Concepts for Advanced Mathematics (C2)

Friday 22 May 2009
Duration: 1 hour 30 minutes

Section A (36 marks)

1 Use an isosceles right-angled triangle to show that $\cos 45° = \frac{1}{\sqrt{2}}$. **[2]**

2 Find $\int_1^2 (12x^5 + 5)\,dx$. **[4]**

3 **(i)** Find $\sum_{k=3}^{8} (k^2 - 1)$. **[2]**

(ii) State whether the sequence with kth term $k^2 - 1$ is convergent or divergent, giving a reason for your answer. **[1]**

4 A sector of a circle of radius 18.0 cm has arc length 43.2 cm.

(i) Find in radians the angle of the sector. **[2]**

(ii) Find this angle in degrees, giving your answer to the nearest degree. **[2]**

5 **(i)** On the same axes, sketch the graphs of $y = \cos x$ and $y = \cos 2x$ for values of x from 0 to 2π. **[3]**

(ii) Describe the transformation which maps the graph of $y = \cos x$ onto the graph of $y = 3\cos x$. **[2]**

6 Use calculus to find the x-coordinates of the turning points of the curve $y = x^3 - 6x^2 - 15x$.

Hence find the set of values of x for which $x^3 - 6x^2 - 15x$ is an increasing function. **[5]**

7 Show that the equation $4\cos^2\theta = 4 - \sin\theta$ may be written in the form

$$4\sin^2\theta - \sin\theta = 0.$$

Hence solve the equation $4\cos^2\theta = 4 - \sin\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

8 The gradient of a curve is $3\sqrt{x} - 5$. The curve passes through the point (4, 6). Find the equation of the curve. **[5]**

9 Simplify

(i) $10 - 3\log_a a$, **[1]**

(ii) $\frac{\log_{10} a^5 + \log_{10}\sqrt{a}}{\log_{10} a}$. **[2]**

Section B (36 marks)

10 **Answer part (i) of this question on the insert provided.**

Ash trees grow quickly for the first years of their life, then more slowly. This table shows the height of a tree at various ages.

Age (t years)	4	7	10	15	20	40
Height (h m)	4	9	12	17	19	26

The height, h m, of an ash tree when it is t years old may be modelled by an equation of the form

$$h = a \log_{10} t + b.$$

(i) **On the insert**, complete the table and plot h against $\log_{10} t$, drawing by eye a line of best fit. **[3]**

(ii) Use your graph to find an equation for h in terms of $\log_{10} t$ for this model. **[3]**

(iii) Find the height of the tree at age 100 years, as predicted by this model. **[1]**

(iv) Find the age of the tree when it reaches a height of 29 m, according to this model. **[3]**

(v) Comment on the suitability of the model when the tree is very young. **[2]**

11 **(i)** In a 'Make Ten' quiz game, contestants get £10 for answering the first question correctly, then a further £20 for the second question, then a further £30 for the third, and so on, until they get a question wrong and are out of the game.

(A) Haroon answers six questions correctly. Show that he receives a total of £210. **[1]**

(B) State, in a simple form, a formula for the total amount received by a contestant who answers n questions correctly.

Hence find the value of n for a contestant who receives £10 350 from this game. **[4]**

(ii) In a 'Double Your Money' quiz game, contestants get £5 for answering the first question correctly, then a further £10 for the second question, then a further £20 for the third, and so on doubling the amount for each question until they get a question wrong and are out of the game.

(A) Gary received £75 from the game. How many questions did he get right? **[1]**

(B) Bethan answered 9 questions correctly. How much did she receive from the game? **[2]**

(C) State a formula for the total amount received by a contestant who answers n questions correctly.

Hence find the value of n for a contestant in this game who receives £2 621 435. **[4]**

[Question 12 is printed overleaf.]

12 **(i)** Calculate the gradient of the chord joining the points on the curve $y = x^2 - 7$ for which $x = 3$ and $x = 3.1$. **[2]**

(ii) Given that $\mathrm{f}(x) = x^2 - 7$, find and simplify $\dfrac{\mathrm{f}(3+h) - \mathrm{f}(3)}{h}$. **[3]**

(iii) Use your result in part **(ii)** to find the gradient of $y = x^2 - 7$ at the point where $x = 3$, showing your reasoning. **[2]**

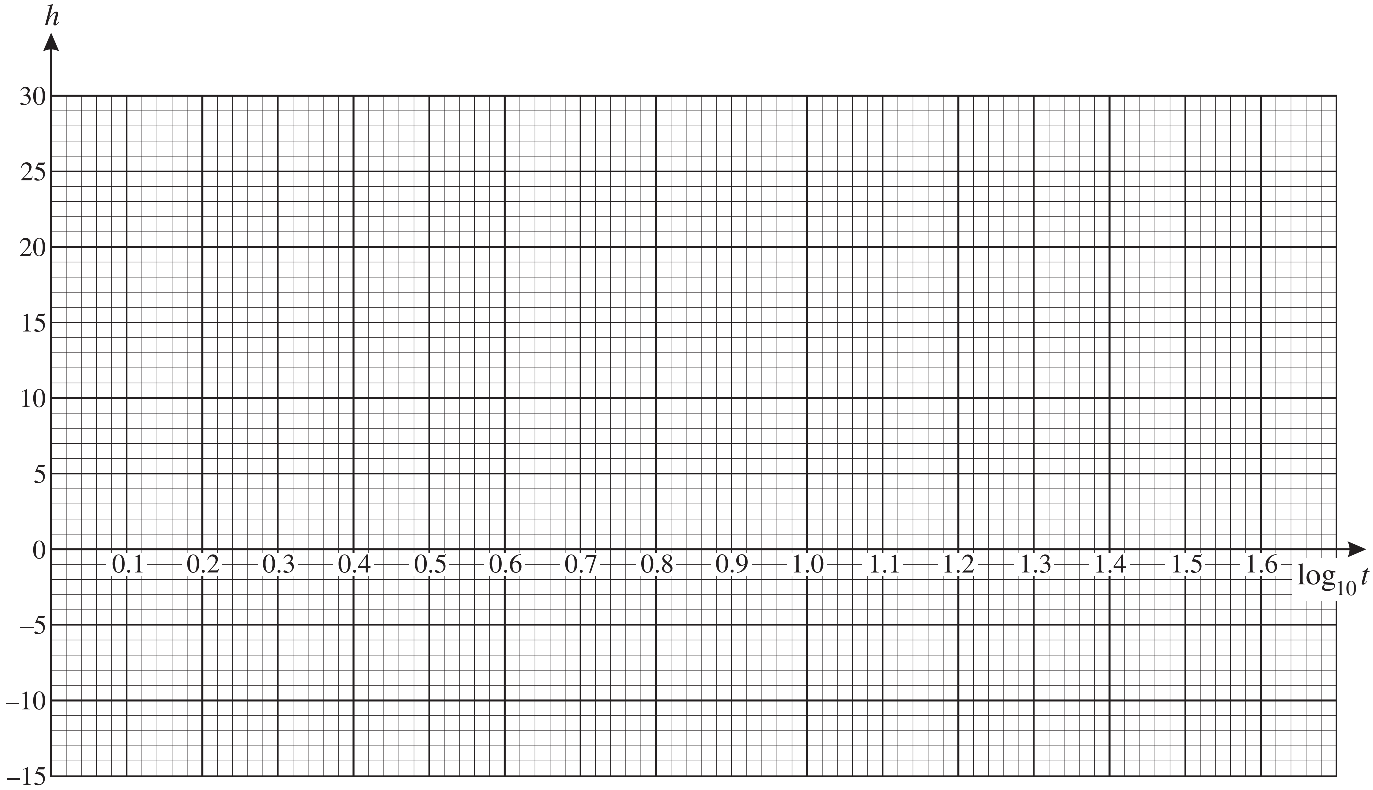
(iv) Find the equation of the tangent to the curve $y = x^2 - 7$ at the point where $x = 3$. **[2]**

(v) This tangent crosses the x-axis at the point P. The curve crosses the positive x-axis at the point Q. Find the distance PQ, giving your answer correct to 3 decimal places. **[3]**

INSERT for Question 10

10 (i)

Age (t years)	4	7	10	15	20	40
$\log_{10} t$			1			
Height (h m)	4	9	12	17	19	26



RECOGNISING ACHIEVEMENT

4752 (C2) Concepts for Advanced Mathematics

Section A

1	using Pythagoras to show that hyp. of right angled isos. triangle with sides a and a is $\sqrt{2}a$ completion using definition of cosine	M1 A1	www a any letter or a number NB answer given	2
2	$2x^6 + 5x$ value at 2 − value at 1 131	M2 M1 A1	M1 if one error ft attempt at integration only	4
3	(i) 193 (ii) divergent + difference between terms increasing o.e.	2 1	M1 for 8 + 15 + …+ 63	3
4	(i) 2.4 (ii) 138	2 2	M1 for 43.2 ÷ 18 M1 for their (i) × $\frac{180}{\pi}$ or θ = $\frac{43.2 \times 360}{36\pi}$ o.e. or for other rot versions of 137.50…	4
5	(i)sketch of cos$x$; one cycle, sketch of cos2x; two cycles, Both axes scaled correctly (ii) (1-way) stretch parallel to y axis sf 3	1 1 D1 1 D1		5
6	$y' = 3x^2 - 12x - 15$ use of $y' = 0$, s.o.i. ft $x = 5, -1$ c.a.o. $x < -1$ or $x > 5$ f.t.	M1 M1 A1 A1 A1	for two terms correct	5
7	use of $\cos^2\theta = 1 - \sin^2\theta$ at least one correct interim step in obtaining $4\sin^2\theta - \sin\theta = 0$. θ = 0 and 180, 14.(47…) 165 - 166	M1 M1 B1 B1 B1	NB answer given r.o.t to nearest degree or better -1 for extras in range	5
8	attempt to integrate $3\sqrt{x} - 5$ [y=] $2x^{\frac{3}{2}} - 5x + c$ subst of (4, 6) in their integrated eqn c = 10 or [y=] $2x^{\frac{3}{2}} - 5x + 10$	M1 A2 M1 A1	A1 for two terms correct	5
9	(i) 7 (ii) 5.5 o.e.	1 2	M1 for at least one of 5 $\log_{10}a$ or ½ $\log_{10}a$ or $\log_{10}a^{5.5}$ o.e.	3

Section B

10	**i**	0.6(0..), 0.8(45..), [1], 1.1(76..) 1.3(0..), 1.6(0..) points plotted correctly f.t. ruled line of best fit	T 1 P1 L1	Correct to 2 d.p. Allow 0.6, 1.3 and 1.6 tol. 1 mm	3
	ii	b = their intercept a = their gradient $-11 \le b \le -8$ *and* $21 \le a \le 23.5$	M1 M1 A1		3
	iii	34 to 35 m	1		1
	iv	29 = "22"logt – "9" t = $10^{\text{"}1.727..\text{"}}$ 55 [years] approx	M1 M1 A1	 accept 53 to 59	3
	v	For small t the model predicts a negative height (or h = 0 at approx 2.75) Hence model is unsuitable	1 D1		2
11	**iA**	10+20+30+40+50+60	B1	or $\frac{6}{2}(2\times10+5\times10)$ or $\frac{6}{2}(10+60)$	1
	iB	correct use of AP formula with a = 10 and d = 10 $n(5 + 5n)$ or $5n(n + 1)$ or $5(n^2 + n)$ or $(5n^2 + 5n)$ $10n^2 + 10n - 20700 = 0$ 45 c.a.o.	M1 A1 M1 A1	 Or better	4
	iiA	4	1		1
	iiB	£2555	2	M1 for $5(1 + 2 + \ldots 2^8)$ or $5(2^9 - 1)$ o.e.	2
	iiC	correct use of GP formula with a =5, r = 2 $5(2^n - 1)$ o.e.= 2621435 2^n = 524288 www 19 c.a.o.	M1 DM1 M1 A1	 "S" need not be simplified	4
12	**i**	6.1	2	M1 for $\frac{(3.1^2-7)-(3^2-7)}{3.1-3}$ o.e.	2
	ii	$\frac{\left((3+h)^2-7\right)-(3^2-7)}{h}$ numerator = $6h + h^2$ $6 + h$	M1 M1 A1	s.o.i.	3
	iii	as h tends to 0, grad. tends to 6 o.e. f.t.from "6"+h	M1 A1		2
	iv	$y - 2$ = "6" $(x - 3)$ o.e. $y = 6x - 16$	M1 A1	6 may be obtained from $\frac{dy}{dx}$	2
	v	At P, x = 16/6 o.e. or ft At Q, $x = \sqrt{7}$ 0.021 cao	M1 M1 A1		3