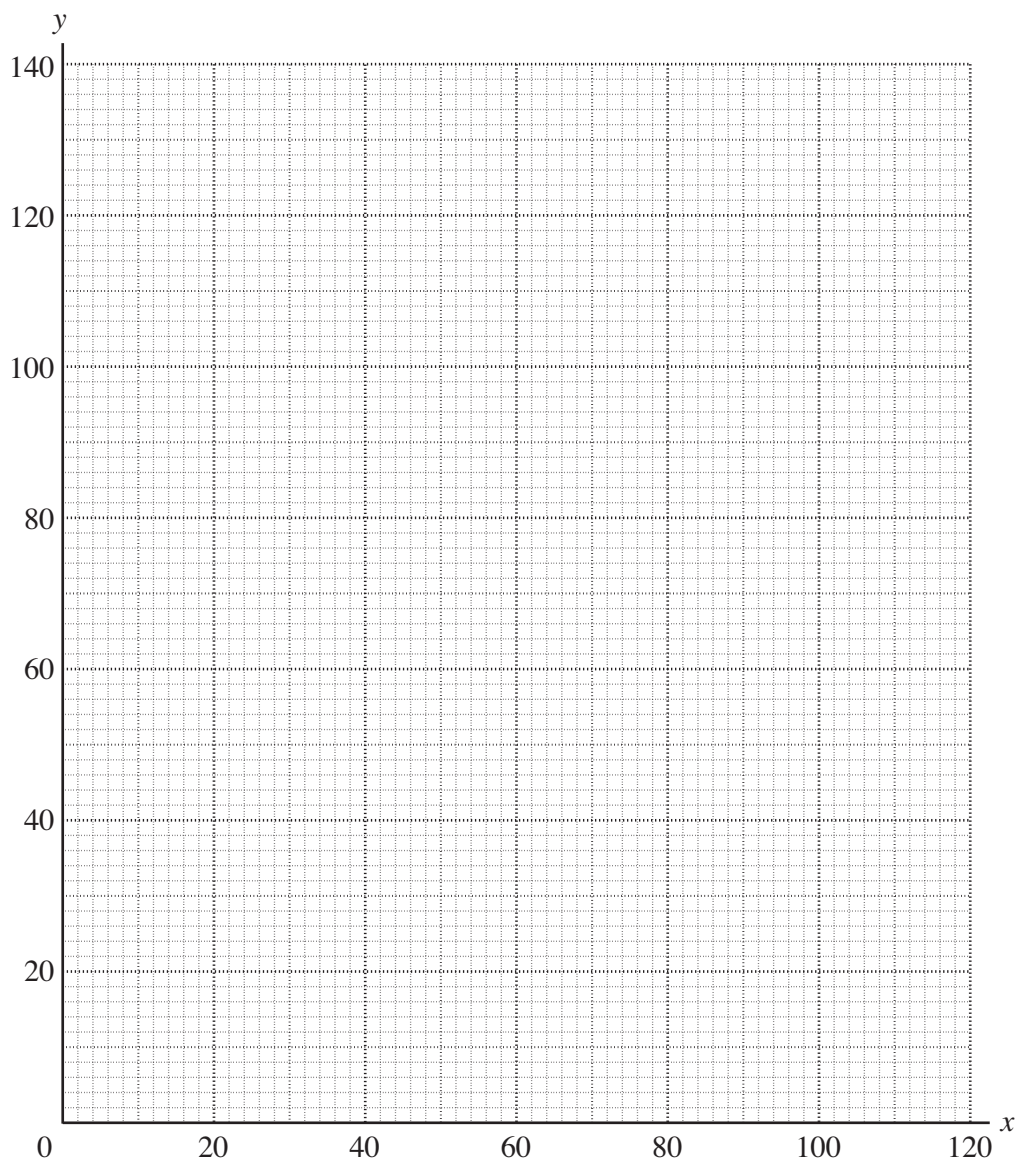
A to E

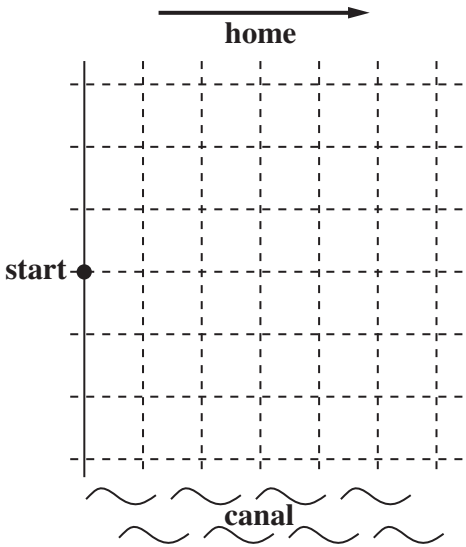
A to F

1 (ii)

2 (i)	Left-hand number	Right-hand number
Result =		
2 (ii)	Left-hand number	Right-hand number
Result =		
2 (iii)		

3 (i)**3 (ii)****3 (iii)**

4 (i)**4 (ii)****4 (iii)****THERE IS A SPARE COPY OF THIS GRAPH PAPER ON PAGE 12**

5 (i)		
5 (ii)	Random digits:	
	9 1 4 8 0 2	
5 (iii)		
5 (iv)		

5 (v)

Random digits:

run 1	4	2	7	2	0	4	5	3	9	7
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run 2	6	9	2	9	3	5	0	5	4	1
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run 3	7	4	1	2	1	2	1	8	6	8
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run 4	1	0	4	0	3	8	0	4	9	2
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run 5	5	6	6	7	7	6	0	1	2	9
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run 6	1	3	3	9	6	9	6	3	8	5
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run 7	5	8	1	5	4	5	0	2	7	6
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run 8	7	6	0	8	5	6	2	9	0	0
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run 9	4	9	8	9	6	7	4	2	0	8
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run 10	1	4	5	2	6	6	1	5	9	8
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run 1										
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run 2										
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run 3										
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run 4										
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run 5										
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run 6										
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run 7										
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run 8										
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run 9										
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run 10										
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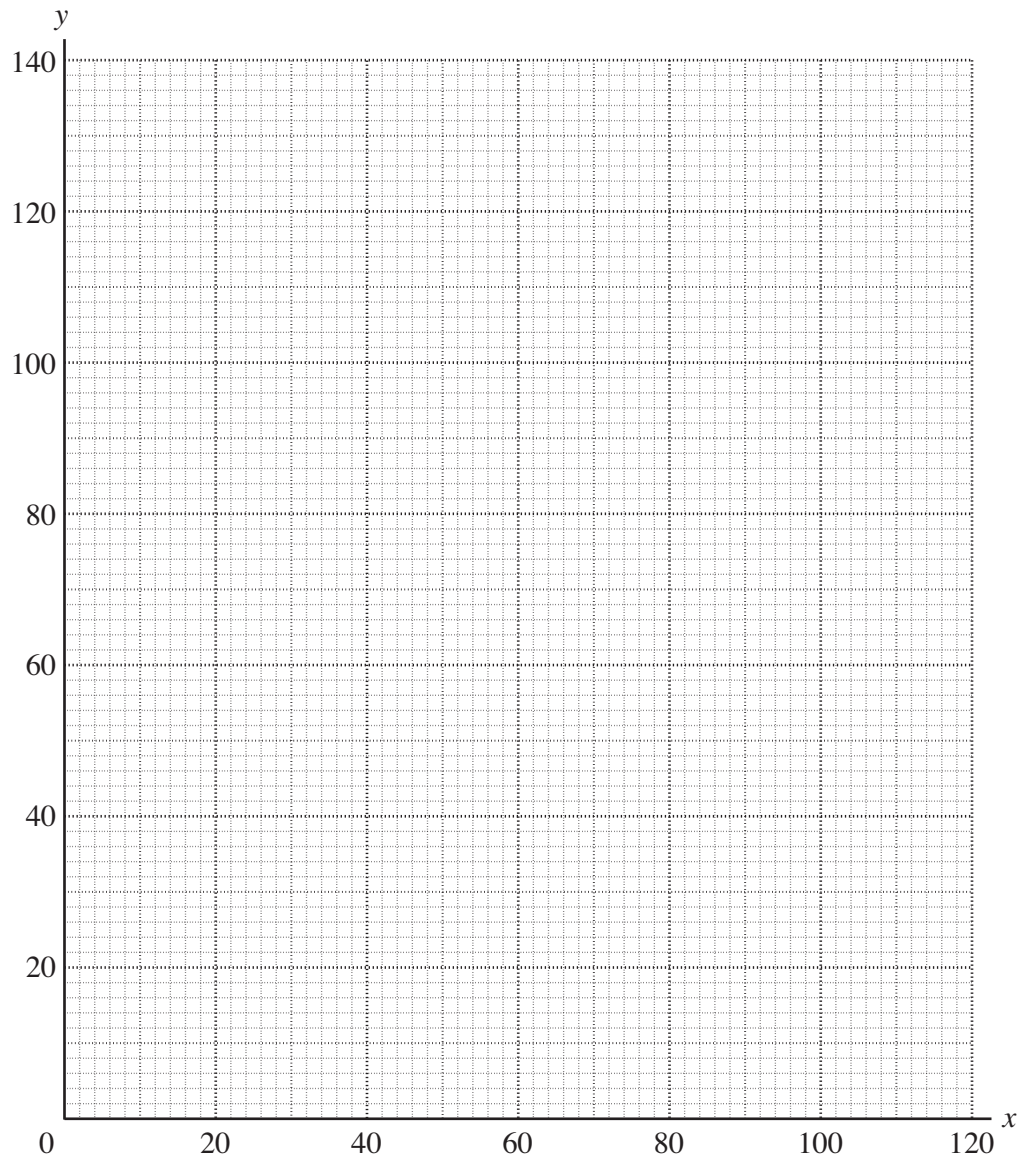
Estimate of probability =

**6 (i) &
(ii)**

0	0
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Project duration: _____

Critical activities: _____

SPARE GRAPH PAPER FOR QUESTION 4(iii)**Copyright Information**

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1.

(i)

Graph structure and edge weights:

- AB: 12
- AC: 13
- AD: 29
- AE: 35
- AF: 22
- BC: 17
- CD: 6
- DE: 6
- EF: 45
- FD: 9

Vertex labels and their associated values:

- A: 1, 0
- B: 2, 12
- C: 3, 13
- D: 5, 29
- E: 6, 35
- F: 4, 22

Path analysis for shortest paths from A:

- AB: 12
- AC: 13
- AD: 29
- AE: 35
- AF: 22
- ABD: 29
- ACF: 22

(ii) 5

M1 Dijkstra
A1 working values
B1 order of labelling
B1 labels

B1 AB and AC
B1 AD and AF
B1 AE

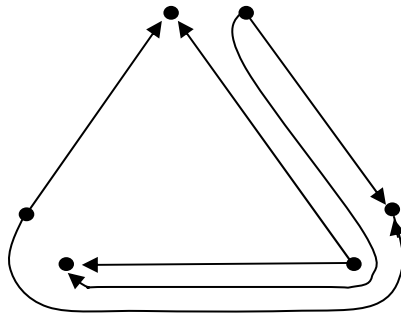
B1

2.

<p>(i)</p> <table> <tr><td>3</td><td>8</td></tr> <tr><td>6</td><td>4</td></tr> <tr><td>12</td><td>2</td></tr> <tr><td>24</td><td>1</td></tr> <tr><td>24</td><td></td></tr> </table>	3	8	6	4	12	2	24	1	24		<p>M1 doubling and halving M1 deleting and summing A1 cao</p>				
3	8														
6	4														
12	2														
24	1														
24															
<p>(ii)</p> <table> <tr><td>26</td><td>42</td></tr> <tr><td>52</td><td>21</td></tr> <tr><td>104</td><td>10</td></tr> <tr><td>208</td><td>5</td></tr> <tr><td>416</td><td>2</td></tr> <tr><td>832</td><td>1</td></tr> <tr><td>1092</td><td></td></tr> </table>	26	42	52	21	104	10	208	5	416	2	832	1	1092		<p>M1 doubling and halving M1 deleting DM summing A1 cao</p>
26	42														
52	21														
104	10														
208	5														
416	2														
832	1														
1092															
<p>(iii) multiplication</p>	<p>B1</p>														

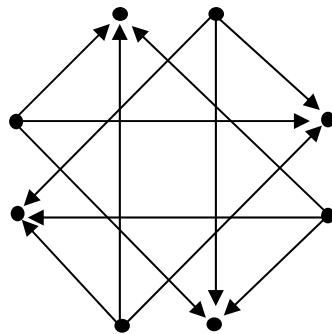
3.

(i)



B1
B1
B1

(ii)

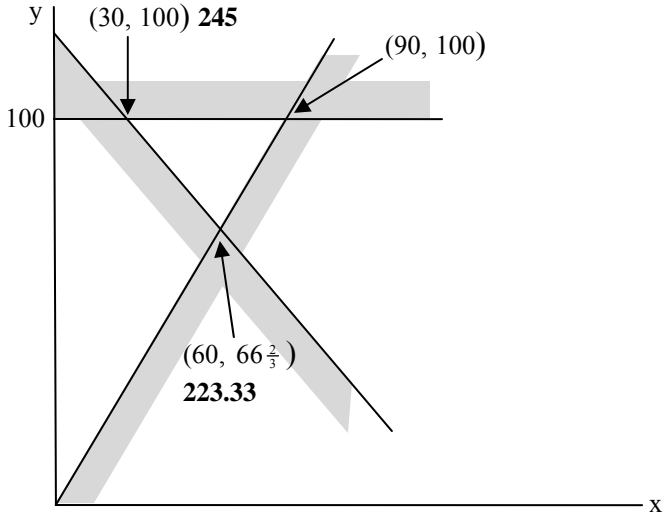


B1 12 arcs
B1 connectivity
B1 3 out of each in vertex
B1 3 into each out vertex

- (iii) The graphs represent traffic flows within the junctions. They do not take account of flows approaching or leaving the junctions.
(Graphs are not planar if these flows are added, so traffic flows have to cross.)

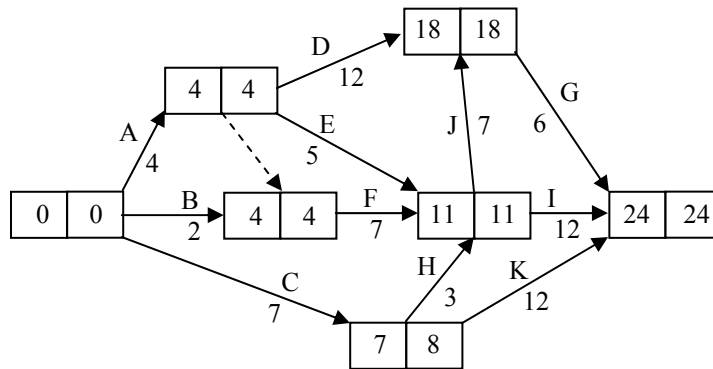
B1

4.

<p>(i) Each small tile has area 100 cm^2 so $1000x$ Similarly $900y$ So $1000x + 900y \geq 400 \times 300 = 120000$</p> <p>(ii) $y \leq 100$ $10x \leq 9y$</p> <p>(iii) e.g. minimise $1.5x + 2y$</p>  <p>Integer solution required, so $x=60$, $y=67$, cost = 224</p> <p>(iv) wastage or design</p>	<p>M1 areas A1 tile areas A1</p> <p>B1 B1 B1</p> <p>B1</p> <p>B3 lines B1 shading</p> <p>M1 solving A1 $x = 59-61$ $y = 66-68$ A1 220-228</p> <p>B2</p>
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6.

(i) & (ii)



Duration = 24 months

Critical : A; F; J; G

(iii) Crash F by 1 month and G by 1 month at a cost of £6m.

(iv) Crash G by 2 months at a cost of £8m.

M1 activity-on-arc
 A1 D, E, H and K
 A1 F
 A1 I and J
 A1 G

M1 forward pass
 A1
 M1 backward pass
 A1

B1 cao

B1 cao

B1 F by 1 month
 B1 G by 1 month
 B1 £6m

M1 G only
 A1 £8m

4771 Decision Mathematics 1

General comments

The mean score on this paper was lower than was the case in 2009 (46 compared to 50). Performance at the lower end was comparable to 2009, with a grade E boundary of 36 (also 36 in 2009). Marks at the top end were harder to obtain (grade A boundary 55 against 63 in 2009). There was one mark which was recognised in advance as being difficult, Q3(iii).

Some candidates made mistakes in producing diagrams, rubbed their initial effort out, and then redrew an improved version on top of this. (This was seen most often on questions 6 and 3.) This caused significant problems. The scanning process is sensitive and what appears on screen is often the original diagram superimposed on the new one making it very difficult to read or follow. Candidates should instead cross out the old working and draw a new diagram, using an additional sheet of paper if necessary.

Comments on individual questions

- 1) *(Networks)*
This question on Dijkstra proved to be an easy starter, although few scored the single mark in part (ii).
- 2) *(Algorithms)*
This was also an easy question. Most candidates were successful with parts (i) and (ii), but some failed with part (iii). “Lowest Common Multiple” was the commonest error. Candidates should note that an algorithm describes a set of rules that need to be followed exactly. A minority of candidates failed to delete rows when asked, forfeiting marks.
- 3) *(Graph Theory)*
A large proportion of candidates failed with part (i). Most succeeded with part (ii). Hardly anyone collected the single mark for part (iii).

There were hints within the question about part (iii). In part (i) candidates were required to show that the graph was planar. In part (ii) they were told that the graph was planar. Few candidates were able to see that if this modelling was correct, then we could do without flyovers, since flows could be organised not to intersect in two dimensions. We know this is not the case, so something must be amiss. It can be seen, for instance, that the inevitable “loop around the outside” is not possible in reality because of incoming and outgoing traffic flows, but only a very few candidates could see that.

4) *(Linear Programming)*

Compared with recent LP questions, answers to this were disappointing.

In part (i) it was enormously disappointing to see just how few candidates could cope with the area computations which were required. One would really have expected that AS candidates would have been able to give the area covered by a large tile, the area covered by a box of small tiles, and the area of the wall. This was not the case for the majority of candidates.

Most managed the easier inequality in part (ii). Few were able correctly to sort out the area issues for the second inequality; if they could not manage part (i) then part (ii) would clearly be closed to them.

There were plenty of marks available in part (iii), with lots of follow-through marks, including for $x \leq y$. However, the majority of candidates did not manage to achieve any sensible optimisation, which is de rigueur for this question. Few obtained the correct objective function. A disappointing number of students failed even to draw their own linear functions accurately, and many failed to shade accurately.

The question in part (iv) was very specific; it asked for two other factors which would *HAVE* to be taken into account in deciding how many of each tile to purchase. Only answers which related to wastage or design were deemed to be acceptable; others were regarded as flights of fancy, and some were very fanciful indeed! Future candidates would do well to appreciate that this sort of question aims to elicit a comment on the modelling; they are not intended to test the candidates' powers of imagination.

5) *(Simulation)*

Part (i) created more of a problem than had been expected. Candidates were asked how to simulate the classic "drunkard's walk". Almost to a person they described how to simulate a step, but neglected to accumulate those steps into a walk.

The majority then negotiated part (ii) successfully.

In part (iii) many then displayed a similar inadequacy to that seen in part (i). Readers are reminded that the question was how to estimate the probability of the drunkard falling into the canal within six steps. The often seen answer can be paraphrased as "Repeat the simulation and work out the probability from the repetitions".

Parts (iv) and (v) were generally good sources of marks.

6) *(Critical Path Analysis)*

There were the inevitable few who produced activity-on-node digraphs, or activity-on-node digraphs which masqueraded as activity-on-arc, or activity-on-arc masquerading as activity-on-node, or graphs. But most candidates did very well in parts (i) and (ii). Note that activity-on-node scored zero in part (i) and a maximum of two in part (ii).

Hardly any correct answers were seen to part (iii). Again it had been anticipated that these three marks would be difficult and discriminating, but not to the extent which they proved.

Finally, it was anticipated that the last two marks, in part (iv), would be found to be easy. Indeed more candidates scored them than scored those in part (iii), but again, far fewer than expected.