

**ADVANCED SUBSIDIARY GCE UNIT
MATHEMATICS (MEI)**

Decision Mathematics 1

MONDAY 18 JUNE 2007

4771/01

Morning
Time: 1 hour 30 minutes

Additional materials:
Printed Answer Book
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the printed answer book.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Answer all the questions in the printed answer book provided.

Section A (24 marks)

- 1** Bus routes connect towns A and B and towns A and C.

Train lines connect towns B and D, towns C and D, and towns A and C.

John represents this information in a graph with four nodes, one for each town, in which an arc is drawn for each connection, giving five arcs in all.

(i) Draw John's graph. [2]

(ii) Is John's graph simple? Justify your answer. [2]

Jamil represents the information in a graph with five nodes. He uses one node for each of towns A, B and D. Because in town C the bus station and train station are some distance apart, he uses a node labelled C(bus) and a node labelled C(train). Again there are 5 arcs, each representing a connection.

(iii) Draw Jamil's graph. [2]

(iv) Is Jamil's graph a tree? Justify your answer. [2]

- 2** Two hikers each have a 25 litre rucksack to pack. The items to be packed have volumes of 14, 6, 11, 9 and 6 litres.

(i) Apply the first fit algorithm to the items in the order given and comment on the outcome. [3]

(ii) Write the five items in descending order of volume. Apply the first fit decreasing algorithm to find a packing for the rucksacks. [3]

(iii) The hikers argue that the first fit decreasing algorithm does not produce a fair allocation of volumes to rucksacks. Produce a packing which gives a fairer allocation of volumes between the two rucksacks. Explain why the hikers might **not** want to use this packing. [2]

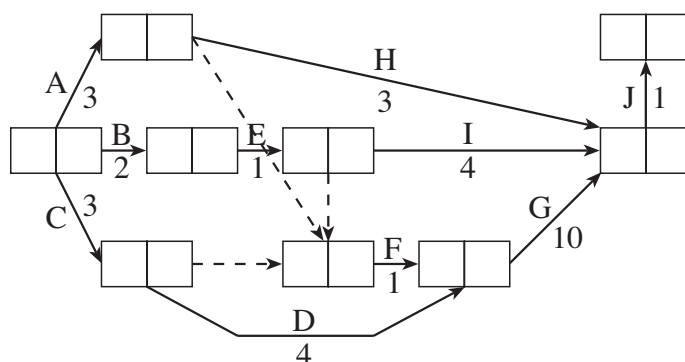
- 3** Use a graphical approach to solve the following LP. [8]

$$\begin{array}{ll}
 \text{Maximise} & 2x + 3y \\
 \text{subject to} & x + 5y \leq 14 \\
 & x + 2y \leq 8 \\
 & 3x + y \leq 21 \\
 & x \geq 0 \\
 & y \geq 0
 \end{array}$$

Section B (48 marks)

- 4 Colin is setting off for a day's sailing. The table and the activity network show the major activities that are involved, their durations and their precedences.

A	Rig foresail
B	Lower sprayhood
C	Start engine
D	Pump out bilges
E	Rig mainsail
F	Cast off mooring ropes
G	Motor out of harbour
H	Raise foresail
I	Raise mainsail
J	Stop engine and start sailing



- (i) Complete the table in your answer book showing the immediate predecessors for each activity. [3]

- (ii) Find the early time and the late time for each event. Give the project duration and list the critical activities. [6]

When he sails on his own Colin can only do one thing at a time with the exception of activity G, motoring out of the harbour.

- (iii) Use the activity network to determine which activities Colin can perform whilst motoring out of the harbour. [1]
- (iv) Find the minimum time to complete the activities when Colin sails on his own, and give a schedule for him to achieve this. [2]
- (v) Find the minimum time to complete the activities when Colin sails with one other crew member, and give a schedule for them to achieve this. [4]

- 5 The table shows the weights on the arcs of a network.

	A	B	C	D	E	F	G
A	–	11	–	–	10	3	5
B	11	–	8	5	–	–	14
C	–	8	–	2	–	7	–
D	–	5	2	–	6	–	–
E	10	–	–	6	–	6	–
F	3	–	7	–	6	–	–
G	5	14	–	–	–	–	–

- (i) Draw the network. [3]

- (ii) Apply Dijkstra's algorithm to find the least weight route from G to D. (Do this on the network you drew for part (i).)

Give your route and its total weight. [6]

- (iii) Find by inspection the route from G to D such that the minimum of the weights for arcs on the route is as large as possible. Give your route and its minimum arc weight. Give an application in which this might be needed. [3]

- (iv) Consider how Dijkstra's algorithm could be modified to solve the problem in part (iii). Explain how to update working values. Explain how to select the next vertex to be permanently labelled. [4]

- 6 In winter in Metland the weather each day can be classified as dry, wet or snowy. The table shows the probabilities for the next day's weather given the current day's weather.

		next day's weather		
		dry	wet	snowy
current day's weather	dry	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{3}{10}$
	wet	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{3}{10}$
	snowy	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{3}{7}$

You are to use two-digit random numbers to simulate the winter weather in Metland.

- (i) Give an efficient rule for using two-digit random numbers to simulate tomorrow's weather if today is

(A) dry, [2]

(B) wet, [2]

(C) snowy. [3]

- (ii) Today is a dry winter's day in Metland. Use the following two-digit random numbers to simulate the next 7 days' weather in Metland. [4]

23 85 98 99 56 47 82 14 03 12

- (iii) Use your simulation from part (ii) to estimate the proportion of dry days in a Metland winter. [1]

- (iv) Explain how you could use simulation to produce an improved estimate of the proportion of dry days in a Metland winter. [2]

- (v) Give two criticisms of this model of weather. [2]

**ADVANCED SUBSIDIARY GCE UNIT
MATHEMATICS (MEI)**

Decision Mathematics 1

ANSWER BOOK

MONDAY 18 JUNE 2007

4771/01

Morning
Time: 1 hour 30 minutes

Candidate
Name

Centre
Number

--	--	--	--	--

Candidate
Number

--	--	--	--

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided above.
- Write your answers in the spaces provided on the answer book. If extra space is required use the blank page making sure that you label your work clearly.

For Examiner's Use	
Qu.	Mark
1	
2	
3	
4	
5	
6	
Total	

This answer book consists of **7** printed pages and **1** blank page.

1 (i)

(ii)

(iii)

(iv)

2 (i) Rucksack 1:

Rucksack 2:

Comment:

(ii) Order:

Rucksack 1:

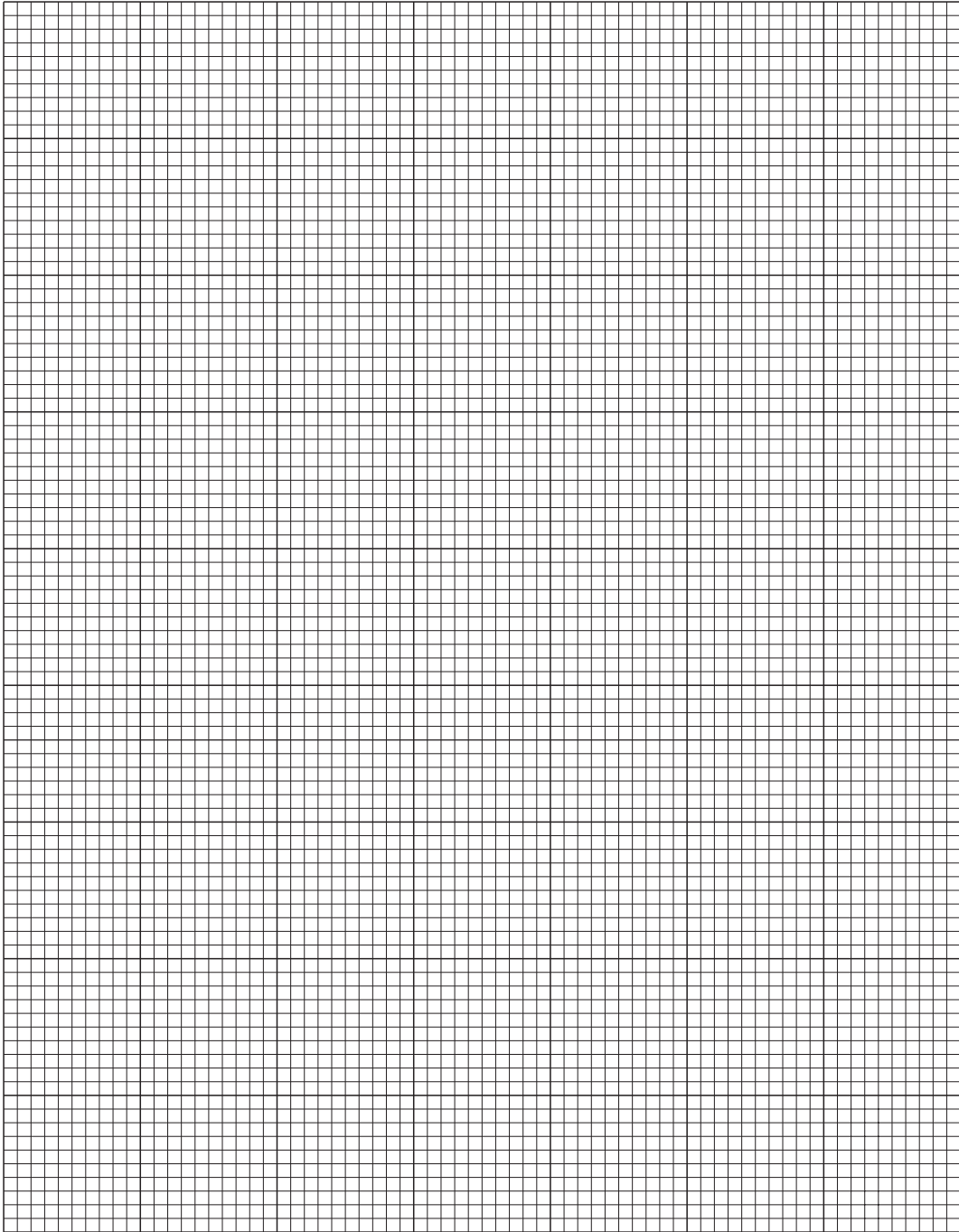
Rucksack 2:

(iii) Rucksack 1:

Rucksack 2:

Explanation:

3



Maximum value of $2x + 3y$:

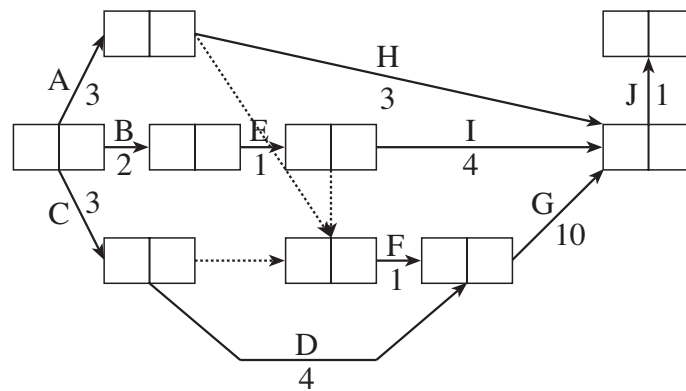
This occurs where $x =$

and where $y =$

4 (i)

Activity		Duration (minutes)	Immediate predecessors
A	Rig foresail	3	
B	Lower sprayhood	2	
C	Start engine	3	
D	Pump out bilges	4	
E	Rig mainsail	1	
F	Cast off mooring ropes	1	
G	Motor out of harbour	10	
H	Raise foresail	3	
I	Raise mainsail	4	
J	Stop engine and start sailing	1	

(ii)



Project duration:

Critical activities:

(iii) Activities whilst motoring:

(iv) Minimum time:

Schedule for Colin:

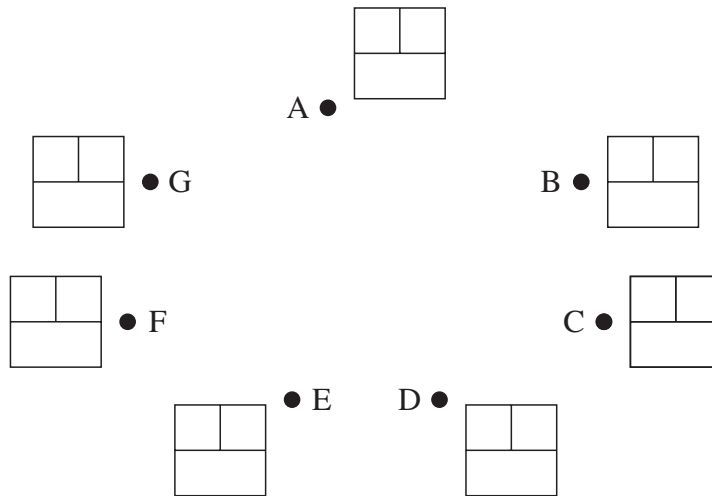
(v) Minimum time:

Schedule for Colin:

Schedule for crew member:

6

5 (i) & (ii)



Route:

Weight:

(iii) Route:

Weight:

Application:

(iv) Working value update

Next vertex to label

6 (i) (A)

(B)

(C)

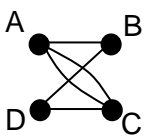
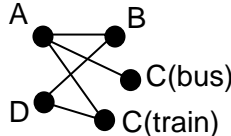
(ii)

(iii)

(iv)

(v)

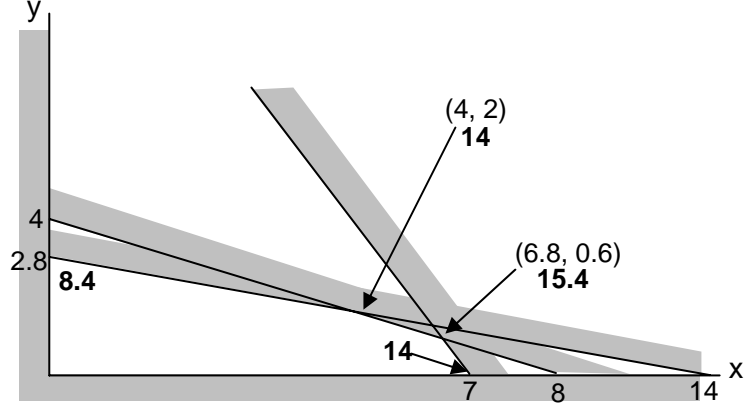
1.

<p>(i)</p> 	<p>M1 4 nodes and 5 arcs A1</p>
<p>(ii) No. Two arcs AC.</p>	<p>M1 A1</p>
<p>(iii)</p> 	<p>M1 5 nodes and 5 arcs A1</p>
<p>(iv) No. ABDC(train)A is a cycle.</p>	<p>M1 A1</p>

2.

<p>(i) Rucksack 1: 14; 6 Rucksack 2: 11; 9 final item will not fit.</p>	<p>M1 6 must be in R1 A1 B1</p>
<p>(ii) Order: 14, 11, 9, 6, 6 Rucksack 1: 14; 11 Rucksack 2: 9; 6; 6</p>	<p>B1 ordering M1 11 in R1 A1</p>
<p>(iii) Rucksack 1: 14; 9 Rucksack 2: 11; 6; 6 e.g. weights.</p>	<p>B1 B1</p>

3.

 <p>Optimum of 15.4 at $x = 6.8$ and $y = 0.6$.</p>	<p>B1 axes scaled & used M1 lines A1 B1 shading M1 two intersection A1 points M1 solution A1 (or by using the objective gradient to identify the optimal point)</p>
--	---

4.

(i)

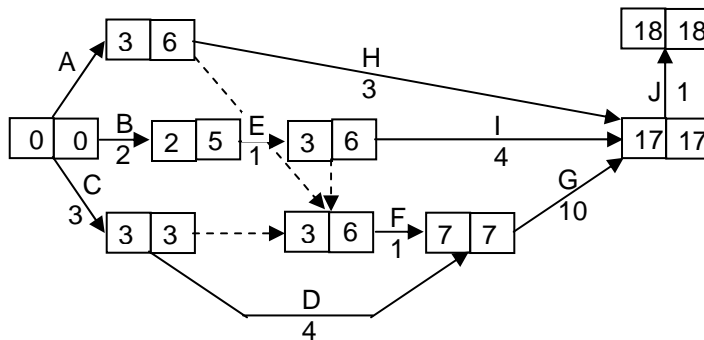
Activity		Duration (minutes)	Immediate predecessors
A	Rig foresail	3	—
B	Lower sprayhood	2	—
C	Start engine	3	—
D	Pump out bilges	4	C
E	Rig mainsail	1	B
F	Cast off mooring ropes	1	A, C, E
G	Motor out of harbour	10	D, F
H	Raise foresail	3	A
I	Raise mainsail	4	E
J	Stop engine and start sailing	1	G, H, I

B1 A, B, C,
D, E, H & I

B1 F

B1 G and J

(ii)



Critical activities: C; D; G; J
 Project duration: 18 minutes

M1 A1 forward pass

M1 A1 backward pass

(iii) H and I

B1

B1

(iv) 25 mins

B1

B1

Must do A, B, E, C, F, D (in appropriate order) then H and I with G, then J.

B1

(v) 18 mins

B1

e.g. Colin does C, D
 Crew does A, B, E, F
 Thence G et al

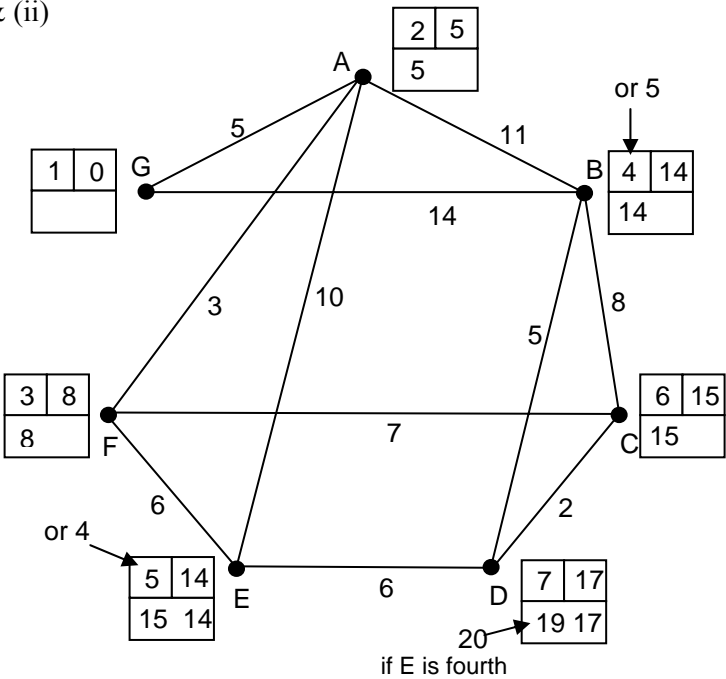
B1

B1

B1

5.

(i) & (ii)



Route: G A F C D Weight: 17

(iii) Route: G B C F E D or G B A E D Weight: 6
Any capacitated route application.

(iv) Compute $\min(\text{label}, \text{arc})$ and update working value if result is larger than current working value.
Label unlabelled vertex with largest working value.

M1
A1 arcs
A1 arc weights

M1 Dijkstra
A1 labels
A1 order of labelling
A2 working values

B1 B1

B1 B1
B1

B1 B1

B1

6.

(i)(a) e.g. Dry: 00 – 39 Wet: 40 – 69 Snowy: 70 – 99	M1 proportions A1 efficient
(b) e.g. Dry: 00 – 19 Wet: 20 – 69 Snowy: 70 – 99	M1 proportions A1 efficient
(c) e.g. Dry: 00 – 27 Wet: 28 – 55 Snowy: 56 – 97 Reject: 98 & 99	M1 reject some A1 proportions A1 reject 2
(ii) D (today) \rightarrow D \rightarrow S \rightarrow S \rightarrow W \rightarrow S \rightarrow D \rightarrow D	M1 applying their rules sometimes A1 dry rules A1 wet rules A1 snowy rules
(iii) 3/7 (or 4/8)	B1
(iv) a (much) longer simulation run, with a "settling in" period ignored.	B1 B1
(v) Defining days as dry, wet or snowy is problematical. Assuming that the transition probabilities remain constant. Weather depends on more than just previous day's weather	B1 B1