Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43165A

1. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

(*b*) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

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2. Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \dots$$

(*a*) find the value of the constant k,

(2)

(*b*) find the value of the constant A, giving your answer in its simplest form.

(3)

3.

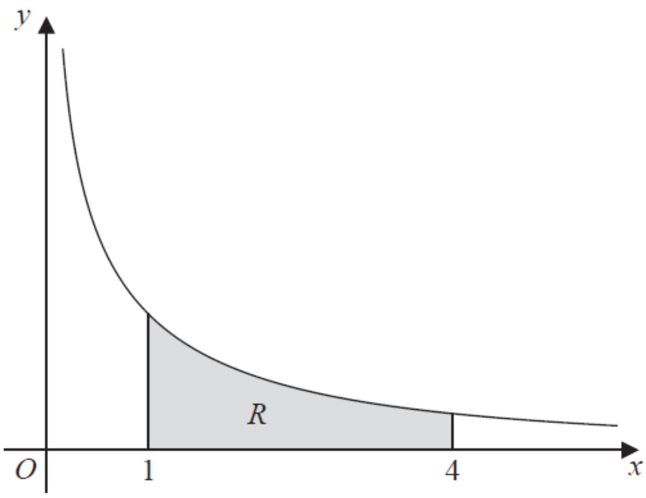


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$.

x	1	2	3	4
y	1.42857	0.90326		0.55556

(*a*) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (*b*) is an overestimate or an underestimate for the area of R.

(1)

(*d*) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,dx$$

(6)

4.

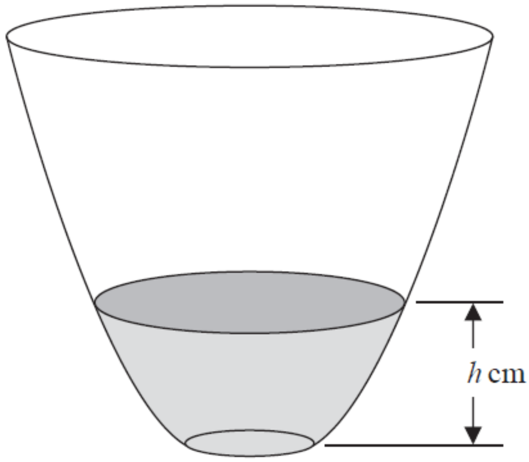


Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm^3 is given by

$$V = 4\pi h(h + 4), \qquad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of 80π $\text{cm}^3\ \text{s}^{-1}$.

Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 6$.

(5)

5.

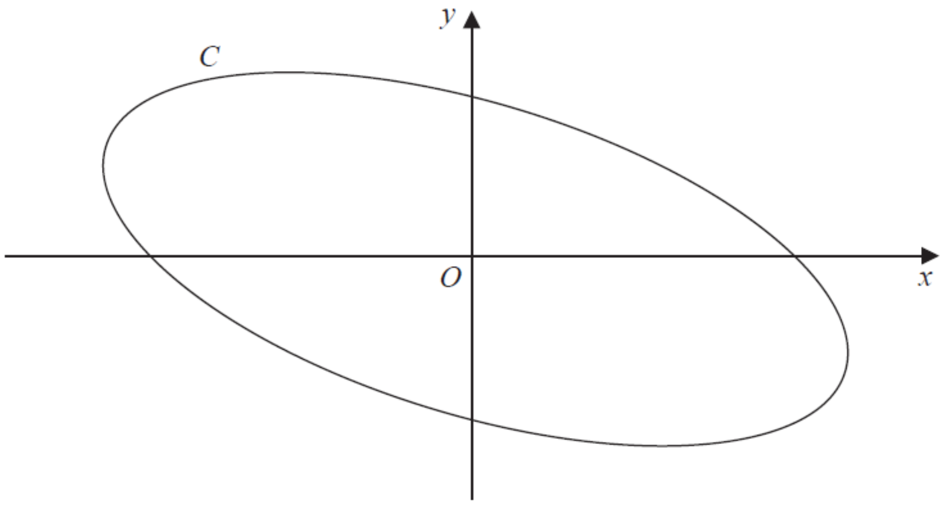


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \qquad y = 2\sin t, \qquad 0 \leq t \leq 2\pi$$

(*a*) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(*b*) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be determined.

(2)

6. (i) Find

$$\int x\mathrm{e}^{4x}\,\mathrm{d}x$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\,\mathrm{d}x\,, \qquad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \dfrac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{x}\operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

7.

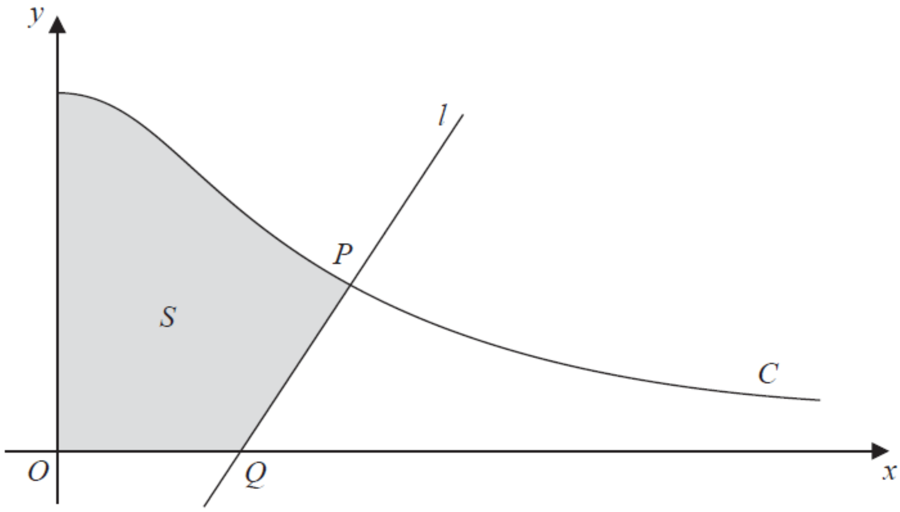


Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\tan\theta, \quad y = 4\cos^2\theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates (3, 2).

The line l is the normal to C at P. The normal cuts the x-axis at the point Q.

(a) Find the x coordinate of the point Q.

(6)

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis, the y-axis and the line l. This shaded region is rotated 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact value of the volume of the solid of revolution, giving your answer in the form $p\pi + q\pi^2$, where p and q are rational numbers to be determined.

[*You may use the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(9)

8. Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Hence find a vector equation for the line l_1.

(1)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$.

Given that angle PBA is θ,

(c) show that $\cos\theta = \frac{1}{3}$.

(3)

The line l_2 passes through the point P and is parallel to the line l_1.

(d) Find a vector equation for the line l_2.

(2)

The points C and D both lie on the line l_2.

Given that $AB = PC = DP$ and the x coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D.

(3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$x^3 + 2xy - x - y^3 - 20 = 0$	
(a)	$\left\{\frac{\cancel{\mathrm{d}y}}{\cancel{\mathrm{d}x}} \cancel{=}\right\} \quad \underline{3x^2} + \underline{\underline{\left(2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}\right)}} \underline{- 1 - 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$3x^2 + 2y - 1 + \left(2x - 3y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]
(b)	At $P(3, -2)$, $\mathrm{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \frac{22}{6}$ or $\frac{11}{3}$	
	and either T: $y - -2 = \frac{"11"}{3}(x - 3)$ **see notes**	M1
	or $(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \ldots,$	
	T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 **cso**
		[2] **7**
(a)	**<u>Alternative method for part (a)</u>**	
	$\left\{\frac{\cancel{\mathrm{d}x}}{\cancel{\mathrm{d}y}} \cancel{=}\right\} \quad \underline{3x^2\frac{\mathrm{d}x}{\mathrm{d}y}} + \underline{\underline{\left(2y\frac{\mathrm{d}x}{\mathrm{d}y} + 2x\right)}} \underline{- \frac{\mathrm{d}x}{\mathrm{d}y} - 3y^2 = 0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$2x - 3y^2 + \left(3x^2 + 2y - 1\right)\frac{\mathrm{d}x}{\mathrm{d}y} = 0$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]

		Question 1 Notes
(a) **General**	**Note**	Writing down $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ from **no working** is full marks.
	Note	Writing down $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\frac{1 - 3x^2 - 2y}{3y^2 - 2x}$ from **no working is** M1A0B0M1A0
	Note	Few candidates will write $3x^2 + 2y + 2x\mathrm{d}y - 1 - 3y^2\mathrm{d}y = 0$ leading to $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$, o.e. This should get full marks.
1. (a)	**M1**	Differentiates implicitly to include either $2x\frac{\mathrm{d}y}{\mathrm{d}x}$ or $-y^3 \to \pm k y^2\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).
	A1	$x^3 \to 3x^2$ **and** $-x - y^3 - 20 = 0 \to -1 - 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
	B1	$2xy \to 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	If an extra term appears then award 1st A0.

1. (a) ctd	**Note**	$3x^2 + 2y + 2x\frac{dy}{dx} - 1 - 3y^2\frac{dy}{dx} \rightarrow 3x^2 + 2y - 1 = 3y^2\frac{dy}{dx} - 2x\frac{dy}{dx}$ will get 1st A1 (implied) as the $"=0"$ can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.**
		An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$.
		ie. $... + (2x - 3y^2)\frac{dy}{dx} = ...$
	Note	Placing an extra $\frac{dy}{dx}$ at the beginning and then including it in their factorisation is fine for dM1.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$
		cso: If the candidate's solution is not completely correct, then do not give this mark.
		isw: You can, however, ignore subsequent working following on from correct solution.
1. (b)	**M1**	**Some** attempt to substitute **both** $x = 3$ **and** $y = -2$ into their $\frac{dy}{dx}$ which contains both x and y to find m_T **and**
		• **either** applies $y - -2 = (\text{their } m_T)(x - 3)$, where m_T is a numerical value.
		• **or** finds c by solving $(-2) = (\text{their } m_T)(3) + c$, where m_T is a numerical value.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ is M0).
	A1	Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.
	cso	A correct solution is required from a correct $\frac{dy}{dx}$.
	isw	You can ignore subsequent working following a correct solution.
	<u>Alternative method for part (a): Differentiating with respect to *y*</u>	
1. (a)	**M1**	Differentiates implicitly to include either $2y\frac{dx}{dy}$ or $x^3 \rightarrow \pm kx^2\frac{dx}{dy}$ or $-x \rightarrow -\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$).
	A1	$x^3 \rightarrow 3x^2\frac{dx}{dy}$ **and** $-x - y^3 - 20 = 0 \rightarrow -\frac{dx}{dy} - 3y^2 = 0$
	B1	$2xy \rightarrow 2y\frac{dx}{dy} + 2x$
	dM1	**dependent on the first method mark being awarded.**
		An attempt to factorise out **all the terms in** $\frac{dx}{dy}$ as long as there are ***at least two terms*** in $\frac{dx}{dy}$.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$
		cso: If the candidate's solution is not completely correct, then do not give this mark.

Question Number	Scheme		Marks
2.	$\left\{(1+kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + ...\right\}$		
(a)	**Either** $(-4)k = -6$ **or** $(1+kx)^{-4} = 1 + (-4)(kx)$	**see notes**	M1
	leading to $k = \frac{3}{2}$	$k = \frac{3}{2}$ or 1.5 or $\frac{6}{4}$	A1
			[2]
(b)	$\frac{(-4)(-5)}{2}(k)^2$	**Either** $\frac{(-4)(-5)}{2!}$ **or** $(k)^2$ **or** $(kx)^2$	M1
		Either $\frac{(-4)(-5)}{2!}(k)^2$ **or** $\frac{(-4)(-5)}{2!}(kx)^2$	M1
	$\left\{A = \frac{(-4)(-5)}{2!}\left(\frac{3}{2}\right)^2\right\} \Rightarrow A = \frac{45}{2}$	$\frac{45}{2}$ or 22.5	A1
			[3]
			5

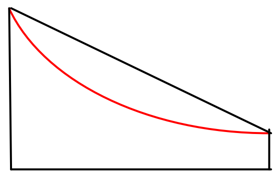
Question 2 Notes

Note		
Note		**In this question ignore part labelling and mark part (a) and part (b) together.**
	Note	Writing down $\left\{(1+kx)^{-4}\right\} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + ...$ gets all the method marks in Q2. i.e. (a) M1 and (b) M1M1
(a)	**M1**	Award M1 for • **either** writing down $(-4)k = -6$ or $4k = 6$ • **or** expanding $(1+kx)^{-4}$ to give $1 + (-4)(kx)$ • **or** writing down $(-4)kx = -6$ or $(-4k) = -6x$ or $-4kx = -6x$
	A1	$k = \frac{3}{2}$ or 1.5 or $\frac{6}{4}$ **from no incorrect sign errors.**
	Note	The M1 mark can be implied by a candidate writing down the correct value of k.
	Note	Award M1 for writing down $4k = 6$ and then A1 for $k = 1.5$ (or equivalent).
	Note	Award M0 for $4k = -6$ (if there is no evidence that $(1+kx)^{-4}$ expands to give $1 + (-4)(kx) + ...$)
	Note	$1 + (-4)(kx)$ leading to $(-4)k = 6$ leading to $k = \frac{3}{2}$ is M1A0.
(b)	**M1**	For **either** $\frac{(-4)(-4-1)}{2!}$ **or** $\frac{(-4)(-5)}{2!}$ **or** 10 **or** $(k)^2$ **or** $(kx)^2$
	M1	**Either** $\frac{(-4)(-4-1)}{2!}(k)^2$ **or** $\frac{(-4)(-5)}{2!}(k)^2$ **or** $\frac{(-4)(-5)}{2!}(kx)^2$ **or** $\frac{(-4)(-5)}{2!}(\text{their } k)^2$ **or** $10k^2$
	Note	Candidates are allowed to use 2 instead of 2!
	A1	Uses $k = 1.5$ to give $A = \frac{45}{2}$ or 22.5
	Note	$A = \frac{90}{4}$ which has not been simplified is A0.
	Note	Award A0 for $A = \frac{45}{2}x^2$.
	Note	Allow A1 for $A = \frac{45}{2}x^2$ followed by $A = \frac{45}{2}$
	Note	$k = -1.5$ leading to $A = \frac{45}{2}$ or 22.5 is A0.

Question Number	Scheme		Marks
3.	x: 1, 2, 3, 4; y: 1.42857, 0.90326, ***0.682116...***, 0.55556	$y = \dfrac{10}{2x + 5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y = 0.68212$ (5 dp)	0.68212	B1 **cao**
			[1]
(b)	$\frac{1}{2}\times 1\times\underline{\left[1.42857 + 0.55556 + 2\left(0.90326 + \text{their } 0.68212\right)\right]}$	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$	B1 aef
		<u>For structure of</u> $\underline{[\ldots\ldots\ldots\ldots]}$	M1
	$\left\{= \frac{1}{2}(5.15489)\right\} = 2.577445 = 2.5774$ (4 dp)	anything that rounds to 2.5774	A1
			[3]
(c)	• Overestimate **and** a reason such as • {top of} <u>trapezia lie above the curve</u> • a diagram which gives reference to the extra area • concave or convex • $\dfrac{d^2y}{dx^2} > 0$ (can be implied) • bends inwards • curves downwards		B1
			[1]
(d)	$\left\{u = \sqrt{x} \Rightarrow\right\} \dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{dx}{du} = 2u$		B1
	$\displaystyle\int \frac{10}{2u^2 + 5u}.\ 2u\ du$	Either $\left\{\displaystyle\int\right\}\dfrac{\pm ku}{\alpha u^2 \pm \beta u}\ \{du\}$ or $\left\{\displaystyle\int\right\}\dfrac{\pm k}{u\left(\alpha u^2 \pm \beta u\right)}\ \{du\}$	M1
	$\left\{= \displaystyle\int \frac{20}{2u + 5}\ du\right\} = \dfrac{20}{2}\ln(2u + 5)$	$\pm\lambda\ln(2u + 5)$ or $\pm\lambda\ln\left(u + \frac{5}{2}\right),\ \lambda \neq 0$ **with no other terms.**	M1
		$\dfrac{20}{2u + 5} \to \dfrac{20}{2}\ln(2u + 5)$ or $10\ln\left(u + \frac{5}{2}\right)$	A1 **cso**
	$\left\{\left[\dfrac{20}{2}\ln(2u + 5)\right]_1^2\right\} = 10\ln\left(2(2) + 5\right) - 10\ln\left(2(1) + 5\right)$	Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9 - 10\ln 7$ or $10\ln\left(\dfrac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$		A1 oe **cso**
			[6] **11**

Question 3 Notes

3. (a)	**B1**	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$ or equivalent.
	M1	<u>For structure of trapezium rule</u> $[\ \underline{\ldots\ldots\ldots\ldots}\ ]$
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 2.5774
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428…)

3. (b) contd	**Note**	Award B1M1A1 for $\frac{1}{2}(1.42857 + 0.55556) + (0.90326 + \text{their } 0.68212) = 2.577445$ <u>**Bracketing mistake**</u>**: Unless the final answer implies that the calculation has been done correctly** award B1M0A0 for $\frac{1}{2}\times 1 + 1.42857 + 2(0.90326 + \text{their } 0.68212) + 0.55556$ (nb: answer of 5.65489). award B1M0A0 for $\frac{1}{2}\times 1\ (1.42857 + 0.55556) + 2(0.90326 + \text{their } 0.68212)$ (nb: answer of 4.162825). <u>**Alternative method: Adding individual trapezia**</u> Area $\approx 1\times\left[\frac{1.42857+0.90326}{2} + \frac{0.90326 + "0.68212"}{2} + \frac{"0.68212" + 0.55556}{2}\right] = 2.577445$
	B1	**B1:** 1 and a divisor of 2 on all terms inside brackets.
	M1	**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1	**A1:** anything that rounds to 2.5774
(c)	**B1**	Overestimate **and** either trapezia lie above curve **or** a diagram that gives reference to the extra area **eg.** This diagram is sufficient. It must show the top of a trapezium lying above the curve.  **or** concave or convex **or** $\frac{d^2y}{dx^2} > 0$ (can be implied) **or** bends inwards **or** curves downwards.
	Note	Reason of "gradient is negative" by itself is B0.
(d)	**B1**	$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $du = \frac{1}{2\sqrt{x}}dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2u\,du$ or $\frac{dx}{du} = 2u$ o.e.
	M1	Applying the substitution and achieving $\left\{\int\right\}\frac{\pm ku}{\alpha u^2 \pm \beta u}\{du\}$ or $\left\{\int\right\}\frac{\pm k}{u(\alpha u^2 \pm \beta u)}\{du\}$, $k, \alpha, \beta \neq 0$. Integral sign and du not required for this mark.
	M1	Cancelling u and integrates to achieve $\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**
	A1	**cso**. Integrates $\frac{20}{2u+5}$ to give $\frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$, un-simplified or simplified.
	Note	BE CAREFUL! Candidates must be integrating $\frac{20}{2u+5}$ or equivalent. So $\int \frac{10}{2u+5}\,du = 10\ln(2u+5)$ WOULD BE A0 and final A0.
	M1	Applies limits of 2 and 1 in u or 4 and 1 in x in their (i.e. any) changed function and subtracts the correct way round.
	A1	Exact answers of either $10\ln 9 - 10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$ or $20\ln\left(\frac{3}{\sqrt{7}}\right)$ or $\ln\left(\frac{9^{10}}{7^{10}}\right)$ or equivalent. **Correct solution only.**
	Note	You can ignore subsequent working which follows from a correct answer.
	Note	A decimal answer of 2.513144283... (without a correct **exact** answer) is A0.

Question Number	Scheme	Marks
4.	$\frac{\mathrm{d}V}{\mathrm{d}t} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$,	
	$\frac{\mathrm{d}V}{\mathrm{d}h} = 8\pi h + 16\pi$ $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$	M1
	$8\pi h + 16\pi$	A1
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \Rightarrow\right\}$ $(8\pi h + 16\pi)\frac{\mathrm{d}h}{\mathrm{d}t} = 80\pi$ $\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 80\pi$	
	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}h} \Rightarrow\right\}$ $\frac{\mathrm{d}h}{\mathrm{d}t} = 80\pi \times \frac{1}{8\pi h + 16\pi}$ or $80\pi \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}$	M1 oe
	When $h = 6$, $\left\{\frac{\mathrm{d}h}{\mathrm{d}t} =\right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi$ $\left\{= \frac{80\pi}{64\pi}\right\}$ **dependent on the previous M1 see notes**	dM1
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \underline{1.25}$ (cm s^{-1}) 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	A1 oe
		[5] 5
	<u>Alternative Method for the first M1A1</u>	
	Product rule: $\left\{\begin{array}{ll} u = 4\pi h & v = h + 4 \\ \frac{\mathrm{d}u}{\mathrm{d}h} = 4\pi & \frac{\mathrm{d}v}{\mathrm{d}h} = 1 \end{array}\right\}$	
	$\frac{\mathrm{d}V}{\mathrm{d}h} = 4\pi(h+4) + 4\pi h$ $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$	M1
	$4\pi(h+4) + 4\pi h$	A1

Question 4 Notes

M1	An expression of the form $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$. **Can be simplified or un-simplified.**
A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent.
Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
Note	$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
M1	$\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}$
Note	Also allow 2nd M1 for $\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}$
Note	Give 2nd M0 for $\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}$
dM1	**which is dependent on the previous M1 mark.** Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and 80π (or 80)
A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
Note	Substituting $h = 6$ into a correct $\frac{\mathrm{d}V}{\mathrm{d}h}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)

Question Number	Scheme		Marks
5.	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
	Main Scheme		
(a)	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**Alternative Method 1**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
	Main Scheme		
(b)	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
		$\{a = 3, b = 12\}$	**[2]**
(b)	**Alternative Method 1**		
	$(x+y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x+y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**Alternative Method 2**		
	$(x+y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x+y)^2 + 3y^2 = 12$		M1, A1
			[2]
			5

		Question 5 Notes
5. (a)	M1	$\cos\left(t+\dfrac{\pi}{6}\right) \to \cos t\cos\left(\dfrac{\pi}{6}\right) \pm \sin t\sin\left(\dfrac{\pi}{6}\right)$ or $\cos\left(t+\dfrac{\pi}{6}\right) \to \left(\dfrac{\sqrt{3}}{2}\right)\cos t \pm \left(\dfrac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1. **<u>Awarding the dM1 mark which is dependent on the first method mark</u>**
Main	dM1 **Note**	Adds their expanded x (which is in terms of t) to $2\sin t$ Writing $x + y = ...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.
	A1*	Evidence of $\cos\left(\dfrac{\pi}{6}\right)$ and $\sin\left(\dfrac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x+y\} = 4\cos\left(t+\dfrac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1 A1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's. leading $(x+y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x+y)^2 + 3y^2 = 12$ from no working • $a = 3,\ b = 12$, but **<u>does not provide a correct proof</u>**.
	Note **Note**	Alternative method 2 is fine for M1 A1 Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3,\ b = 12$ is SC: B1B0
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a = 3,\ b = 12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

Question Number	Scheme		Marks
6. (i)	$\int x\mathrm{e}^{4x}\,\mathrm{d}x = \frac{1}{4}x\mathrm{e}^{4x} - \int \frac{1}{4}\mathrm{e}^{4x}\,\{\mathrm{d}x\}$	$\pm\alpha x\mathrm{e}^{4x} - \int \beta\mathrm{e}^{4x}\{\mathrm{d}x\},\quad \alpha\neq 0,\ \beta>0$	M1
		$\frac{1}{4}x\mathrm{e}^{4x} - \int \frac{1}{4}\mathrm{e}^{4x}\,\{\mathrm{d}x\}$	A1
	$= \frac{1}{4}x\mathrm{e}^{4x} - \frac{1}{16}\mathrm{e}^{4x}\,\{+c\}$	$\frac{1}{4}x\mathrm{e}^{4x} - \frac{1}{16}\mathrm{e}^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,\mathrm{d}x = \frac{8(2x-1)^{-2}}{(2)(-2)}\,\{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\,\{+c\}\}$	*{Ignore subsequent working}.*	**[2]**
(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x\operatorname{cosec}2y\operatorname{cosec}y \qquad y=\frac{\pi}{6}$ at $x=0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ **or** $\int \sin 2y\sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$		B1 oe
	$\int 2\sin y\cos y\sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = \mathrm{e}^x\,\{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$\mathrm{e}^x \to \mathrm{e}^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = \mathrm{e}^0 + c$ **or** $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y=\frac{\pi}{6}$ and $x=0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec}2y\operatorname{cosec}y}\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ **or** $\int \sin 2y\sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$	$\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = \mathrm{e}^x\,\{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$\mathrm{e}^x \to \mathrm{e}^x$ ***as part of solving their DE.***	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = \mathrm{e}^0 + c$ **or** $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y=\frac{\pi}{6}$ and $x=0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$	A1
			[7]
			12

		Question 6 Notes
6. (i)	**M1**	Integration by parts is applied in the form $\pm\alpha x\mathrm{e}^{4x} - \int \beta\mathrm{e}^{4x}\,\{\mathrm{d}x\}$, where $\alpha \neq 0,\ \beta > 0$. (must be in this form).
	A1	$\frac{1}{4}x\mathrm{e}^{4x} - \int \frac{1}{4}\mathrm{e}^{4x}\,\{\mathrm{d}x\}$ or equivalent.
	A1	$\frac{1}{4}x\mathrm{e}^{4x} - \frac{1}{16}\mathrm{e}^{4x}$ with/without + c. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \mathrm{e}^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	**M1**	$\pm\lambda(2x-1)^{-2}$, $\lambda \neq 0$. **Note** that λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without + c. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	**B1**	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\operatorname{cosec} 2y\ \operatorname{cosec} y} = \int \mathrm{e}^x$ **or** $\int \sin 2y \sin y = \int \mathrm{e}^x$
	M1	$\frac{1}{\operatorname{cosec} 2y} \to 2\sin y\cos y$ **or** $\sin 2y \to 2\sin y\cos y$ **or** $\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm\mu\sin^3 y,\ \mu \neq 0$ **or** $\pm\alpha\sin 3y \pm \beta\sin y$, $\alpha \neq 0,\ \beta \neq 0$
	A1	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) **or** integrates to give $-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x ***as part of solving their DE.***
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$ **or** $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

Alternative Method 2 (Using integration by parts twice)

$\int \sin 2y \sin y\ \mathrm{d}y = \int \mathrm{e}^x\ \mathrm{d}x$		B1 oe
	Applies integration by parts **twice** to give $\pm\alpha\cos y\sin 2y \pm \beta\sin y\cos 2y$	**M2**
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = \mathrm{e}^x\ \{+c\}$	$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y$ (simplified or un-simplified)	A1
	$\mathrm{e}^x \to \mathrm{e}^x$ ***as part of solving their DE.***	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y = \mathrm{e}^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$	A1
		[7]

Question Number	Scheme		Marks
7.	$x = 3\tan\theta$, $y = 4\cos^2\theta$ or $y = 2 + 2\cos 2\theta$, $0 \leqslant \theta < \frac{\pi}{2}$.		
(a)	$\frac{dx}{d\theta} = 3\sec^2\theta$, $\frac{dy}{d\theta} = -8\cos\theta\sin\theta$ **or** $\frac{dy}{d\theta} = -4\sin 2\theta$		
	$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ $\left\{= -\frac{8}{3}\cos^3\theta\sin\theta = -\frac{4}{3}\sin 2\theta\cos^2\theta\right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$	M1 A1 oe
	At $P(3, 2)$, $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{8}{3}\cos^3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$ $\left\{= -\frac{2}{3}\right\}$ So, $m(\mathbf{N}) = \frac{3}{2}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1 M1
	Either N: $y - 2 = "\frac{3}{2}"(x - 3)$ **or** $2 = \left("\frac{3}{2}"\right)(3) + c$	**see notes**	M1
	$\{$At Q, $y = 0$, so, $-2 = \frac{3}{2}(x - 3)\}$ **giving** $\underline{x = \frac{5}{3}}$	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67	A1 **cso** **[6]**
(b)	$\left\{\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta\right\} = \left\{\int\right\}(4\cos^2\theta)^2\, 3\sec^2\theta\ \{d\theta\}$	**see notes**	M1
	So, $\pi\int y^2 dx = \pi\int(4\cos^2\theta)^2 3\sec^2\theta\,\{d\theta\}$	**see notes**	A1
	$\int y^2 dx = \int 48\cos^2\theta\, d\theta$	$\int 48\cos^2\theta\{d\theta\}$	A1
	$= \{48\}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta$ $\left\{= \int(24 + 24\cos 2\theta)d\theta\right\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$	M1
	$= \{48\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$ $\{= 24\theta + 12\sin 2\theta\}$	**Dependent on the first method mark.** For $\pm\alpha\theta \pm \beta\sin 2\theta$ $\cos^2\theta \to \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	dM1 A1
	$\int_0^{\frac{\pi}{4}} y^2 dx \left\{= 48\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}\right\} = \{48\}\left(\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0 + 0)\right)$ $\{= 6\pi + 12\}$	**Dependent on the third method mark.**	dM1
	$\left\{\text{So } V = \pi\int_0^{\frac{\pi}{4}} y^2 dx = 6\pi^2 + 12\pi\right\}$		
	$V_{cone} = \frac{1}{3}\pi(2)^2\left(3 - \frac{5}{3}\right)$ $\left\{= \frac{16\pi}{9}\right\}$	$V_{cone} = \frac{1}{3}\pi(2)^2(3 - \text{their } (a))$	M1
	$\left\{\text{Vol}(S) = 6\pi^2 + 12\pi - \frac{16\pi}{9}\right\} \Rightarrow \text{Vol}(S) = \underline{\frac{92}{9}\pi + 6\pi^2}$	$\frac{92}{9}\pi + 6\pi^2$ $\left\{p = \frac{92}{9},\ q = 6\right\}$	A1 **[9]**
			15

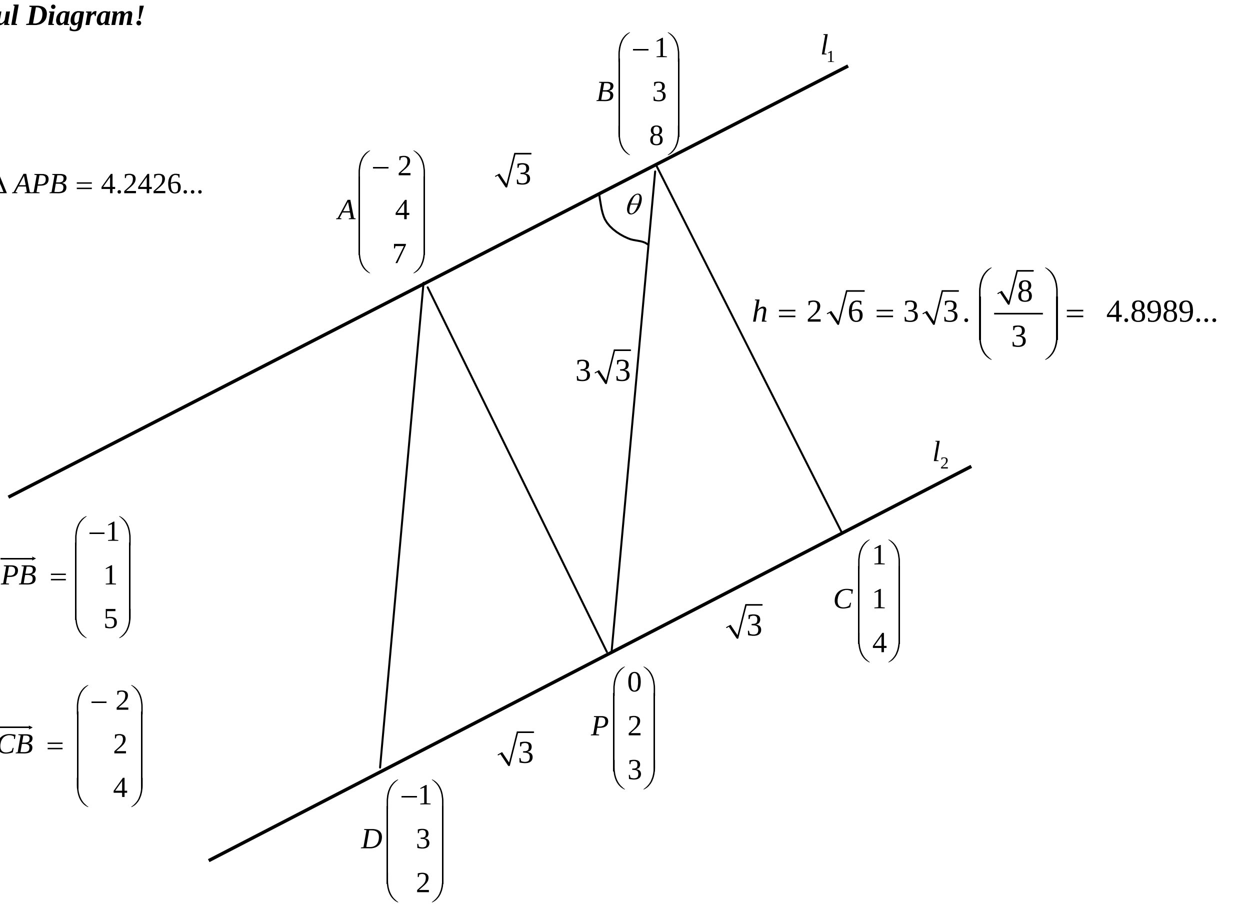
		Question 7 Notes
7. (a)	**1st M1**	Applies their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ **or** applies $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ multiplied by their $\frac{\mathrm{d}\theta}{\mathrm{d}x}$
	SC	Award **Special Case 1st M1** if **both** $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ **and** $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ are both correct.
	1st A1	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ i.e. $\frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ or $-\frac{8}{3}\cos^3\theta\sin\theta$ or $-\frac{4}{3}\sin 2\theta\cos^2\theta$ or any equivalent form.
	2nd M1	***Some evidence*** of substituting $\theta = \frac{\pi}{4}$ or $\theta = 45^\circ$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	For 3rd M1 and 4th M1, $m(\mathbf{T})$ must be found by using $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	3rd M1	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$. Numerical value for $m(\mathbf{N})$ is required here.
	4th M1	• Applies $y - 2 = (\text{their } m_N)(x-3)$, where $\mathrm{m}(\mathbf{N})$ is a numerical value, • or ***finds c*** by solving $2 = (\text{their } m_N)3 + c$, where $\mathrm{m}(\mathbf{N})$ is a numerical value, and $m_N = -\frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = -\text{their m}(\mathbf{T})$.
	Note	This mark can be implied by subsequent working.
	2nd A1	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 **from a correct solution only.**
(b)	**1st M1**	Applying $\int y^2 \mathrm{d}x$ as $y^2\frac{\mathrm{d}x}{\mathrm{d}\theta}$ with their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$. Ignore π or $\frac{1}{3}\pi$ outside integral.
	Note	You can ignore the omission of an integral sign and/or $\mathrm{d}\theta$ for the 1st M1.
	Note	Allow 1st M1 for $\int(\cos^2\theta)^2 \times$ "their $3\sec^2\theta$" $\mathrm{d}\theta$ or $\int 4(\cos^2\theta)^2 \times$ "their $3\sec^2\theta$ " $\mathrm{d}\theta$
	1st A1	Correct expression $\left\{\pi\int y^2\mathrm{d}x\right\} = \pi\int(4\cos^2\theta)^2\, 3\sec^2\theta\,\{\mathrm{d}\theta\}$ (Allow the omission of $\mathrm{d}\theta$)
	Note	**IMPORTANT:** The π can be recovered later, **but as a correct statement only**.
	2nd A1	$\left\{\int y^2\mathrm{d}x\right\} = \int 48\cos^2\theta\{\mathrm{d}\theta\}$. (Ignore $\mathrm{d}\theta$). **Note**: 48 can be written as 24(2) for example.
	2nd M1	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral. (Seen or **implied**.)
	3rd dM1*	**which is dependent on the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\pm\alpha\theta \pm \beta\sin 2\theta$, $\alpha \neq 0, \beta \neq 0$, un-simplified or simplified.
	3rd A1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$, un-simplified or simplified. This can be implied by $k\cos^2\theta$ giving $\frac{k}{2}\theta + \frac{k}{4}\sin 2\theta$**,** un-simplified or simplified.
	4th dM1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Some evidence of applying limits of $\frac{\pi}{4}$ and 0 (0 can be implied) to an integrated function in θ
	5th M1	Applies $V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their part}\,(a)\text{ answer})$.
	Note	Also allow the 5th M1 for $V_{\text{cone}} = \pi\int_{\text{their }\frac{5}{3}}^{3}\left(\frac{3}{2}x - \frac{5}{2}\right)^2\{dx\}$, which includes the correct limits.
	4th A1	$\frac{92}{9}\pi + 6\pi^2$ or $10\frac{2}{9}\pi + 6\pi^2$
	Note	A decimal answer of 91.33168464... (without a correct **exact** answer) is A0.
	Note	The π in the volume formula is only needed for the 1st A1 mark and the final accuracy mark.

7.		**<u>Working with a Cartesian Equation</u>** A cartesian equation for C is $y = \dfrac{36}{x^2 + 9}$
(a)	**1st M1** **1st A1** **2nd dM1**	$\dfrac{dy}{dx} = \pm\lambda x\left(\pm\alpha x^2 \pm \beta\right)^{-2}$ **or** $\dfrac{dy}{dx} = \dfrac{\pm\lambda x}{\left(\pm\alpha x^2 \pm \beta\right)^2}$ $\dfrac{dy}{dx} = -36(x^2+9)^{-2}(2x)$ **or** $\dfrac{dy}{dx} = \dfrac{-72x}{(x^2+9)^2}$ un-simplified or simplified. **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ into their $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $\dfrac{dy}{dx} = \dfrac{-72(3)}{(3^2+9)^2}\left\{= -\dfrac{2}{3}\right\}$ From this point onwards the original scheme can be applied.
(b)	**1st M1** **A1**	For $\displaystyle\int\left(\frac{\pm\lambda}{\pm\alpha x^2 \pm \beta}\right)^2 \{dx\}$ (π not required for this mark) For $\pi\displaystyle\int\left(\frac{36}{x^2+9}\right)^2 \{dx\}$ (π required for this mark) To integrate, a substitution of $x = 3\tan\theta$ is required which will lead to $\displaystyle\int 48\cos^2\theta\, d\theta$ and so from this point onwards the original scheme can be applied.
(a)	**1st M1** **1st A1** **2nd dM1**	Another cartesian equation for C is $x^2 = \dfrac{36}{y} - 9$ $\pm\alpha x = \pm\dfrac{\beta}{y^2}\dfrac{dy}{dx}$ or $\pm\alpha x\dfrac{dx}{dy} = \pm\dfrac{\beta}{y^2}$ $2x = -\dfrac{36}{y^2}\dfrac{dy}{dx}$ or $2x\dfrac{dx}{dy} = -\dfrac{36}{y^2}$ **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ to find $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $2(3) = -\dfrac{36}{4}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = ...$ From this point onwards the original scheme can be applied.

Question Number	Scheme		Marks
8.	$\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm((-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}));= \mathbf{i} - \mathbf{j} + \mathbf{k}$		M1; A1
			[2]
(b)	$\{l_1 : \mathbf{r}\ \} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$		B1ft
			[1]
(c)	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$		M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{PB}\rvert} = \dfrac{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} . \sqrt{(-1)^2 + (1)^2 + (5)^2}}$	Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{PB}$ or $\overrightarrow{BP})$.	M1
	$\{\cos\theta\} = \dfrac{-1-1+5}{\sqrt{3}.\sqrt{27}} = \dfrac{3}{9} = \underline{\dfrac{1}{3}}$	Correct proof	A1 cso
			[3]
(d)	$\{l_2 : \mathbf{r} =\} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	$\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$, $\mathbf{p} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$.	M1
		Correct vector equation.	A1 ft
			[2]
(e)	$\overrightarrow{OC} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$	M1
		At least one set of coordinates are correct.	A1 ft
	$\{C(1, 1, 4),\ D(-1, 3, 2)\}$	Both sets of coordinates are correct.	A1 ft
			[3]
(f) **Way 1**	$\dfrac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin\theta$	$\dfrac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$	M1
	$h = \sqrt{27}\sin(70.5...) \left\{ = \sqrt{27}\dfrac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9 \right\}$	$\sqrt{27}\sin(70.5...)$ or $\sqrt{27}.\dfrac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent	A1 oe
	Area $ABCD = \dfrac{1}{2} 2\sqrt{6}\left(\sqrt{3} + 2\sqrt{3}\right)$	$\dfrac{1}{2}(\text{their } h)(\text{their } AB + \text{their } CD)$	dM1
	$\left\{= \dfrac{1}{2} 2\sqrt{6}\left(3\sqrt{3}\right) = 3\sqrt{18}\right\} = \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cao**
			[4] **15**

8. (f) ***Helpful Diagram!***

Area $\Delta\,APB = 4.2426...$



$\overrightarrow{DA} = \overrightarrow{PB} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$

$\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$

	$\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$, so $BC \perp AB$	Candidates do not need to prove this result for part (f)	
8. (f) **Way 2**	$h = \lvert\overrightarrow{CB}\rvert = \sqrt{(-2)^2 + (2)^2 + (4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989...$	Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$	M1
		$\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$	A1 oe
	Area $ABCD = \frac{1}{2}\sqrt{24}\left(\sqrt{3} + 2\sqrt{3}\right)$ or $\frac{1}{2}\sqrt{24}\sqrt{3} + \sqrt{24}\sqrt{3}$	$\frac{1}{2}h(\text{their } AB + \text{their } CD)$	dM1 oe
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cso**
			[4]

Way3	<u>**Finds the area of either triangle *APB* or *APD* or *BCP* and triples the result.**</u>		
8. (f)	Area $\Delta\,APB = \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\theta$	Attempts $\frac{1}{2}$ (their AB)(their PB) $\sin\theta$	M1
	$= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$	$\frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$ or $3\sqrt{2}$ or awrt 4.24 or equivalent	A1
	Area $ABCD = 3\left(3\sqrt{2}\right)$	$3 \times$ Area of $\Delta\,APB$	dM1
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cso**
			[4]

	Question 8 Notes	
8. (a)	**M1**	Finding the difference (either way) between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	A1	$\mathbf{i}-\mathbf{j}+\mathbf{k}$ or $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $(1, -1, 1)$ or benefit of the doubt $\begin{matrix}1\\-1\\1\end{matrix}$
(b)	**B1ft**	$\{\mathbf{r}\}=\begin{pmatrix}-2\\4\\7\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ **or** $\{\mathbf{r}\}=\begin{pmatrix}-1\\3\\8\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$, with $\overrightarrow{AB}$ or $\overrightarrow{BA}$ correctly followed through from (a).
	Note	$\mathbf{r} =$ is not needed.
(c)	**M1**	An attempt to find either the vector $\overrightarrow{PB}$ or $\overrightarrow{BP}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	M1	Applies dot product formula between their $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB}\text{ or }\overrightarrow{BP}\right)$.
	A1	Obtains $\{\cos\theta\}=\frac{1}{3}$ ***by correct solution only.***
	Note	If candidate starts by applying $\dfrac{\overrightarrow{AB}\bullet\overrightarrow{PB}}{\left\lvert\overrightarrow{AB}\right\rvert.\left\lvert\overrightarrow{PB}\right\rvert}$ correctly (without reference to $\cos\theta = ...$) they can gain both 2nd M1 and A1 mark.
	Note	Award the final A1 mark if candidate achieves $\{\cos\theta\}=\frac{1}{3}$ by either taking the dot product between (i) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ **or** (ii) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly.
	Note	Award final A0, cso for those candidates who take the dot product between (iii) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$ or (iv) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ They will usually find $\{\cos\theta\}=-\frac{1}{3}$ or may fudge $\{\cos\theta\}=\frac{1}{3}$. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso

(c)	**Alternative Method 1: The Cosine Rule**		
	$\overrightarrow{PB}=\overrightarrow{OB}-\overrightarrow{OP}=\begin{pmatrix}-1\\3\\8\end{pmatrix}-\begin{pmatrix}0\\2\\3\end{pmatrix}=\begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP}=\begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Note $\left\lvert\overrightarrow{PB}\right\rvert=\sqrt{27}$, $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{3}$ and $\left\lvert\overrightarrow{PA}\right\rvert=\sqrt{24}$		
	$\left(\sqrt{24}\right)^2=\left(\sqrt{27}\right)^2+\left(\sqrt{3}\right)^2-2\left(\sqrt{27}\right)\left(\sqrt{3}\right)\cos\theta$	Applies the cosine rule the correct way round	M1 oe
	$\cos\theta = \dfrac{27+3-24}{18}=\dfrac{1}{3}$	Correct proof	A1 cso
			[3]

8. (c)	**Alternative Method 2: Right-Angled Trigonometry**		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Either $\left(\sqrt{24}\right)^2 + \left(\sqrt{3}\right)^2 = \left(\sqrt{27}\right)^2$ or $\overrightarrow{AB} \bullet \overrightarrow{PA} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} = -2 - 2 + 4 = 0$	Confirms ΔPAB is right-angled	M1
	So, $\left\{\cos\theta = \frac{AB}{PB} \Rightarrow \right\} \cos\theta = \frac{\sqrt{3}}{\sqrt{27}} = \frac{1}{3}$	Correct proof	A1 cso
			[3]

(d)	**M1**	Writing down a line in the form $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$ with either $\mathbf{a} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$ $\mathbf{d} =$ their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$ found in part (a).
	A1ft	Writing $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d} =$ their $\overrightarrow{AB}$ or a multiple of their $\overrightarrow{AB}$ found in part (a).
	Note	$\mathbf{r} =$ is not needed.
	Note	Using the same scalar parameter as in part (b) is fine for A1.
(e)	**M1**	Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ – their $\overrightarrow{AB}$.
	Note	This can be implied at least two out of three correct components for either their *C* or their *D*.
	A1ft	At least one set of coordinates are correct. Ignore labelling of C, D
	A1ft	Both sets of coordinates are correct. Ignore labelling of C, D
	Note	You can follow through either or both accuracy marks in this part using their $\overrightarrow{AB}$ from part (a).
(f)	**M1**	Way 1: $\frac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$ Way 2: Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$ Way 3: Attempts $\frac{1}{2}$ (their PB)(their AB)$\sin\theta$
	Note	Finding *AD* by itself is M0.
	A1	Either • $h = \sqrt{27}\sin(70.5...)$ or $\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$ or equivalent. (See Way 1 and Way 2) or • the area of either triangle *APB* or *APD* or *BDP* $= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\left(70.5...\right)$ o.e. (See Way 3).
	dM1	**which is dependent on the 1st M1 mark.** A full method to find the area of trapezium *ABCD*. (See Way 1, Way 2 and Way 3).
	A1	$9\sqrt{2}$ from a correct solution only.
	Note	A decimal answer of 12.7279... (without a correct **exact** answer) is A0.