OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4752

Concepts for Advanced Mathematics (C2)

Wednesday **12 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- There is an **insert** for use in Question **11**.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages, 3 blank pages and an insert.

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[Turn over

Section A (36 marks)

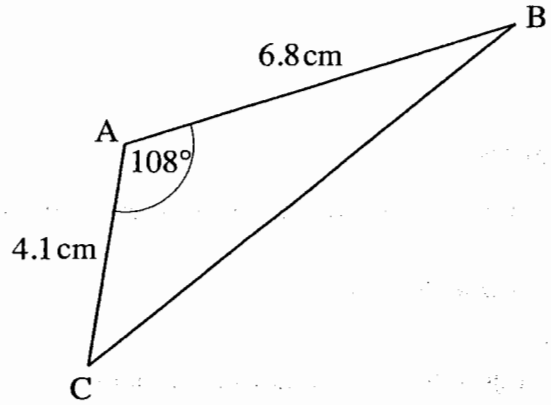
1 Find $\frac{dy}{dx}$ when $y = x^6 + \sqrt{x}$. [3]

2 Find $\int \left(x^3 + \frac{1}{x^3} \right) dx$. [4]

3 Sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

Solve the equation $\sin x = -0.2$ for $0° \leqslant x \leqslant 360°$. [4]

4



Not to scale

Fig. 4

For triangle ABC shown in Fig. 4, calculate

(i) the length of BC, [3]

(ii) the area of triangle ABC. [2]

5 The first three terms of a geometric progression are 4, 2, 1.

Find the twentieth term, expressing your answer as a power of 2.

Find also the sum to infinity of this progression. [5]

6 A sequence is given by

$$a_1 = 4,$$

$$a_{r+1} = a_r + 3.$$

Write down the first 4 terms of this sequence.

Find the sum of the first 100 terms of the sequence. [5]

7

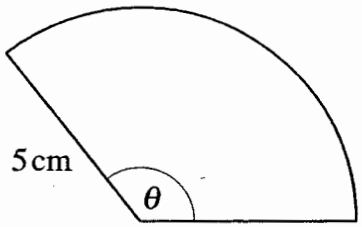


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Fig. 7

Fig. 7 shows a sector of a circle of radius 5 cm which has angle θ radians. The sector has area 30 cm^2.

(i) Find θ. [3]

(ii) Hence find the perimeter of the sector. [2]

8 **(i)** Solve the equation $10^x = 316$. [2]

(ii) Simplify $\log_a\left(a^2\right) - 4\log_a\left(\frac{1}{a}\right)$. [3]

Section B (36 marks)

9 **(i)** A tunnel is 100 m long. Its cross-section, shown in Fig. 9.1, is modelled by the curve

$$y = \tfrac{1}{4}(10x - x^2),$$

where x and y are horizontal and vertical distances in metres.

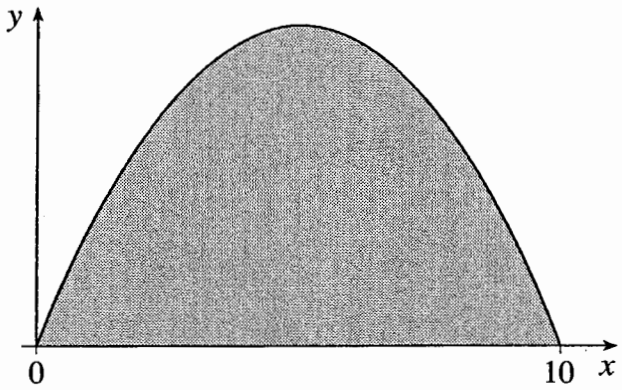

Figure 9.1

Using this model,

(*A*) find the greatest height of the tunnel, [2]

(*B*) explain why $100\int_0^{10} y\,dx$ gives the volume, in cubic metres, of earth removed to make the tunnel. Calculate this volume. [5]

(ii) The roof of the tunnel is re-shaped to allow for larger vehicles. Fig. 9.2 shows the new cross-section.

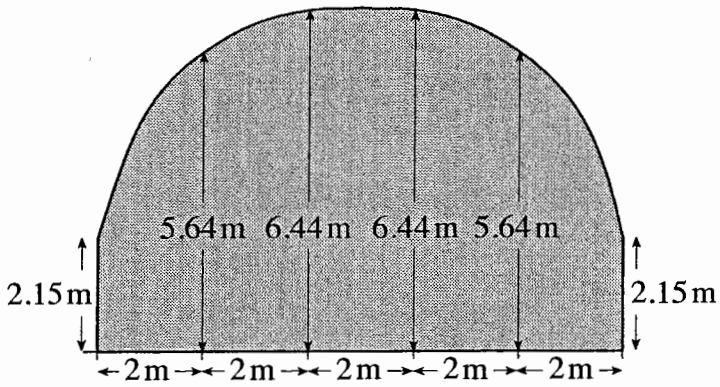

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Fig. 9.2

Use the trapezium rule with 5 strips to estimate the new cross-sectional area.

Hence estimate the volume of earth removed when the tunnel is re-shaped. [5]

10 A curve has equation $y = x^3 - 6x^2 + 12$.

(i) Use calculus to find the coordinates of the turning points of this curve. Determine also the nature of these turning points. [7]

(ii) Find, in the form $y = mx + c$, the equation of the normal to the curve at the point $(2, -4)$. [4]

11 Answer part (iii) of this question on the insert provided.

A hot drink is made and left to cool. The table shows its temperature at ten-minute intervals after it is made.

Time (minutes)	10	20	30	40	50
Temperature (°C)	68	53	42	36	31

The room temperature is 22 °C. The difference between the temperature of the drink and room temperature at time t minutes is z °C. The relationship between z and t is modelled by

$$z = z_0 10^{-kt},$$

where z_0 and k are positive constants.

(i) Give a physical interpretation for the constant z_0. [2]

(ii) Show that $\log_{10} z = -kt + \log_{10} z_0$. [2]

(iii) On the insert, complete the table and draw the graph of $\log_{10} z$ against t.

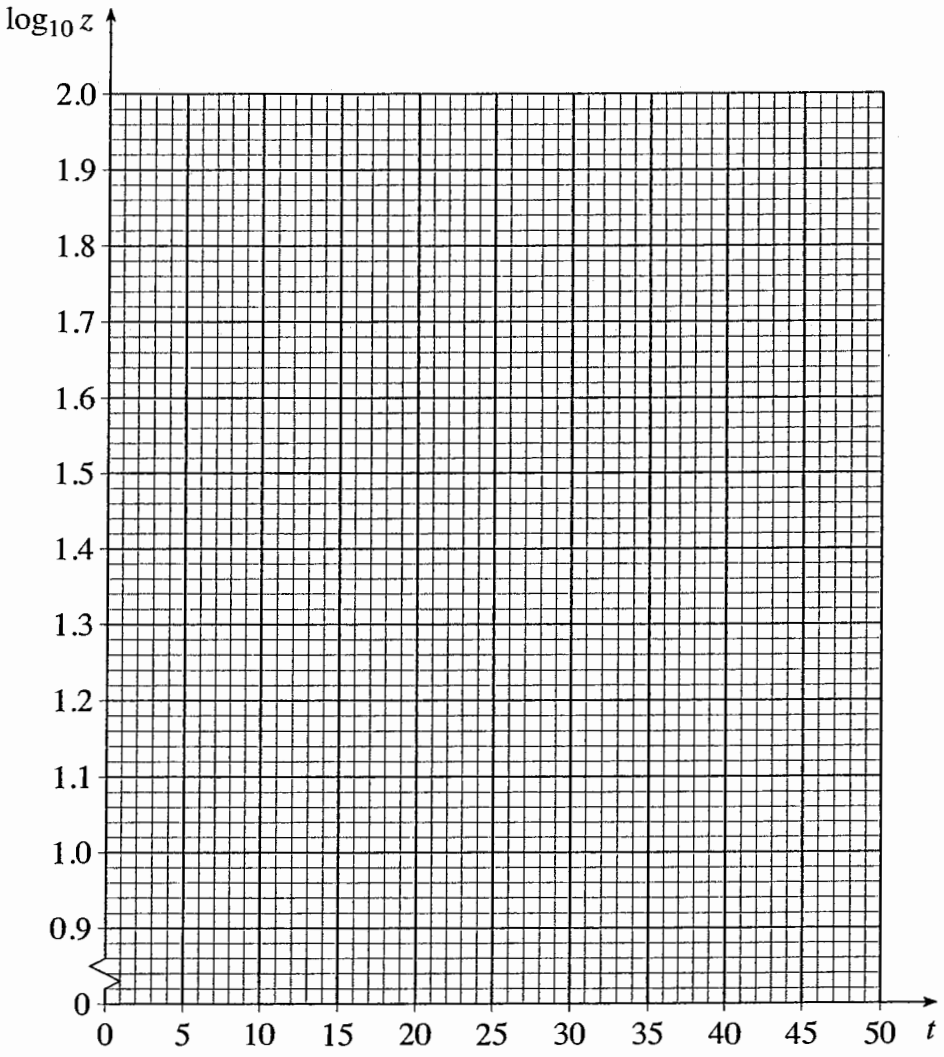
Use your graph to estimate the values of k and z_0.

Hence estimate the temperature of the drink 70 minutes after it is made. [9]

This insert is to be used for question 11

11 (iii)

t	10	20	30	40	50
z	46				
$\log_{10} z$					



Mark Scheme 4752 MEI PURE MATHS C2 JANUARY 2005

Section A

1	$6x^5 + \frac{1}{2}x^{-\frac{1}{2}}$ o.e.	B1 B1 B1	$6x^5$ $x^{\frac{1}{2}}$ soi $\frac{1}{2}x^{-\frac{1}{2}}$ isw	3
2	$x^4/4$ $\frac{x^{-2}}{-2}$ c	B1 B2 B1	 B1 for kx^{-2}	4
3	At least 1 period of sine curve Sine curve from 0 to 360 191.537 rot to 3 or more sf 348.463 rot to 3 or more sf	G1 G1 B1 B1	 ± 1 indicated After B1 B1, -1 for extras in the range SC1 for 192.8 and 347.2 (grads) SC1 for 180.2 and 359.8 (radians)	4
4	9.0 or 8.96 or 8.960 13.2577	B3 B2	M1 for $[BC^2=]6.8^2+4.1^2-2\times4.1\times6.8\times\cos108$ A1 for 80.2(8..), 8.37(grads), 6.49 (rads) Correctly rounded to 3 or more sf M1 for $0.5\times4.1\times6.8\times\sin108$ For complete long methods using BC, allow M1 and A1 for 13.2 to 13.3	5 **[16]**
5	a = 4, r = 1/2 identified 2^{-17} 8	B1 T2 S2	Stated or identified by correct use M1 20th term = their(a)x(their r)^{19} M1 S = their (a) / (1–their (r))	5
6	4, 7, 10, 13, 16 ignore extras 15250	B1 B4	For showing 1st four or 2nd four terms B1 for d = 3 soi B1 for a = 4 soi M1 for use of 100/2[2a + 99d] o.e.	5
7	(i) 2.4, $2\frac{2}{5}$, $\frac{12}{5}$ (ii) 22	B3 P2	M1 for 30 = ½ × 25 × q o.e. M1 for q = (2 x 30) / 5^2 M1 for (arc =)5 x their 2.4	5
8	(i) 2.5, 2.50, 2.500, 2.499.. (ii) 6 www	B2 B3	M1 for $\log_{10} 316$ or ln 316/ln 10 B2 for 6 $\log_a a$ or $\log_a(a^6)$ Or B1 for $2\log_a(a)$ or $-\log_a a^{-4}$ SC1 Using a=10 ⇒ 6 SC2 Using numerical a, not 10 ⇒ 6	5 **[20]**
			Total for section A	36

Section B

9	**i*A***	6.25	B2	M1 for x = 5 used to find y	2
	i*B*	(V =) area of cross-section × length	E1		
		$(\frac{100}{4})[\frac{10}{2}x^2 - \frac{1}{3}x^3]$ o.e.	M1		
		[val at x = 10] – [val at x = 0]	M1	Subs of correct limits into their integrand	
		4166 to 4167 or 4170	A2	A1 for 166.6... or 16666.6... or 41.6…rot to 3 sf or more	5
	ii	52.62	B4	M3 for- 2/2×[2.15x2+2(5.64x2+6.44x2)]oe Or M2 if one slip Or M1 if 2 slips or one trap evaluated	5 **[12]**
		Their(5262) – their (4167)	M1	Must be >0	
10	**i**	$y' = 3x^2 - 12x$	B1B1		
		use of $y' = 0$	M1		
		x = 0 and 4	A1		
		(0, 12) and (4, –20)	A1	Allow y = 12 and y = -20	
		$y'' = 6x - 12$ used	M1	y' used each side of TP or good sketch	7
		max when x = 0, min when x = 4	A1	Both stated, only one needs testing	
	ii	when x = 2 y' = –12	B1		
		grad of normal = 1/12	B1ft	from their y'	
		$y + 4 = 1/12(x - 2)$	M1ft	accept any numerical m Or -4 = their(m) x 2 + c	4 **[11]**
		$y = \frac{1}{12}x - 4\frac{1}{6}$	A1	Any recognisable 25/6, at worst 4.1	
11	**i**	Excess temperature	B1		2
		At t =0 oe	B1		
	ii	log z = log z_0 + log (10^{-kt})	B1		
		= log z_0 – ktlog 10	B1	nb AG	2
	iii	Z= 46 31 20 14 9		If z= 68, 53…P1, L1,M1, M1, M1 available	
		log z : 1.66 1.49 1.30 1.15 0.95	T1		
		correctly plotted	P1	ft their values, within 2mm	
		line of best fit	L1	Ruled, using their points	
		k = 0.017 to 0.019 or 0.02	G2	M1 for attempting +/- gradient	9
		z_0= 66 to 73	B2	M1 for (log) z_0= 1.82 to 1.86	
		temp of drink = 25 to 27	C2	M1 3 to 5 or their 69 × $10^{-70 \text{ x their } k}$	**[13]**