Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Friday 18 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1.

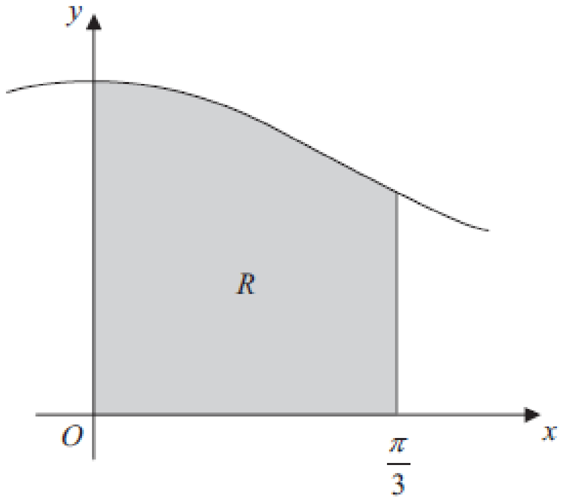

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(*a*) Copy and complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(*b*) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R.

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places.

(6)

2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x+1} \sin x \ \ dx = e(e-1).$$

(6)

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3. A curve C has equation

$$2^x + y^2 = 2xy.$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).

(7)

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4. A curve C has parametric equations

$$x = \sin^2 t, \ y = 2 \tan t \ , \ 0 \leq t < \frac{\pi}{2}.$$

(*a*) Find $\frac{dy}{dx}$ in terms of t.

(4)

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(*b*) Find the x-coordinate of P.

(6)

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5. $$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2.

Give each coefficient as a simplified fraction.

(7)

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6. $$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(*a*) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(*b*) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta\, f(\theta)\ d\theta$.

(7)

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7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(*a*) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(*b*) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(*c*) Hence, or otherwise, find the area of the triangle ABC.

(5)

8.

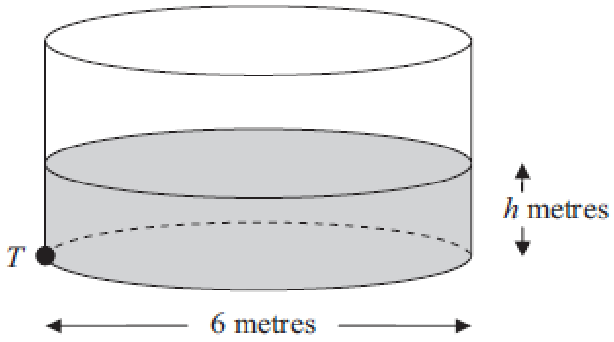

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(a) Show that, t minutes after the tap has been opened,

$$75\frac{\mathrm{d}h}{\mathrm{d}t} = (4 - 5h).$$

(5)

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$

(6)

TOTAL FOR PAPER: 75 MARKS

END

June 2010
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks	
1.	(a) $y\left(\frac{\pi}{6}\right) \approx 1.2247, \quad y\left(\frac{\pi}{4}\right) = 1.1180$	accept awrt 4 d.p.	B1 B1	**(2)**
	(b)(i) $I \approx \left(\frac{\pi}{12}\right)\left(1.3229 + 2 \times 1.2247 + 1\right)$	B1 for $\frac{\pi}{12}$	B1 M1	
	≈ 1.249	cao	A1	
	(ii) $I \approx \left(\frac{\pi}{24}\right)\left(1.3229 + 2 \times \left(1.2973 + 1.2247 + 1.1180\right) + 1\right)$	B1 for $\frac{\pi}{24}$	B1 M1	
	≈ 1.257	cao	A1	**(6)** **[8]**

Question Number	Scheme		Marks	
2.	$\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$		B1	
	$\int \sin x \, \mathrm{e}^{\cos x + 1} \mathrm{d}x = -\int \mathrm{e}^{u} \mathrm{d}u$		M1 A1	
	$= -\mathrm{e}^{u}$	ft sign error	A1ft	
	$= -\mathrm{e}^{\cos x + 1}$			
	$\left[-\mathrm{e}^{\cos x + 1}\right]_{0}^{\frac{\pi}{2}} = -\mathrm{e}^{1} - \left(-\mathrm{e}^{2}\right)$	or equivalent with u	M1	
	$= \mathrm{e}\left(\mathrm{e} - 1\right)$ ∗	cso	A1	**(6)**
				[6]

Question Number	Scheme	Marks
3.	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2^x\right) = \ln 2.2^x$	B1
	$\ln 2.2^x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + 6\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4\ln 2 - 2$ Accept exact equivalents	M1 A1 **(7)**
		[7]

Question Number	Scheme		Marks
4.	(a) $\dfrac{dx}{dt} = 2\sin t\cos t,\ \dfrac{dy}{dt} = 2\sec^2 t$		B1 B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 t}{\sin t\cos t}\left(= \dfrac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1 **(4)**
	(b) At $t = \dfrac{\pi}{3},\ \ x = \dfrac{3}{4},\ \ y = 2\sqrt{3}$		B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \dfrac{16}{\sqrt{3}}$		M1 A1
	$y - 2\sqrt{3} = \dfrac{16}{\sqrt{3}}\left(x - \dfrac{3}{4}\right)$		M1
	$y = 0 \quad \Rightarrow \quad x = \dfrac{3}{8}$		M1 A1 **(6)**
			[10]

Question Number	Scheme	Marks
5.	(a) $A = 2$	B1
	$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$	
	$x \to 1$ $\quad -3 = 3B \Rightarrow B = -1$	M1 A1
	$x \to -2$ $\quad -12 = -3C \Rightarrow C = 4$	A1 **(4)**
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$	M1
	$(1-x)^{-1} = 1 + x + x^2 + \ ...$	B1
	$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ ...$	B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ ...$	M1
	$= 5 + \ ...$ ft their $A - B + \frac{1}{2}C$	A1 ft
	$= \ ... + \dfrac{3}{2}x^2 + \ ...$ $0x$ stated or implied	A1 A1 **(7)**
		[11]

Question Number	Scheme	Marks
6.	(a) $\mathrm{f}(\theta) = 4\cos^2\theta - 3\sin^2\theta$	
	$= 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$	M1 M1
	$= \frac{1}{2} + \frac{7}{2}\cos 2\theta$ ∗ cso	A1 **(3)**
	(b) $\int \theta \cos 2\theta \, \mathrm{d}\theta = \frac{1}{2}\theta \sin 2\theta - \frac{1}{2}\int \sin 2\theta \, \mathrm{d}\theta$	M1 A1
	$= \frac{1}{2}\theta \sin 2\theta + \frac{1}{4}\cos 2\theta$	A1
	$\int \theta \mathrm{f}(\theta) \, \mathrm{d}\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta \sin 2\theta + \frac{7}{8}\cos 2\theta$	M1 A1
	$\left[\ \ldots \ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8}\right] - \left[0 + 0 + \frac{7}{8}\right]$	M1
	$= \frac{\pi^2}{16} - \frac{7}{4}$	A1 **(7)**
		[10]

Question Number	Scheme	Marks
7.	(a) **j** components $3+2\lambda=9 \Rightarrow \lambda=3$ $(\mu=1)$	M1 A1
	Leading to $C:(5,9,-1)$ accept vector forms	A1 **(3)**
	(b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$	M1
	$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$ use of scalar product	M1 A1
	$\angle ACB=57.95°$ awrt $57.95°$	A1 **(4)**
	(c) $A:(2,3,-4)$ $B:(-5,9,-5)$	
	$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$	
	$AC^2=3^2+6^2+3^2 \Rightarrow AC=3\sqrt{6}$	M1 A1
	$BC^2=10^2+4^2 \Rightarrow BC=2\sqrt{29}$	A1
	$\triangle ABC=\frac{1}{2}AC\times BC\sin\angle ACB$	
	$=\frac{1}{2}3\sqrt{6}\times2\sqrt{29}\sin\angle ACB\approx 33.5$ $15\sqrt{5}$, awrt 34	M1 A1 **(5)**
		[12]
	Alternative method for (b) *and* (c)	
	(b) $A:(2,3,-4)$ $B:(-5,9,-5)$ $C:(5,9,-1)$	
	$AB^2=7^2+6^2+1^2=86$	
	$AC^2=3^2+6^2+3^2=54$	
	$BC^2=10^2+0^2+4^2=116$ Finding all three sides	M1
	$\cos\angle ACB=\dfrac{116+54-86}{2\sqrt{116}\sqrt{54}}$ $(=0.53066\ ...)$	M1 A1
	$\angle ACB=57.95°$ awrt $57.95°$	A1 **(4)**
	If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c).	

Question Number	Scheme		Marks
8.	(a) $\dfrac{dV}{dt} = 0.48\pi - 0.6\pi h$		M1 A1
	$V = 9\pi h \quad \Rightarrow \dfrac{dV}{dt} = 9\pi \dfrac{dh}{dt}$		B1
	$9\pi \dfrac{dh}{dt} = 0.48\pi - 0.6\pi h$		M1
	Leading to $\quad 75\dfrac{dh}{dt} = 4 - 5h$ $\ast$	cso	A1 **(5)**
	(b) $\displaystyle\int \frac{75}{4-5h}\,dh = \int 1\,dt$	separating variables	M1
	$-15\ln(4-5h) = t \ (+C)$		M1 A1
	$-15\ln(4-5h) = t + C$		
	When $t = 0$, $h = 0.2$		
	$-15\ln 3 = C$		M1
	$t = 15\ln 3 - 15\ln(4-5h)$		
	When $h = 0.5$		
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	M1 A1
	Alternative for last 3 *marks*		
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$		
	$= -15\ln 1.5 + 15\ln 3$		M1 M1
	$= 15\ln\left(\dfrac{3}{1.5}\right) = 15\ln 2$	awrt 10.4	A1 **(6)**