Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 20 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38160A

1. $$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}.$$

Find the values of the constants A, B and C.

(4)

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2. $$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}.$$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

3.

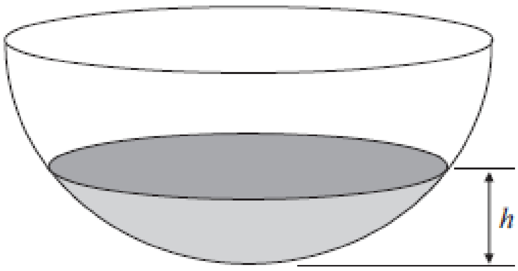

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.

When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3-4h), \qquad 0 \leq h \leq 0.25.$$

(*a*) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$.

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^3 s^{-1}.

(*b*) Find the rate of change of h, in m s^{-1}, when $h = 0.1$.

(2)

4.

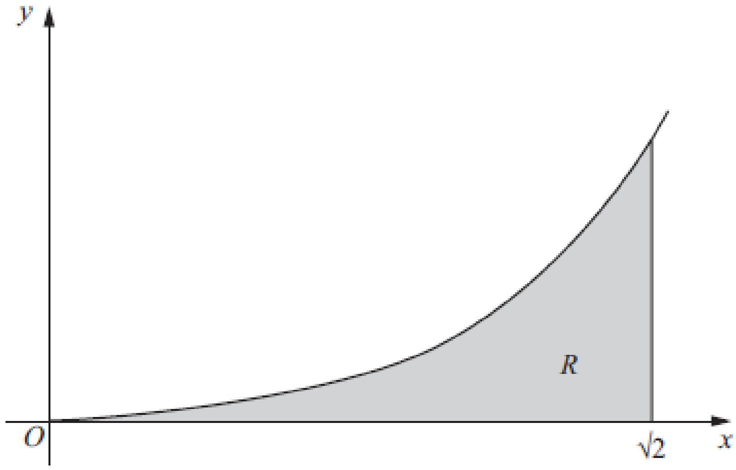

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln (x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln (x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(*a*) Complete the table above giving the missing values of y to 4 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(*c*) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_{2}^{4} (u-2)\ln u \ \mathrm{d}u .$$

(4)

(*d*) Hence, or otherwise, find the exact area of R.

(6)

5. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \qquad x > 0, \quad y > 0,$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

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6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 \colon \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2 \colon \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where μ and λ are scalar parameters.

(*a*) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.

(6)

(*b*) Find, to the nearest 0.1°, the acute angle between l_1 and l_2.

(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(*c*) Show that B lies on l_1.

(1)

(*d*) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.

(4)

7.

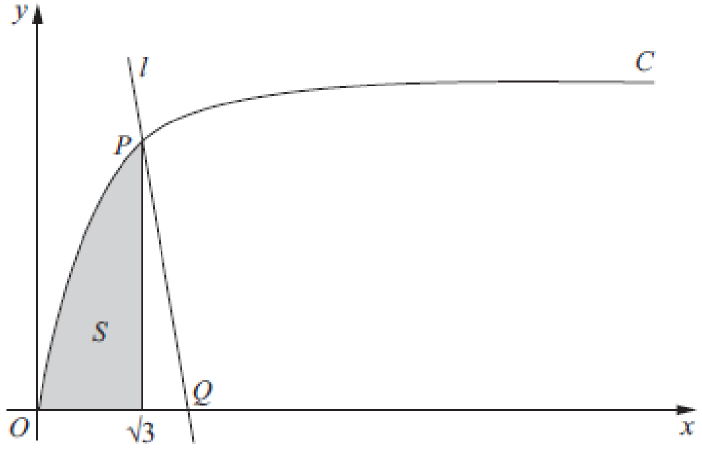


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leq \theta < \frac{\pi}{2}.$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

8. (*a*) Find $\int (4y+3)^{-\frac{1}{2}} \, dy$.

(2)

(*b*) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{(4y+3)}}{x^2},$$

giving your answer in the form $y = f(x)$.

(6)

TOTAL FOR PAPER: 75 MARKS

END

June 2011
Core Mathematics C4 6666
Mark Scheme

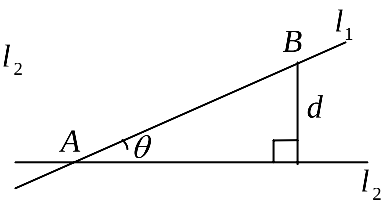
Question Number	Scheme		Marks	
1.	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$		B1	
	$x \to 1$ $\quad 9 = 3B \Rightarrow B = 3$		M1	
	$x \to -\frac{1}{2}$ $\quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	Any two of *A*, *B*, *C*	A1	
	x^2 terms $\quad 9 = 2A + C \Rightarrow A = 4$	All three correct	A1	**(4)** **[4]**
	Alternatives for finding A. x terms $\quad 0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $\quad 0 = -A + B + C \Rightarrow A = 4$			

Question Number	Scheme		Marks	
2.	$\text{f}(x) = (\ \ldots\ +\ \ldots)^{-\frac{1}{2}}$		M1	
	$= 9^{-\frac{1}{2}}(\ \ldots\ +\ \ldots)^{\cdots}$	$3^{-1},\ \frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$	B1	
	$(1+kx^2)^n =\ 1+nkx^2+\ \ldots$	n not a natural number, $k \neq 1$	M1	
	$(1+kx^2)^{-\frac{1}{2}} =\ \ldots\ + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$	ft their $k \neq 1$	A1 ft	
	$\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1-\frac{2}{9}x^2+\frac{2}{27}x^4$		A1	
	$\text{f}(x) = \frac{1}{3}-\frac{2}{27}x^2+\frac{2}{81}x^4$		A1	**(6)**
				[6]

Question Number	Scheme		Marks	
3.	(a) $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{2}\pi h - \pi h^2$	or equivalent	M1 A1	
	At $h = 0.1$, $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$	M1 A1	**(4)**
	(b) $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031	A1	**(2)**
				[6]

Question Number	Scheme	Marks
4.	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 **(2)**
	(b) $\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}[\ \ldots]$	B1
	$\approx \ \ldots \left[0+2(0.0333+0.3240+1.3596)+3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 **(3)**
	(c) $u = x^2+2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$	B1
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,\mathrm{d}x$	B1
	$\int x^3 \ln(x^2+2)\,\mathrm{d}x = \int x^2 \ln(x^2+2)\,x\,\mathrm{d}x = \int (u-2)(\ln u)\tfrac{1}{2}\,\mathrm{d}u$	M1
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,\mathrm{d}u$ ✱ cso	A1 **(4)**
	(d) $\int (u-2)\ln u\,\mathrm{d}u = \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u^2}{2}-2u\right)\frac{1}{u}\,\mathrm{d}u$	M1 A1
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u}{2}-2\right)\mathrm{d}u$	
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right) \quad (+C)$	M1 A1
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\right]_2^4$	
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - \left((2-4)\ln 2 - 1 + 4\right)\right]$	M1
	$= \tfrac{1}{2}(2\ln 2 + 1)$ $\ln 2 + \tfrac{1}{2}$	A1 **(6)**
		[15]

Question Number	Scheme	Marks
5.	$\frac{1}{y}\frac{dy}{dx} = \ldots$	B1
	$\ldots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1
	At $x = 2$, $\quad \ln y = 2(2)\ln 2$	M1
	leading to $\quad y = 16 \quad$ Accept $y = e^{4\ln 2}$	A1
	At $(2, 16)$ $\quad \frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$	M1
	$\frac{dy}{dx} = 16(2 + 2\ln 2)$	A1 **(7)**
		[7]
	Alternative	
	$y = e^{2x\ln x}$	B1
	$\frac{d}{dx}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1
	$\frac{dy}{dx} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$	M1 A1
	At $x = 2$, $\quad \frac{dy}{dx} = (2\ln 2 + 2)e^{4\ln 2}$	M1
	$= 16(2 + 2\ln 2)$	A1 **(7)**

Question Number	Scheme	Marks
6.	(a) **i**: $6-\lambda=-5+2\mu$	
	j: $-3+2\lambda=15-3\mu$ Any two equations	M1
	leading to $\lambda=3,\ \mu=4$	M1 A1
	$\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$	M1 A1
	k: LHS $=-2+3(3)=7$, RHS $=3+4(1)=7$	B1 **(6)**
	(As LHS = RHS, lines intersect)	
	Alternatively for B1, showing that $\lambda=3$ and $\mu=4$ both give $\begin{pmatrix}3\\3\\7\end{pmatrix}$	
	(b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\sqrt{14}\sqrt{14}\cos\theta$ $(\theta\approx 110.92°)$	M1 A1
	Acute angle is $69.1°$ awrt 69.1	A1 **(3)**
	(c) $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix}$ $(\Rightarrow B$ lies on $l_1)$	B1 **(1)**
	(d) Let d be shortest distance from B to l_2	
		
	$\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$	M1
	$\lvert\overrightarrow{AB}\rvert=\sqrt{\left(2^2+(-4)^2+(-6)^2\right)}=\sqrt{56}$ awrt 7.5	A1
	$\dfrac{d}{\sqrt{56}}=\sin\theta$	M1
	$d=\sqrt{56}\sin 69.1°\approx 6.99$ awrt 6.99	A1 **(4)**
		[14]

Question Number	Scheme	Marks
7.	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$	M1
	$\theta = \frac{\pi}{3}$ awrt 1.05	A1 **(2)**
	(b) $\frac{dx}{d\theta} = \sec^2\theta, \quad \frac{dy}{d\theta} = \cos\theta$	
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta} \quad \left(= \cos^3\theta\right)$	M1 A1
	At *P*, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$ Can be implied	A1
	Using $mm' = -1$, $m' = -8$	M1
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	M1
	At *Q*, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$ 1.0625	A1 **(6)**
	(c) $\int y^2\,dx = \int y^2 \frac{dx}{d\theta}\,d\theta = \int \sin^2\theta \sec^2\theta\,d\theta$	M1 A1
	$= \int \tan^2\theta\,d\theta$	A1
	$= \int \left(\sec^2\theta - 1\right)d\theta$	M1
	$= \tan\theta - \theta \quad (+C)$	A1
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0 - 0)\right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2 \qquad \left(p = 1, q = -\frac{1}{3}\right)$	A1 **(7)**
		[15]

Question Number	Scheme	Marks
8.	(a) $\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}x = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1 **(2)**
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
	(b) $\displaystyle\int \frac{1}{\sqrt{(4y+3)}}\,\mathrm{d}y = \int \frac{1}{x^2}\,\mathrm{d}x$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}y = \int x^{-2}\,\mathrm{d}x$	
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \dfrac{1}{2}(4\times 1.5+3)^{\frac{1}{2}} = -\dfrac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \dfrac{2}{x}$	M1
	$y = \dfrac{1}{4}\left(2 - \dfrac{2}{x}\right)^2 - \dfrac{3}{4}$ or equivalent	A1 **(6)**
		[8]