OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4771/01**

MATHEMATICS (MEI)

Decision Mathematics 1

THURSDAY 12 JUNE 2008

Morning
Time: 1 hour 30 minutes

Additional materials: Printed Answer Book (enclosed)
MEI Examination Formulae and Tables (MF2)

CUP/T44383

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks for each question is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **8** printed pages.

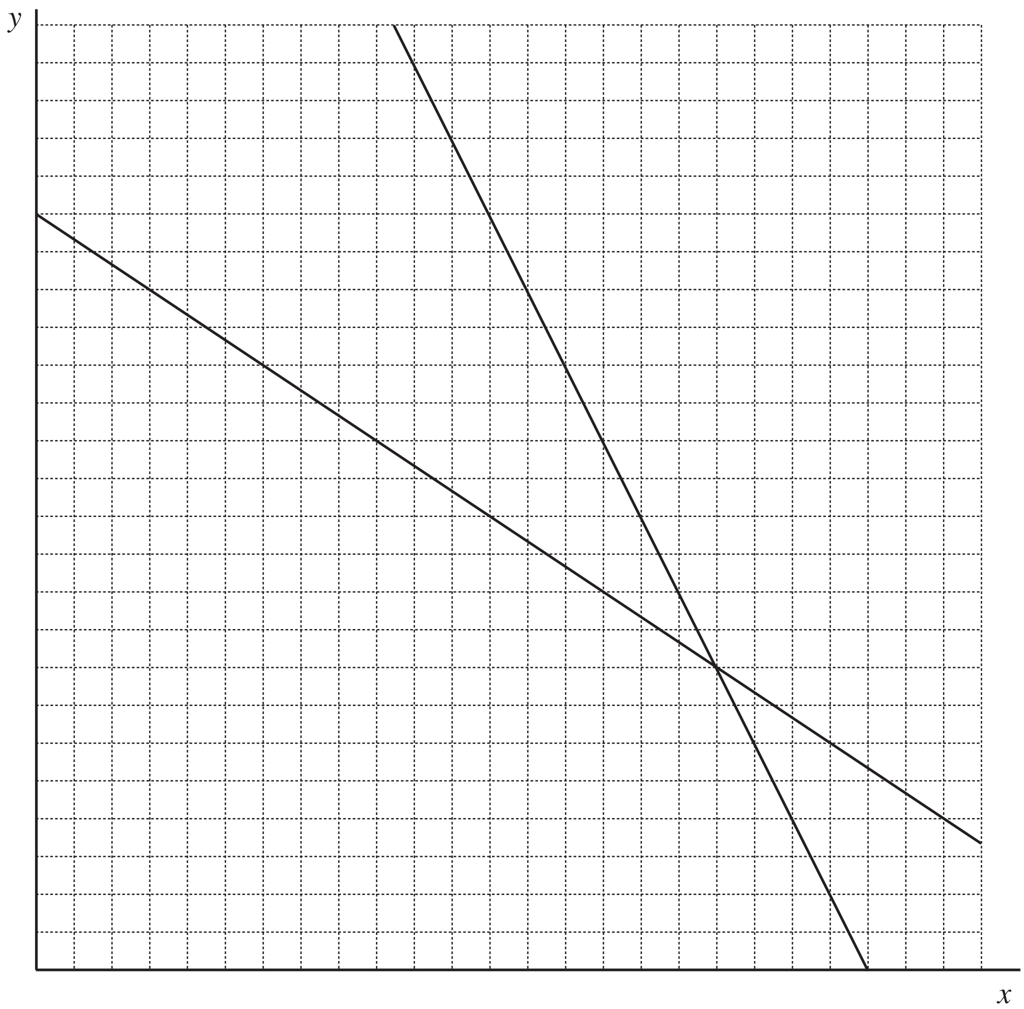
Answer all the questions in the printed answer book provided.

Section A (24 marks)

1 Consider the following LP.
Maximise $x + y$
subject to $2x + y < 44$
$2x + 3y < 60$
$10x + 11y < 244$

Part of a graphical solution is produced below and in your answer book.

Complete this graphical solution in your answer book. [8]



2 The following algorithm acts on a list of three or more numbers.

Step 1: Set both X and Y equal to the first number on the list.

Step 2: If there is no next number then go to Step 5.

Step 3: If the next number on the list is bigger than X then set X equal to it. If it is less than Y then set Y equal to it.

Step 4: Go to Step 2.

Step 5: Delete a number equal to X from the list and delete a number equal to Y from the list.

Step 6: If there is one number left then record it as the answer and stop.

Step 7: If there are two numbers left then record their mean as the answer and stop.

Step 8: Go to Step 1.

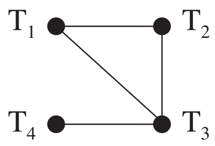
(i) Apply the algorithm to the list 5, 14, 153, 6, 24, 2, 14, 15, counting the number of comparisons which you have to make. [3]

(ii) Apply the algorithm to the list 5, 14, 153, 6, 24, 2, 14, counting the number of comparisons which you have to make. [3]

(iii) Say what the algorithm is finding. [1]

(iv) The order of the algorithm is quadratic. Explain what this means when it is applied to long lists. [1]

3 The graph represents four towns together with (two-way) roads connecting them.



A *path* is a set of connected arcs linking one vertex to another. A path contains no repeated vertex.

$T_1 \rightarrow T_2$ and $T_1 \rightarrow T_3 \rightarrow T_2$ are paths.

(i) There are six paths from T_1. List them. [2]

(ii) List the paths from T_4. [2]

(iii) How many paths are there altogether? [2]

For this question a *route* is defined to be a path in which the direction of the arcs is not relevant. Thus $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$ are the same route. Similarly $T_1 \rightarrow T_3 \rightarrow T_2$ and $T_2 \rightarrow T_3 \rightarrow T_1$ are the same route (but note that $T_1 \rightarrow T_2 \rightarrow T_3$ is different).

(iv) How many routes are there altogether? [2]

Section B (48 marks)

4 Joe is to catch a plane to go on holiday. He has arranged to leave his car at a car park near to the airport. There is a bus service from the car park to the airport, and the bus leaves when there are at least 15 passengers on board. Joe is delayed getting to the car park and arrives needing the bus to leave within 15 minutes if he is to catch his plane. He is the 10^{th} passenger to board the bus, so he has to wait for another 5 passengers to arrive.

The distribution of the time intervals between car arrivals and the distribution of the number of passengers per car are given below.

Time interval between cars (minutes)	1	2	3	4	5
Probability	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

Number of passengers per car	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$

(i) Give an efficient rule for using 2-digit random numbers to simulate the intervals between car arrivals. [3]

(ii) Give an efficient rule for using 2-digit random numbers to simulate the number of passengers in a car. [4]

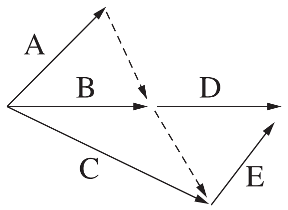
(iii) The incomplete table in your answer book shows the results of nine simulations of the situation. Complete the table, showing in each case whether or not Joe catches his plane. [3]

(iv) Use the random numbers provided in your answer book to run a tenth simulation. [4]

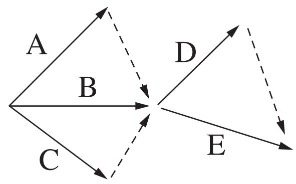
(v) Estimate the probability of Joe catching his plane. State how you could improve your estimate. [2]

5 **(a)** The graphs below illustrate the precedences involved in running two projects, each consisting of the same activities A, B, C, D and E.

Project 1



Project 2



(i) For one activity the precedences in the two projects are different. State which activity and describe the difference. [3]

(ii) The table below shows the durations of the five activities.

Activity	A	B	C	D	E
Duration	2	1	x	3	2

Give the total time for project 1 for all possible values of x.
Give the total time for project 2 for all possible values of x. [3]

(b) The durations and precedences for the activities in a project are shown in the table.

Activity	Duration	Immediate predecessors
R	2	–
S	1	–
T	5	–
W	3	R, S
X	2	R, S, T
Y	3	R
Z	1	W, Y

(i) Draw an activity on arc network to represent this information. [4]

(ii) Find the early time and the late time for each event. Give the project duration and list the critical activities. [6]

6 The matrix gives the lengths of the arcs of a network.

	A	B	C	D	E	F
A	–	10	7	–	9	5
B	10	–	–	1	–	4
C	7	–	–	–	3	–
D	–	1	–	–	2	–
E	9	–	3	2	–	–
F	5	4	–	–	–	–

(i) Using the copy of the matrix in your answer book, apply the tabular form of Prim's algorithm to find a minimum connector for the network. Start by choosing vertex A and show the order in which you include vertices.
List the arcs in your connector and give its total length. [6]

Serena takes a different approach to find a minimum connector. She first uses Dijkstra's algorithm to find shortest paths from A to each of the other vertices. She then uses the arcs in those paths to construct a connector.

(ii) Draw the network using the vertices printed in your answer book. [2]

(iii) Apply Serena's method to produce a connector.
List the arcs in the connector and give its total length. [6]

Serena adapts her method by starting from each vertex in turn, producing six connectors, from which she chooses the best.

(iv) Serena's approach will not find the minimum connector in all networks, but it is an algorithm. What is its algorithmic complexity? Justify your answer. [2]

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PRINTED ANSWER BOOK

THURSDAY 12 JUNE 2008

Morning

Time: 1 hour 30 minutes

CUP/T44383

Candidate Forename		Candidate Surname	
Centre Number		Candidate Number	

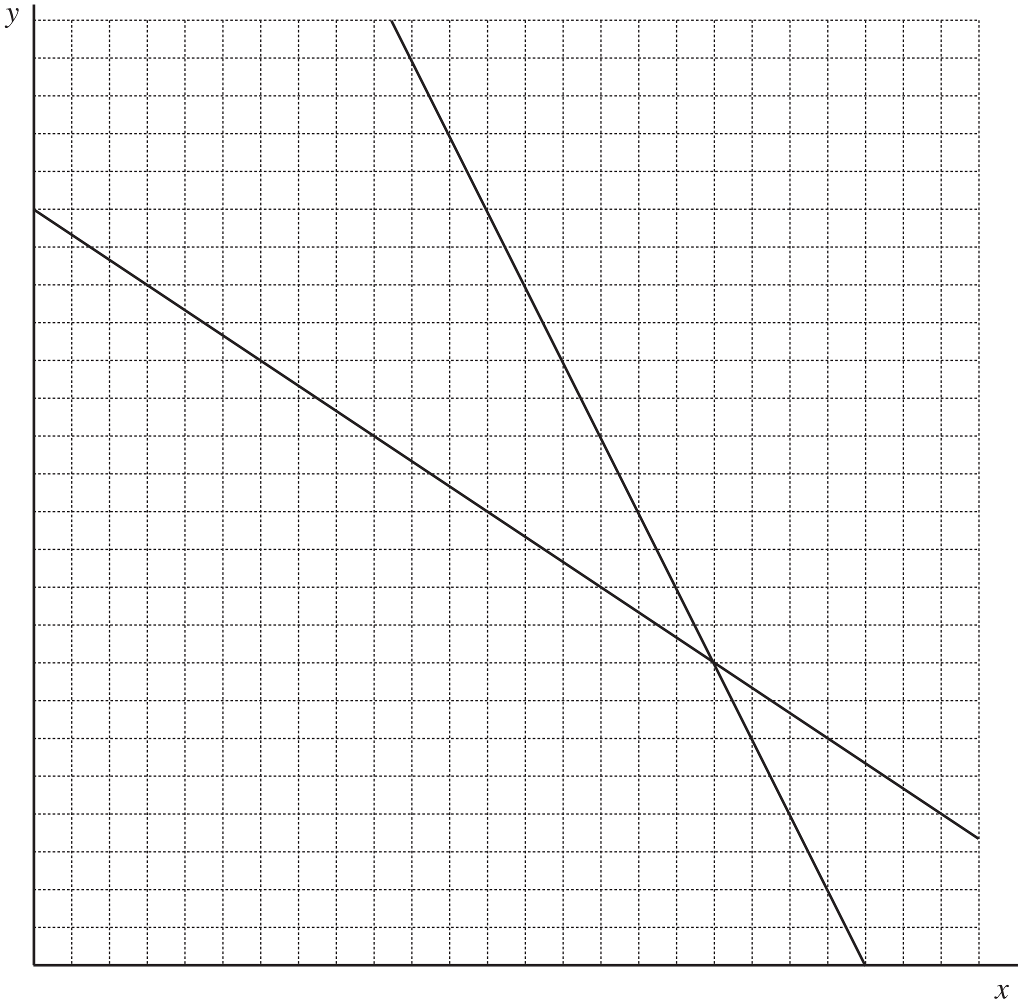
INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- Write your answers in the spaces provided on the printed answer book. If extra paper is required use a 4 page answer booklet making sure that you label your work clearly.

FOR EXAMINER'S USE	
Qu.	**Mark**
1	
2	
3	
4	
5	
6	
TOTAL	

This document consists of **7** printed pages and **1** blank page.

1



Optimal point: ....

Optimal value: ....

2 **(i)** (Do the repeated step 3s all on one line of the table.)

List	X	Y
5, 14, 153, 6, 24, 2, 14, 15		

Answer = ...

Number of comparisons: ....

(ii)

List	X	Y
5, 14, 153, 6, 24, 2, 14		

Answer = ...

Number of comparisons: ....

(iii)

(iv)

3 **(i)**

(ii)

(iii)

(iv)

4 (i)

(ii)

(iii) & (iv)

Simulation number	Cars arriving after Joe – time interval / number of passengers										Time to bus leaving (mins)	Catches plane?
1	3	2	2	1	1	2	2	2	3	1	6	yes
2	3	1	2	2	1	4	1	2	5	1		
3	5	1	2	2	2	1	3	4	2	2		
4	4	6	3	2	4	1	1	2	2	3		
5	5	1	4	1	3	2	5	4	2	2		
6	4	4	4	2	5	3	1	4	1	4		
7	4	1	4	2	3	1	5	4	1	3		
8	2	2	2	2	2	4	3	5	1	2		
9	1	1	1	1	1	1	1	1	1	2		
10												

Random numbers for simulation number 10:

For time intervals – 31 45 36 22 10 48

For the number of passengers – 65 97 47 90 82 07

(v)

[Turn over

5 (a) (i)

(ii)

(b) (i)

(ii)

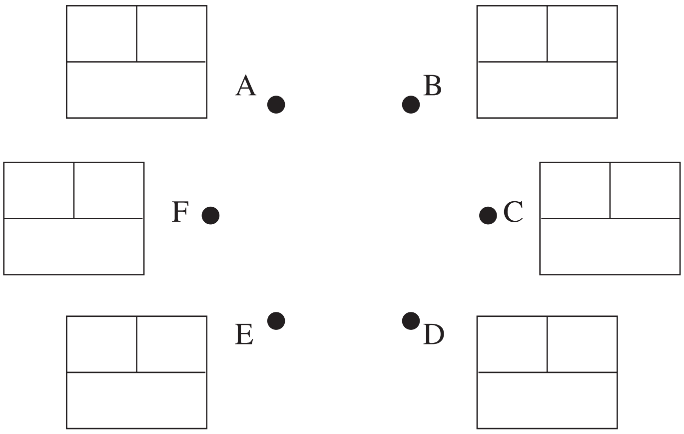
6 (i)

Order of inclusion	1					
	A	B	C	D	E	F
A	–	10	7	–	9	5
B	10	–	–	1	–	4
C	7	–	–	–	3	–
D	–	1	–	–	2	–
E	9	–	3	2	–	–
F	5	4	–	–	–	–

Arcs:

Length:

(ii) & (iii)



Arcs:

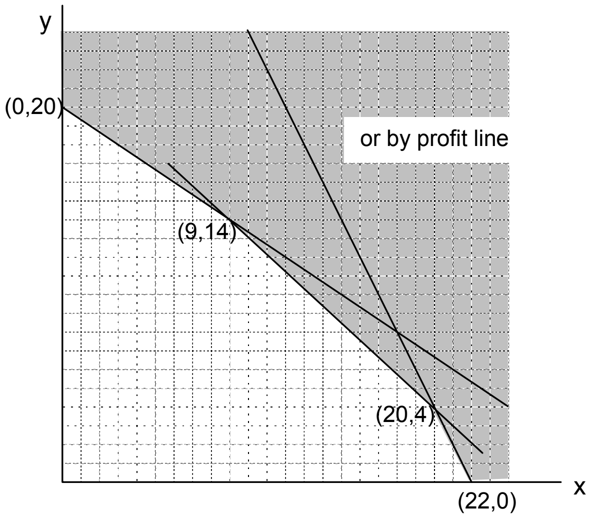
Length:

(iv)

4771 Decision Mathematics 1

Solutions

1.

 Objective has maximum value of 24 at (20,4)	M1 A1 third line B1 shading B1 (0,20) and (22,0) B1 (9,14) B1 (20,4) M1 A1 solution or M1 A1 B1, B1 scale (implied OK), B1 profit line, B1 (20,4) M1 A1 (20,4) A1 (24)

2.

(i)

	X	Y
5, 14, 153, 6, 24, 2, 14, 15	5, 14, 153	5, 2
5, 14, 6, 24,14, 15	5, 14, 24	5
14, 6, 14, 15,	14, 15	14, 6
14, 14		

M1

Answer = 14 — A1
Comparisons = 30 — A1

(ii)

	X	Y
5, 14, 153, 6, 24, 2, 14	5, 14, 153	5, 2
5, 14, 6, 24,14	5, 14, 24	5
14, 6, 14	14	14, 6
14		

M1

Answer = 14 — A1
Comparisons = 24 — A1

(iii) Median — B1

(iv) Time taken approximately proportional to square of length of list (or twice length takes four times the time, or equivalent). — B1

3.

(i)	$T_1 \to T_2$ $T_1 \to T_3 \to T_2$ $T_1 \to T_3$ $T_1 \to T_2 \to T_3$ $T_1 \to T_2 \to T_3 \to T_4$ $T_1 \to T_3 \to T_4$	M1 A1
(ii)	$T_4 \to T_3 \to T_2 \to T_1$ $T_4 \to T_3 \to T_1$ $T_4 \to T_3 \to T_1 \to T_2$ $T_4 \to T_3 \to T_2$ $T_4 \to T_3$	M1 A1
(iii)	22	M1 allow for 23 A1
(iv)	11	M1 halving (not 11.5) A1

4.

(i) e.g.

00–09→1
10–39→2
40–79→3
80–89→4
90–99→5

M1
A1 proportions OK
A1 efficient

(ii) e.g.

00–15→1
16–47→2
48–55→3
56–79→4
80–87→5
88–95→6
96, 97, 98, 99 reject

M1 some rejected
A2 proportions OK (–1 each error)
A1 efficient

(iii) & (iv)

Sim. no.	Cars arriving after Joe – time interval | number of passengers										Time to 15 passengers (minutes)
1	3	2	2	1	1	2	2	2	3	1	6
2	3	1	2	2	1	4	1	2	5	1	6
3	5	1	2	2	2	1	3	4	2	2	12
4	4	6	3	2	4	1	1	2	2	3	4
5	5	1	4	1	3	2	5	4	2	2	17
6	4	4	4	2	5	3	1	4	1	4	8
7	4	1	4	2	3	1	5	4	1	3	16
8	2	2	2	2	2	4	3	5	1	2	6
9	1	1	1	1	1	1	1	1	1	2	5
10	2	4	3	2	2	6	2	5	2	1	5

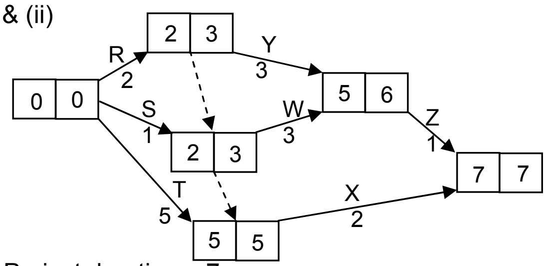
M1
A2 (–1 each error)

M1 simulation
A1 time intervals
A1 passengers
A1 time to wait

(v) 0.8
more runs

B1
B1

5.

(a)(i) Activity D. Depends on A and B in project 1, but on A, B and C in project 2.	M1 A1 A1
(ii) Project 1: Duration is 5 for x<3, thence x+2. Project 2: Duration is 5 for x<2, thence x+3	B1 "5" B1 B1 beyond 5
(b) (i) & (ii)  Project duration – 7 Critical activities – T, X	M1 activity-on-arc A1 single start and single end A2 precedences (–1 each error) M1 A1 forward pass M1 A1 backward pass B1 B1

6.

(i)

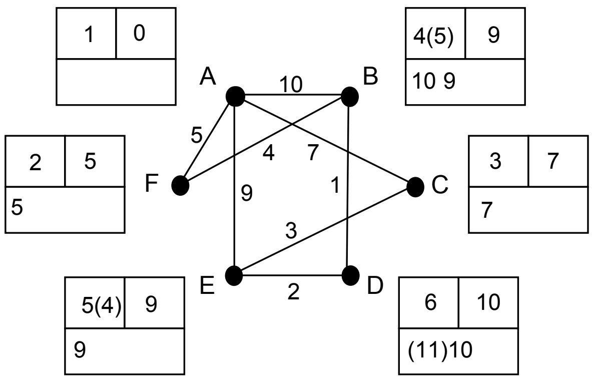
Order of inclusion	1	3	6	4	5	2
	A	B	C	D	E	F
A	–	10	7	–	9	5
B	10	–	–	1	–	4
C	7	–	–	–	3	–
D	–	1	–	–	2	–
E	9	–	3	2	–	–
F	5	4	–	–	–	–

M1
A1 select
A1 delete
A1 order

Arcs: AF, FB, BD, DE, EC — B1

Length: 15 — B1

(ii) & (iii)



B1 arcs
B1 lengths

M1 Dijkstra
A1 working values
A1 order of labelling
A1 labels

Arcs: AF, FB, BD, AC, AE — M1
Length: 26 — A1

(iv) Cubic — B1

n applications of Dijkstra, which is quadratic — B1