Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Wednesday 26 January 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35405A

1. Use integration to find the exact value of $\int_0^{\frac{\pi}{2}} x \sin 2x \quad dx$.

(6)

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2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form ln a, where a is a constant.

(5)

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3. (*a*) Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(*b*) Hence find $\int \frac{5}{(x-1)(3x+2)} \, dx$, where $x > 1$.

(3)

(*c*) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$.

(6)

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4. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(*a*) Find $\overrightarrow{AB}$.

(2)

(*b*) Find a vector equation of l.

(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant.

Given that AC is perpendicular to l, find

(*c*) the value of p,

(4)

(*d*) the distance AC.

(2)

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5. (*a*) Use the binomial theorem to expand

$$(2 - 3x)^{-2}, \quad |x| < \frac{2}{3},$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

$$\mathrm{f}(x) = \frac{a + bx}{(2 - 3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$.

Find

(*b*) the value of a and the value of b,

(5)

(*c*) the coefficient of x^3, giving your answer as a simplified fraction.

(3)

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6. The curve C has parametric equations

$$x = \ln t, \qquad y = t^2 - 2, \qquad t > 0.$$

Find

(a) an equation of the normal to C at the point where $t = 3$,

(6)

(b) a cartesian equation of C.

(3)

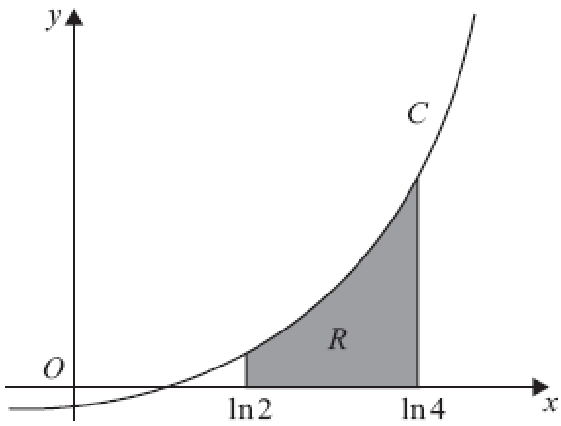

Figure 1

The finite area R, shown in Figure 1, is bounded by C, the x-axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through 360° about the x-axis.

(c) Use calculus to find the exact volume of the solid generated.

(6)

7. $$I = \int_2^5 \frac{1}{4 + \sqrt{(x-1)}}\,dx.$$

(a) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

(c) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I.

(8)

TOTAL FOR PAPER: 75 MARKS

END

January 2011
Core Mathematics C4 6666
Mark Scheme

Question Number	Scheme	Marks
1.	$\int x \sin 2x \, \mathrm{d}x = -\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \mathrm{d}x$	M1 A1 A1
	$= \quad \ldots \quad + \frac{\sin 2x}{4}$	M1
	$\left[\ \ldots \ \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	M1 A1
		[6]
2.	$\frac{\mathrm{d}I}{\mathrm{d}t} = -16 \ln(0.5) 0.5^t$	M1 A1
	At $t = 3$ $\quad \frac{\mathrm{d}I}{\mathrm{d}t} = -16 \ln(0.5) 0.5^3$	M1
	$= -2 \ln 0.5 = \ln 4$	M1 A1
		[5]

Question Number	Scheme	Marks
3. (a)	$\dfrac{5}{(x-1)(3x+2)} = \dfrac{A}{x-1} + \dfrac{B}{3x+2}$	
	$5 = A(3x+2) + B(x-1)$	
	$x \to 1$ $\quad 5 = 5A \Rightarrow A = 1$	M1 A1
	$x \to -\dfrac{2}{3}$ $\quad 5 = -\dfrac{5}{3}B \Rightarrow B = -3$	A1 (3)
(b)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x = \int \left(\frac{1}{x-1} - \frac{3}{3x+2}\right)\mathrm{d}x$	
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	M1 A1ft A1ft
		(3)
(c)	$\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x = \int \left(\frac{1}{y}\right)\mathrm{d}y$	M1
	$\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$	M1 A1
	$y = \dfrac{K(x-1)}{3x+2}$ depends on first two Ms in (c)	M1 dep
	Using $(2, 8)$ $\quad 8 = \dfrac{K}{8}$ depends on first two Ms in (c)	M1 dep
	$y = \dfrac{64(x-1)}{3x+2}$	A1 (6)
		[12]

Question Number	Scheme	Marks
4. **(a)**	$\overrightarrow{AB} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	M1 A1 (2)
(b)	$\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ or $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	M1 A1ft (2)
(c)	$\overrightarrow{AC} = 2\mathbf{i} + p\mathbf{j} - 4\mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ $= \mathbf{i} + (p+3)\mathbf{j} - 6\mathbf{k}$ or $\overrightarrow{CA}$ $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} . \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$ $-3 + 5p + 15 + 18 = 0$ Leading to $p = -6$	 B1 M1 M1 A1 (4)
(d)	$AC^2 = (2-1)^2 + (-6+3)^2 + (-4-2)^2 \quad (= 46)$ $AC = \sqrt{46}$ accept awrt 6.8	M1 A1 (2) **[10]**

Question Number	Scheme	Marks
5.		
(a)	$(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$	B1
	$\left(1-\frac{3}{2}x\right)^{-2} = 1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^2+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^3+\ \dots$	M1 A1
	$=1+3x+\frac{27}{4}x^2+\frac{27}{2}x^3+\ \dots$	
	$(2-3x)^{-2}=\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \dots$	M1 A1 (5)
(b)	$\mathrm{f}(x)=(a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2+\frac{27}{8}x^3+\ \dots\right)$	
	Coefficient of x; $\frac{3a}{4}+\frac{b}{4}=0$ $(3a+b=0)$	M1
	Coefficient of x^2; $\frac{27a}{16}+\frac{3b}{4}=\frac{9}{16}$ $(9a+4b=3)$ A1 either correct	M1 A1
	Leading to $a=-1,\ b=3$	M1 A1 (5)
(c)	Coefficient of x^3 is $\frac{27a}{8}+\frac{27b}{16}=\frac{27}{8}\times(-1)+\frac{27}{16}\times 3$	M1 A1ft
	$=\frac{27}{16}$ cao	A1 (3)
		[13]

Question Number	Scheme	Marks
6.		
(a)	$\dfrac{dx}{dt}=\dfrac{1}{t}, \quad \dfrac{dy}{dt}=2t$	
	$\dfrac{dy}{dx}=2t^2$	M1 A1
	Using $mm'=-1$, at $t=3$	
	$m'=-\dfrac{1}{18}$	M1 A1
	$y-7=-\dfrac{1}{18}(x-\ln 3)$	M1 A1 (6)
(b)	$x=\ln t \Rightarrow t=e^x$	B1
	$y=e^{2x}-2$	M1 A1 (3)
(c)	$V=\pi\int\left(e^{2x}-2\right)^2 dx$	M1
	$\int\left(e^{2x}-2\right)^2 dx=\int\left(e^{4x}-4e^{2x}+4\right)dx$	M1
	$=\dfrac{e^{4x}}{4}-\dfrac{4e^{2x}}{2}+4x$	M1 A1
	$\pi\left[\dfrac{e^{4x}}{4}-\dfrac{4e^{2x}}{2}+4x\right]_{\ln 2}^{\ln 4}=\pi\left[(64-32+4\ln 4)-(4-8+4\ln 2)\right]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6) **[15]**
	Alternative to (c) *using parameters*	
	$V=\pi\int\left(t^2-2\right)^2\dfrac{dx}{dt}dt$	M1
	$\int\left(\left(t^2-2\right)^2\times\dfrac{1}{t}\right)dt=\int\left(t^3-4t+\dfrac{4}{t}\right)dt$	M1
	$=\dfrac{t^4}{4}-2t^2+4\ln t$	M1 A1
	The limits are $t=2$ and $t=4$	
	$\pi\left[\dfrac{t^4}{4}-2t^2+4\ln t\right]_2^4=\pi\left[(64-32+4\ln 4)-(4-8+4\ln 2)\right]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6)

Question Number	Scheme	Marks
7. **(a)**	$x=3 \Rightarrow y=0.1847$ awrt	B1
	$x=5 \Rightarrow y=0.1667$ awrt or $\frac{1}{6}$	B1
		(2)
(b)	$I \approx \frac{1}{\underline{\underline{2}}}\left[0.2+0.1667+2\left(0.1847+0.1745\right)\right]$	$\underline{\underline{\text{B1}}}$ M1 A1ft
	≈ 0.543 0.542 or 0.543	A1 (4)
(c)	$\frac{dx}{du}=2(u-4)$	B1
	$\int \frac{1}{4+\sqrt{(x-1)}}\,dx=\int \frac{1}{u}\times 2(u-4)\,du$	M1
	$=\int\left(2-\frac{8}{u}\right)du$	A1
	$=2u-8\ln u$	M1 A1
	$x=2 \Rightarrow u=5,\ \ x=5 \Rightarrow u=6$	B1
	$\left[2u-8\ln u\right]_5^6=(12-8\ln 6)-(10-8\ln 5)$	M1
	$=2+8\ln\left(\frac{5}{6}\right)$	A1
		(8)
		[14]