OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS   **4771**

Decision Mathematics 1

Friday   **14 JANUARY 2005**   Morning   1 hour 30 minutes

Additional materials:
  Answer booklet
  Graph paper
  MEI Examination Formulae and Tables (MF2)

TIME   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **2, 4** and **5**.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 7 printed pages, 1 blank page and an insert.

Section A

1   The bipartite graph in Fig. 1 represents a board game for two players. At each turn a player tosses a coin and moves their counter. The graph shows which square the counter is moved to if the coin shows heads, and which square if it shows tails. Each player starts with their counter on square 1. Play continues until one player gets their counter to square 9 and wins.
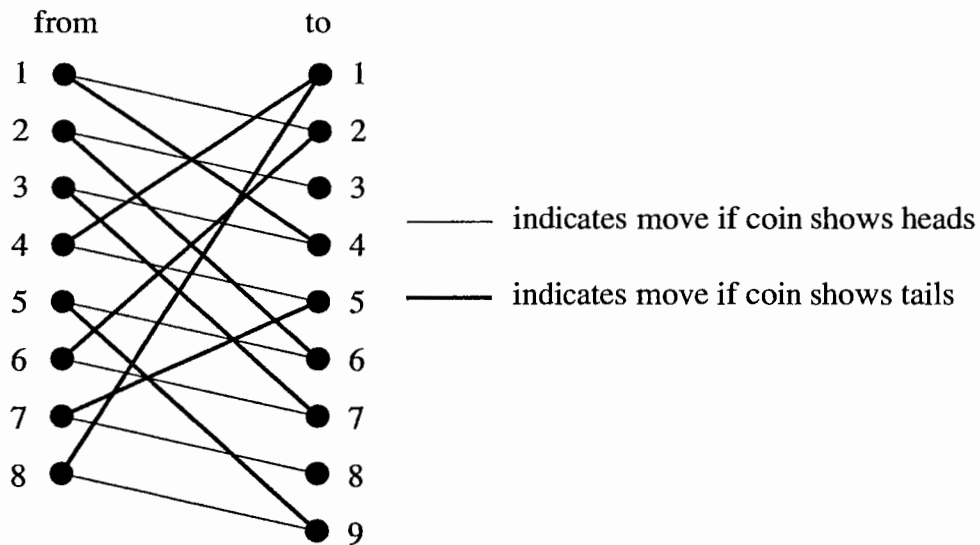

Fig. 1

   (i) Draw a tree to show all of the possibilities for the player's first three moves.   [4]

   (ii) Show how a player can win in 3 turns.   [1]

   (iii) List all squares which it is possible for a counter to occupy after 3 turns.   [2]

   (iv) Show that a game can continue indefinitely.   [1]

2   Answer this question on the insert provided.

(i) Use Dijkstra's algorithm to find the least weight route from A to G in the network shown in Fig. 2.1. Show the order in which you label vertices, give the route and its weight.   [5]
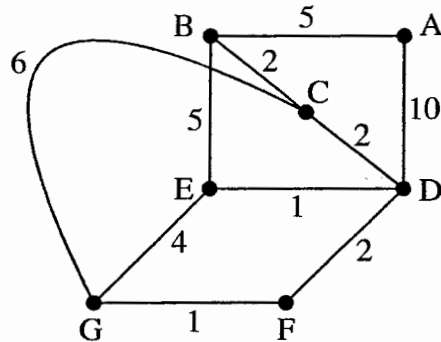

Fig. 2.1

(ii) Fig. 2.2 shows a partially completed application of Kruskal's algorithm to find a minimum spanning tree for the network.
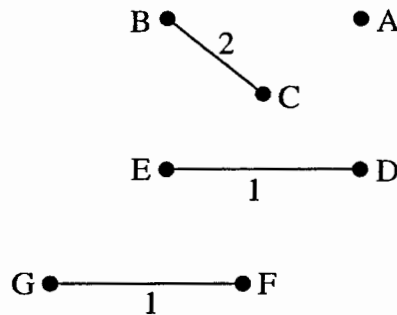

Fig. 2.2

Complete the algorithm and give the total weight of your minimum spanning tree.   [3]

[Turn over

3 The following algorithm finds the highest common factor of two positive integers. ("int (x)" stands for the integer part of x, e.g. int (7.8) = 7.)
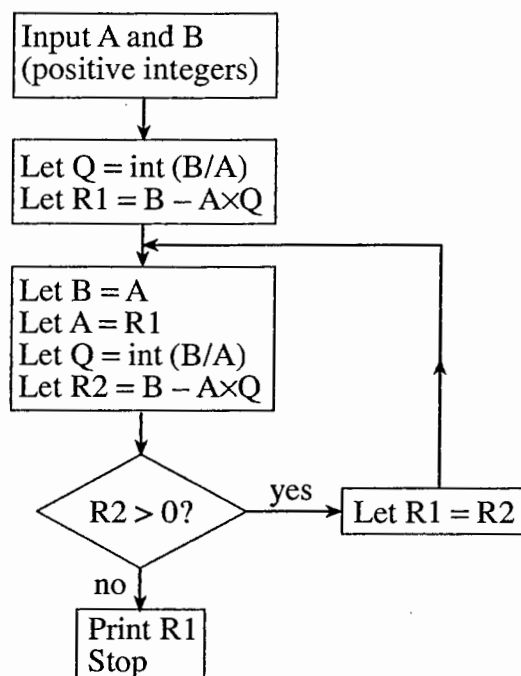
```
Input A and B
(positive integers)
        |
        v
Let Q = int (B/A)
Let R1 = B − A×Q
        |
        v
Let B = A
Let A = R1
Let Q = int (B/A)
Let R2 = B − A×Q
        |
        v
   < R2 > 0? >  --yes-->  Let R1 = R2
        |
       no
        v
   Print R1
   Stop
```

Fig. 3.1

(i) Run the algorithm with A = 84 and B = 660, showing all of your calculations. [3]

(ii) Run the algorithm with A = 660 and B = 84, showing as many calculations as are necessary. [2]

(iii) The algorithm is run with A = 30 and B = 42, and the result is shown in Table 3.2 below.

A	B	Q	R1	R2
30	42	1	12	
12	30	2		6
			6	
6	12	2		0

Print 6

Table 3.2

The first line of the table shows that $12 = 42 - 1 \times 30$.

Use the second line to obtain a similar expression for 6 in terms of 30 and 12.

Hence express 6 in the form $m \times 30 - n \times 42$, where m and n are integers. [3]

Section B

4   **Answer this question on the insert provided.**

The table shows activities involved in a "perm" in a hair salon, their durations and immediate predecessors.

	Activity	Duration (mins)	Immediate predecessor(s)
A	shampoo	5	–
B	prepare perm lotion	2	–
C	make coffee for customer	3	–
D	trim	5	A
E	clean sink	3	A
F	put rollers in	15	D
G	clean implements	3	D
H	apply perm lotion	5	B, F
I	leave to set	20	C, H
J	clean lotion pot and spreaders	3	H
K	neutralise and rinse	10	I, E
L	dry	10	K
M	wash up and clean up	15	K
N	style	4	G, L

Table 4

(i) Complete the activity-on-arc network in the insert to represent the precedences.   [5]

(ii) Perform a forward pass and a backward pass to find early and late event times. Give the critical activities and the time needed to complete the perm.   [6]

(iii) Give the total float time for the activity G.   [1]

Activities D, F, H, K and N require a stylist.

Activities A, B, C, E, G, J and M are done by a trainee.

Activities I and L require no-one in attendance.

A stylist and a trainee are to give a perm to a customer.

(iv) Use the chart in the insert to show a schedule for the activities, assuming that all activities are started as early as possible.   [3]

(v) Which activity would be better started at its latest start time?   [1]

[Turn over

5 **There is an insert for use in parts (iii) and (iv) of this question.**

This question concerns the simulation of cars passing through two sets of pedestrian controlled traffic lights. The time intervals between cars arriving at the first set of lights are distributed according to Table 5.1.

Time interval (seconds)	2	5	15	25
Probability	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Table 5.1

(i) Give an efficient rule for using two-digit random numbers to simulate arrival intervals. [3]

(ii) Use two-digit random numbers from the list below to simulate the arrival times of five cars at the first lights. The first car arrives at the time given by the first arrival interval.

Random numbers: 24, 01, 99, 89, 77, 19, 58, 42 [4]

The two sets of traffic lights are 23 seconds driving time apart. Moving cars are always at least 2 seconds apart. If there is a queue at a set of lights, then when the red light ends the first car in the queue moves off immediately, the second car 2 seconds later, the third 2 seconds after that, etc.

In this simple model there is to be no consideration of accelerations or decelerations, and the lights are either red or green.

Table 5.2 shows the times when the lights are red.

first set	red start time	14	50	105	155
of lights	red end time	29	65	120	170
second set	red start time	10	55	105	150
of lights	red end time	25	70	120	165

Table 5.2

(iii) Complete the table in the insert to simulate the passage of 10 cars through both sets of traffic lights. Use the arrival times given there. [5]

(iv) Find the mean delay experienced by these cars in passing through each set of lights. [3]

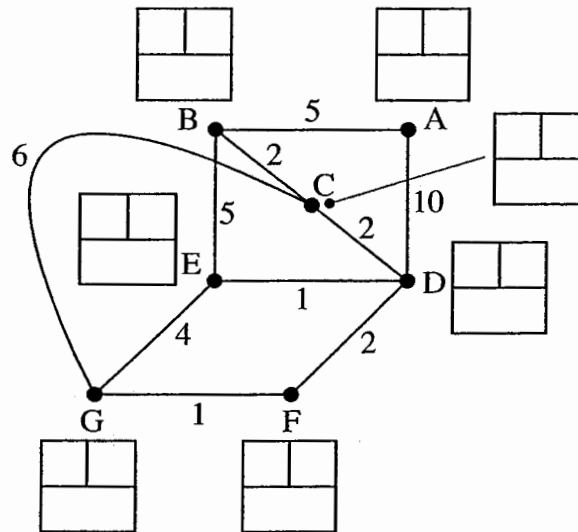(v) How could the output from this simulation model be made more reliable? [1]

6 A recipe for jam states that the weight of sugar used must be between the weight of fruit used and four thirds of the weight of fruit used. Georgia has 10 kg of fruit available and 11 kg of sugar.

(i) Define two variables and formulate inequalities in those variables to model this information.

[5]

(ii) Draw a graph to represent your inequalities. [5]

(iii) Find the vertices of your feasible region and identify the points which would represent the best mix of ingredients under each of the following circumstances.

(A) There is to be as much jam as possible, given that the weight of jam produced is the sum of the weights of the fruit and the sugar.

(B) There is to be as much jam as possible, given that it is to have the lowest possible proportion of sugar.

(C) There is to be as much jam as possible, given that it is to have the highest possible proportion of sugar.

(D) Fruit costs £1 per kg, sugar costs 50p per kg and the objective is to produce as much jam as possible within a budget of £15. [6]

Insert for question 2
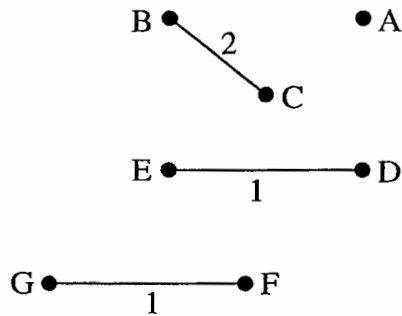
(i)

key: order of
labelling ───► □ ◄── label
            └────── working values

B •———— 5 ————• A
  \   \2
  6\   \
    \   C•
     \   \2   10
   5 │   \
     │   \
    E•——————• D
        1
     \   /
     4\   /2
       \   /
       G•——————• F
          1

Shortest route: _____   Weight: _____

(ii)

B •   • A
   \ 2
    \
     •C

E •————————• D
       1

G •————————• F
       1

Total weight of minimum connector: _____

Insert for question 4

(i) and (ii)

```
        ┌──┐
      B │  │
      2 │  │
        └──┤
   ┌──┐ A │
   │  │ 5 ├──── C
   │  │   │    3
   └──┘   │
```

(iii) Time to complete perm: _____

Critical activities: _____

(iv) Total float for activity G: _____

	10	20	30	40	50	60	70	80
stylist								
trainee								

(v) Activity better started at latest start time: _____

Spare copy

	10	20	30	40	50	60	70	80
stylist								
trainee								

[Turn over

4

Insert for question 5 (iii) and (iv)

Car no.	Arrival time at 1st lights	Departure time from 1st lights	Arrival time at 2nd lights	Departure time from 2nd lights
1	25			
2	27			
3	29			
4	44			
5	46			
6	48			
7	53			
8	78			
9	80			
10	85			
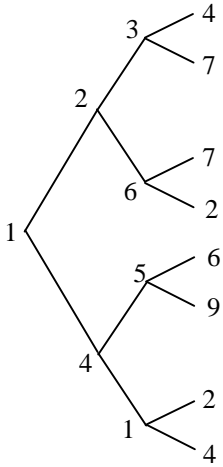
Mean delay at first set of lights: _____

Mean delay at second set of lights: _____

Spare copy

Car no.	Arrival time at 1st lights	Departure time from 1st lights	Arrival time at 2nd lights	Departure time from 2nd lights
1	25			
2	27			
3	29			
4	44			
5	46			
6	48			
7	53			
8	78			
9	80			
10	85			

1.

(i)	M1 A1 first branch A1 second branch A1 third branch
(ii) 1 (T) 4 (H) 5 (T) 9	B1
(iii) 2; 4; 6; 7; 9	M1 at least four different A1 complete and no repeats
(iv) e.g. 1 → 4 → 1 → …	B1

2.

(i)

ABCDFG   12

M1 Dijkstra
A1 order of labelling
A1 labels
A1 working values

B1 route and weight

(ii)

Total weight = 13

M1
A1

B1

3.

(i)	A	B	Q	R1	R2		
	84	660	7	72			M1
	72	84	1		12		A1
				12			
	12	72	6		0		
	Print 12						B1√ their 12
(ii)	A	B	Q	R1	R2		M1
	660	84	0	84			
	84	660	etc.				A1
(iii)	Line 2 says $6 = 30 - 2 \times 12$						B1
	Substituting $6 = 30 - 2 \times (42 - 30) = 3 \times 30 - 2 \times 42$						B1 3
							B1 2

4.

(i) & (ii)
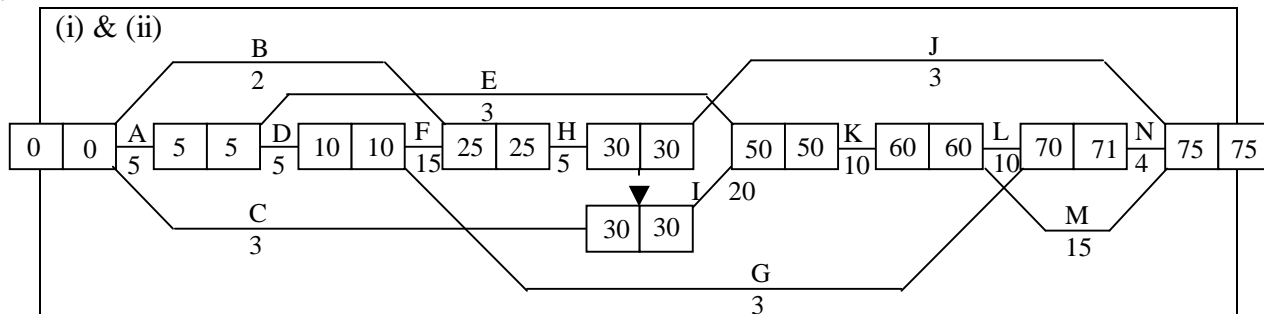
Critical: A, D, F, H, I, K, M   75 minutes

M1
A1 single start/end
A1 I and J
A1 A D F H K L N
A1 B C E G M

M1 A1 forward pass
M1 A1 backward pass
B1 B1 results

(iii)   Total float for G = 58

B1√

(iv)
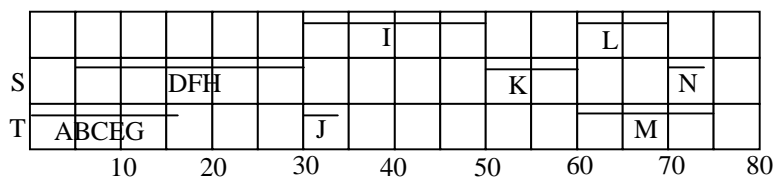
M1
A1 stylist
A1 trainee

(v)   C − make coffee

B1

5.

(i)	eg.	00–41 → 2		
		42–69 → 5		M1 some ignored
		70–83 → 15		A1 proportions
		84–97 → 25		A1 efficient
		98–99 → ignore		

(ii) 2   2   25   15   2   M1 A1 applying rule
      2   4   29   44   46   M1 A1 accumulating

(iii)

Car no.	Arrival time at 1st lights	Departure time from 1st lights	Arrival time at 2nd lights	Departure time from 2nd lights
1	25	29	52	52
2	27	31	54	54
3	29	33	56	70
4	44	44	67	72
5	46	46	69	74
6	48	48	71	76
7	53	65	88	88
8	78	78	101	101
9	80	80	103	103
10	85	85	108	120

M1
A1   departure 1 (1 error allowed)
A1   arrival 2 (ditto)
A1   departure 2 (ditto)
A1   all correct (cao)

(iv)   Mean delay at first lights = 2.4 seconds   M1 A1 (cao)

       Mean delay at second lights = 4.1 seconds   A1 (cao)

(v)   More repetitions (cars)   B1

6.

(i)   Let f be the number of kg of fruit used.
       Let s be the number of kg of sugar used.
 	
       s ≥ f
       s ≤ 4f/3
       f ≤ 10
       s ≤ 11
	B1
	
	B1
	B1
	B1
	B1

(ii)	B1 labels and scales B1 s = f B1 s = 4f/3 (√ ¾) B1 f = 10 and s = 11 B1 shading
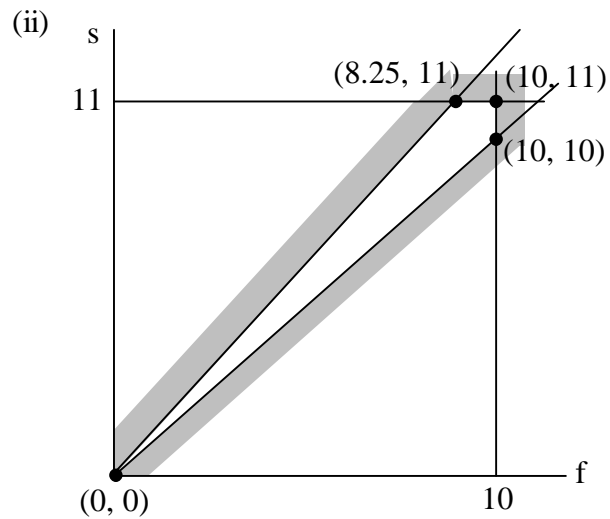
(iii) (0, 0), (8.25, 11), (10, 11), (10, 10)
 	
   (a)   21 kg at (10,11)
 	
   (b)   20 kg at 50% concentration at (10, 10)
 	
   (c)   19.25 kg at 4/7 concentration at (8.25, 11)
 	
   (d)   20.5 kg at (9.5, 11)
	B1√
	
	B1√
	
	B1√
	
	B1√
	
	M1 A1 (cao)