Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 25 January 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (*a*) Find the binomial expansion of

$$\sqrt{(1 - 8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1 - 8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)

2.

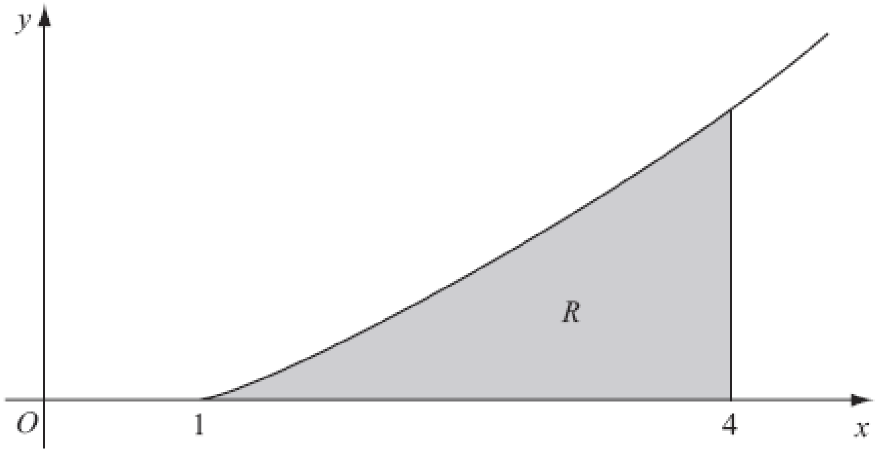

Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(*a*) Copy and complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) (i) Use integration by parts to find $\int x \ln x \; \mathrm{d}x$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

3. The curve C has equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}.$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(*b*) Find the value of y at P.

(3)

(*c*) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)

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4. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(*a*) Write down the coordinates of A.

(1)

(*b*) Find the value of $\cos \theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(*c*) Find the coordinates of X.

(1)

(*d*) Find the vector $\overrightarrow{AX}$.

(2)

(*e*) Hence, or otherwise, show that $\left| \overrightarrow{AX} \right| = 4\sqrt{26}$.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(*f*) find the length of AY, giving your answer to 3 significant figures.

(3)

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5. (*a*) Find $\int \frac{9x+6}{x}\,dx$, $x > 0$.

(2)

(*b*) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

(6)

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6. The area A of a circle is increasing at a constant rate of 1.5 cm^2 s^{-1}. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2.

(5)

7.

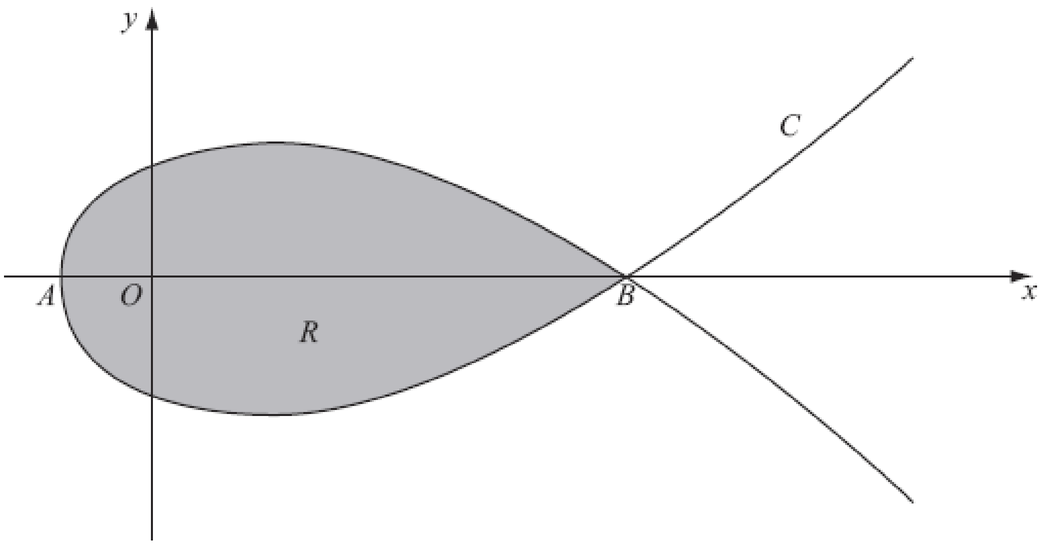


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

8. (*a*) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,dx .$$

(7)

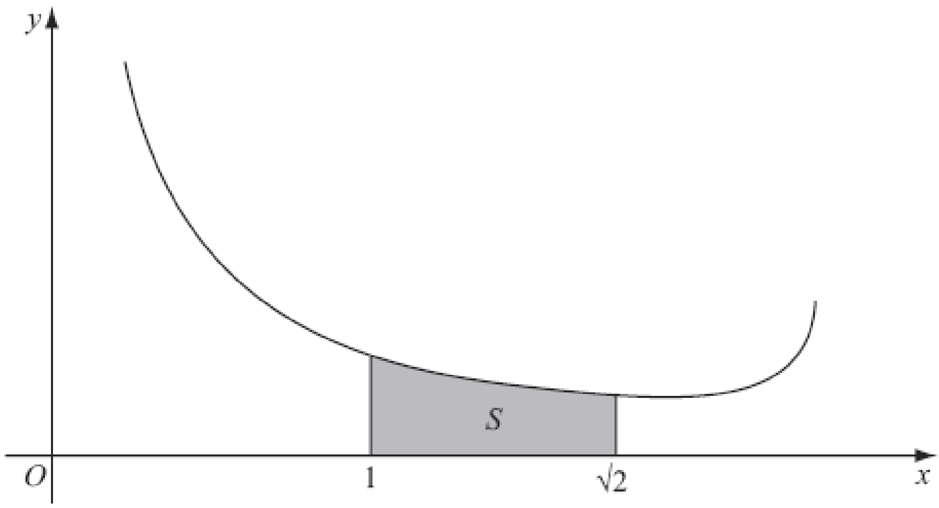


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region S, shown in Figure 3, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Using your answer to part (*a*), find the exact volume of the solid of revolution formed.

(3)

TOTAL FOR PAPER: 75 MARKS

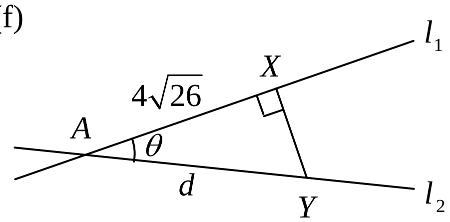
END

January 2010
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks	
Q1	(a) $(1-8x)^{\frac{1}{2}} = 1 + \left(\tfrac{1}{2}\right)(-8x) + \dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)}{2}(-8x)^2 + \dfrac{\left(\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)}{3!}(-8x)^3 + \dots$		M1 A1	
	$= 1 - 4x - 8x^2; -32x^3 - \dots$		A1; A1	(4)
	(b) $\sqrt{(1-8x)} = \sqrt{\left(1 - \dfrac{8}{100}\right)}$		M1	
	$= \sqrt{\dfrac{92}{100}} = \sqrt{\dfrac{23}{25}} = \dfrac{\sqrt{23}}{5}$ *	cso	A1	(2)
	(c) $1 - 4x - 8x^2 - 32x^3 = 1 - 4(0.01) - 8(0.01)^2 - 32(0.01)^3$			
	$= 1 - 0.04 - 0.0008 - 0.000\,032 = 0.959\,168$		M1	
	$\sqrt{23} = 5 \times 0.959\,168$		M1	
	$= 4.795\,84$	cao	A1	(3)
				[9]

Question Number	Scheme		Marks	
Q2	(a) 1.386, 2.291	awrt 1.386, 2.291	B1 B1	(2)
	(b) $A \approx \frac{1}{2} \times 0.5(\ \dots\ )$		B1	
	$= \ \dots\ \left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$		M1	
	$= \ 0.25\left(0+2\left(0.608+1.386+2.291+3.296+4.385\right)+5.545\right)$	ft their (a)	A1ft	
	$= 0.25 \times 29.477 \ \dots\ \approx 7.37$	cao	A1	(4)
	(c)(i) $\int x \ln x \, \mathrm{d}x = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} \mathrm{d}x$		M1 A1	
	$= \frac{x^2}{2} \ln x - \int \frac{x}{2} \mathrm{d}x$			
	$= \frac{x^2}{2} \ln x - \frac{x^2}{4} \ (+C)$		M1 A1	
	(ii) $\left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^4 = \left(8 \ln 4 - 4\right) - \left(-\frac{1}{4}\right)$		M1	
	$= 8 \ln 4 - \frac{15}{4}$			
	$= 8\left(2 \ln 2\right) - \frac{15}{4}$	$\ln 4 = 2 \ln 2$ seen or implied	M1	
	$= \frac{1}{4}\left(64 \ln 2 - 15\right)$	$a = 64, b = -15$	A1	(7)
				[13]

Question Number	Scheme	Marks	
Q3	(a) $-2\sin 2x - 3\sin 3y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}$, $\frac{-2\sin 2x}{3\sin 3y}$	A1	(3)
	(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$	M1	
	$\cos 3y = \frac{1}{2}$	A1	
	$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349	A1	(3)
	(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$	M1	
	$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$	M1	
	Leading to $6x + 9y - 2\pi = 0$	A1	(3)
			[9]

Question Number	Scheme		Marks	
Q4	(a) A: $(-6, 4, -1)$	Accept vector forms	B1	(1)
	(b) $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} . \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = 12 + 4 + 3 = \sqrt{4^2 + (-1)^2 + 3^2}\sqrt{3^2 + (-4)^2 + 1^2}\cos\theta$		M1 A1	
	$\cos\theta = \frac{19}{26}$	awrt 0.73	A1	(3)
	(c) X: $(10, 0, 11)$	Accept vector forms	B1	(1)
	(d) $\overrightarrow{AX} = \begin{pmatrix} 10 \\ 0 \\ 11 \end{pmatrix} - \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$	Either order	M1	
	$= \begin{pmatrix} 16 \\ -4 \\ 12 \end{pmatrix}$	cao	A1	(2)
	(e) $\lvert\overrightarrow{AX}\rvert = \sqrt{16^2 + (-4)^2 + 12^2}$		M1	
	$= \sqrt{416} = \sqrt{16 \times 26} = 4\sqrt{26}$ ∗	Do not penalise if consistent incorrect signs in (d)	A1	(2)
	(f) 	Use of correct right angled triangle	M1	
	$\frac{\lvert\overrightarrow{AX}\rvert}{d} = \cos\theta$		M1	
	$d = \frac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$	awrt 27.9	A1	(3)
				[12]

Question Number	Scheme		Marks	
Q5	(a) $\int \frac{9x+6}{x}\,dx = \int \left(9+\frac{6}{x}\right)dx$		M1	
	$= 9x + 6\ln x \quad (+C)$		A1	(2)
	(b) $\int \frac{1}{y^{\frac{1}{3}}}\,dy = \int \frac{9x+6}{x}\,dx$	Integral signs not necessary	B1	
	$\int y^{-\frac{1}{3}}\,dy = \int \frac{9x+6}{x}\,dx$			
	$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \quad (+C)$	$\pm ky^{\frac{2}{3}} =$ their (a)	M1	
	$\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x \quad (+C)$	ft their (a)	A1ft	
	$y = 8,\ x = 1$			
	$\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$		M1	
	$C = -3$		A1	
	$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$			
	$y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$		A1	(6)
				[8]

Question Number	Scheme		Marks
Q6	$\frac{\mathrm{d}A}{\mathrm{d}t} = 1.5$		B1
	$A = \pi r^2 \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$		B1
	When $A = 2$		
	$2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}} \; (= 0.797\,884\,\ldots)$		M1
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t}$		
	$1.5 = 2\pi r \frac{\mathrm{d}r}{\mathrm{d}t}$		M1
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$	awrt 0.299	A1
			[5]

Question Number	Scheme		Marks	
Q7	(a) $y = 0 \Rightarrow t(9-t^2) = t(3-t)(3+t) = 0$			
	$t = 0, 3, -3$	Any one correct value	B1	
	At $t = 0$, $x = 5(0)^2 - 4 = -4$	Method for finding one value of x	M1	
	At $t = 3$, $x = 5(3)^2 - 4 = 41$			
	$\left(\text{At } t = -3,\ x = 5(-3)^2 - 4 = 41\right)$			
	At A, $x = -4$; at B, $x = 41$	Both	A1	(3)
	(b) $\frac{\mathrm{d}x}{\mathrm{d}t} = 10t$	Seen or implied	B1	
	$\int y\,\mathrm{d}x = \int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t = \int t(9-t^2)10t\,\mathrm{d}t$		M1 A1	
	$= \int (90t^2 - 10t^4)\,\mathrm{d}t$			
	$= \frac{90t^3}{3} - \frac{10t^5}{5}\ (+C) \qquad \left(= 30t^3 - 2t^5\ (+C)\right)$		A1	
	$\left[\frac{90t^3}{3} - \frac{10t^5}{5}\right]_0^3 = 30\times3^3 - 2\times3^5 \quad (= 324)$		M1	
	$A = 2\int y\,\mathrm{d}x = 648 \quad (\text{units}^2)$		A1	(6)
				[9]

Question Number	Scheme	Marks
Q8	(a) $\dfrac{dx}{du} = -2\sin u$	B1
	$\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,du$	M1
	$\displaystyle = \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,du$ Use of $1-\cos^2 u = \sin^2 u$	M1
	$\displaystyle = -\frac{1}{4}\int \frac{1}{\cos^2 u}\,du$ $\displaystyle \pm k\int \frac{1}{\cos^2 u}\,du$	M1
	$= -\frac{1}{4}\tan u \ (+C)$ $\pm k \tan u$	M1
	$x = \sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \frac{\pi}{4}$	
	$x = 1 \Rightarrow 1 = 2\cos u \Rightarrow u = \frac{\pi}{3}$	M1
	$\left[-\frac{1}{4}\tan u\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\frac{1}{4}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{3}\right)$	
	$= -\frac{1}{4}\left(1-\sqrt{3}\right) \quad \left(= \frac{\sqrt{3}-1}{4}\right)$	A1 (7)
	(b) $\displaystyle V = \pi\int_1^{\sqrt{2}} \left(\frac{4}{x\left(4-x^2\right)^{\frac{1}{4}}}\right)^2 dx$	M1
	$\displaystyle = 16\pi\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,dx$ $16\pi \times$ integral in (a)	M1
	$= 16\pi\left(\frac{\sqrt{3}-1}{4}\right)$ $16\pi \times$ their answer to part (a)	A1ft (3)
		[10]