Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 14 June 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40686A

1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

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2. $$\mathrm{f}(x) = x^3 + 3x^2 + 4x - 12$$

(*a*) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3.$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(*b*) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \quad n \geq 0,$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of $\mathrm{f}(x) = 0$ is α.

(*c*) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

3.

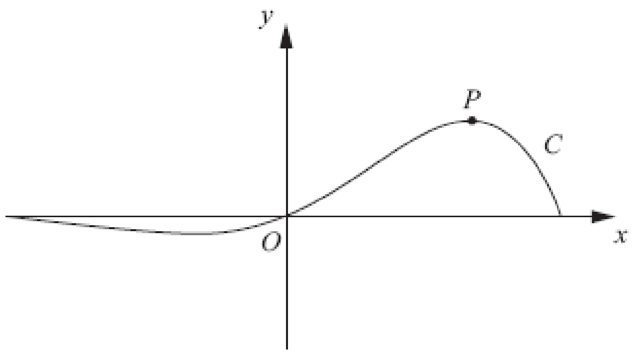

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}.$$

(a) Find the x-coordinate of the turning point P on C, for which $x > 0$.
Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to C at the point where $x = 0$.

(3)

4.

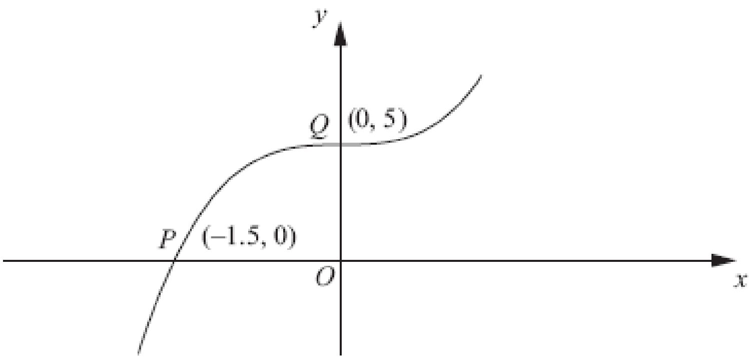

Figure 2

Figure 2 shows part of the curve with equation $y = \mathrm{f}(x)$.
The curve passes through the points $P(-1.5, 0)$and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(*a*) $y = |\mathrm{f}(x)|$

(2)

(*b*) $y = \mathrm{f}(|x|)$

(2)

(*c*) $y = 2\mathrm{f}(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

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5. (*a*) Express $4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$ in terms of $\sin \theta$ and $\cos \theta$.

(2)

(*b*) Hence show that

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \sec^2 \theta .$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = 4$$

giving your answers in terms of π.

(3)

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6. The functions f and g are defined by

$$\text{f}: x \mapsto e^x + 2, \qquad x \in \mathbb{R},$$

$$\text{g}: x \mapsto \ln x, \qquad x > 0.$$

(*a*) State the range of f.

(1)

(*b*) Find fg(x), giving your answer in its simplest form.

(2)

(*c*) Find the exact value of x for which f($2x + 3$) = 6.

(4)

(*d*) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(*e*) On the same axes sketch the curves with equation $y = \text{f}(x)$ and $y = \text{f}^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

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7. (*a*) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln(3x)$,

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(*b*) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

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8. $$f(x) = 7\cos 2x - 24\sin 2x.$$

Given that $f(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α.

(3)

(*b*) Hence solve the equation

$$7\cos 2x - 24\sin 2x = 12.5$$

for $0 \le x < 180°$, giving your answers to 1 decimal place.

(5)

(*c*) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found.

(2)

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14\cos^2 x - 48\sin x\cos x.$$

(2)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$9x^2-4=(3x-2)(3x+2)$ At any stage	B1
	Eliminating the common factor of (3*x*+2) at any stage	
	$\frac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}}=\frac{2}{3x-2}$	B1
	Use of a common denominator	
	$\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\frac{2(3x+1)}{(3x-2)(3x+1)}-\frac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\frac{6}{(3x-2)(3x+1)}$ or $\frac{6}{9x^2-3x-2}$	A1
		(4 marks)

Notes

B1 For factorising $9x^2-4=(3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark

B1 For eliminating/cancelling out a factor of (3*x*+2) at any stage of the answer.

M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$$\frac{2(3x+2)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$$ Only one numerator adapted, separate fractions

$$\frac{2\times 3x+1-2\times 3x-2}{(3x-2)(3x+1)}$$ Invisible brackets, single fraction

A1 $\frac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative method

$$\frac{2(3x+2)}{(9x^2-4)}-\frac{2}{(3x+1)}=\frac{2(3x+2)(3x+1)-2(9x^2-4)}{(9x^2-4)(3x+1)}=\frac{18x+12}{(9x^2-4)(3x+1)}$$ has scored 0,0,1,0 so far

$$=\frac{6\cancel{(3x+2)}}{\cancel{(3x+2)}(3x-2)(3x+1)}$$ is now 1,1,1,0

$$=\frac{6}{(3x-2)(3x+1)}$$ and now 1,1,1,1

Question Number	Scheme	Marks
2.	(a) $x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$	
	$\Rightarrow x^2(x+3)=12-4x$	M1
	$\Rightarrow x^2=\dfrac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1A1*
		(3)
	(b) $x_1=1.41$, *awrt* $x_2=1.20$ $x_3=1.31$	M1A1,A1
		(3)
	(c) Choosing (1.2715,1.2725) or tighter containing root 1.271998323	M1
	$\text{f}(1.2725)=(+)0.00827...$ $\text{f}(1.2715)=-0.00821....$	M1
	Change of sign⇒α=1.272	A1
		(3)
		(9 marks)

Notes

(a) M1 Moves from f(x)=0, which may be implied by subsequent working, to $x^2(x\pm3)=\pm12\pm4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.

dM1 Divides by ‘(x+3)’ term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage

A1* CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The 12-4x needs to have been factorised.

(b) **Note that this appears B1,B1,B1 on EPEN**

M1 An attempt to substitute $x_0=1$ into the iterative formula to calculate x_1.

This can be awarded for the sight of $\sqrt{\dfrac{4(3-1)}{(3+1)}}$, $\sqrt{\dfrac{8}{4}}$, $\sqrt{2}$ and even 1.4

A1 $x_1=1.41$. The subscript is not important. Mark as the first value found, $\sqrt{2}$ is A0

A1 $x_2=\text{awrt }1.20$ $x_3=\text{awrt }1.31$. Mark as the second and third values found. Condone 1.2 for x_2

(c) **Note that this appears M1A1A1 on EPEN**

M1 Choosing the interval (1.2715,1.2725) or tighter containing the root 1.271998323. Continued iteration is not allowed for this question and is M0

M1 Calculates f(1.2715) and f(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated. Also accept f(1.2715) = -0.01 2dp
Accept f(1.2725) = (+)0.008 1sf rounded or truncated. Also accept f(1.2725) = (+)0.01 2dp

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2725)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

Alternative to (a) working backwards

2(a)

	$x=\sqrt{\dfrac{4(3-x)}{(x+3)}}\Rightarrow x^2=\dfrac{4(3-x)}{(x+3)}\Rightarrow x^2(x+3)=4(3-x)$	M1
	$x^3+3x^2=12-4x\Rightarrow x^3+3x^2+4x-12=0$	dM1
	States that this is f(x)=0	A1* (3)

Alternative starting with the given result and working backwards

M1 Square (both sides) and multiply by (x+3)

dM1 Expand brackets and collect terms on one side of the equation =0

A1 A statement to the effect that this is f(x)=0

An acceptable answer to (c) with an example of a tighter interval

M1 Choosing the interval (1.2715, 1.2720). This contains the root 1.2719(98323)

M1 Calculates f(1.2715) and f(1.2720), with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated f(1.2715) = -0.01 2dp
Accept f(1.2720) = (+)0.00003 1sf rounded or f(1.2720) = (+)0.00002 truncated 1sf

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2720)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

x	**f(x)**
1.2715	-0.00821362
1.2716	-0.00656564
1.2717	-0.00491752
1.2718	-0.00326927
1.2719	-0.00162088
1.2720	+0.00002765
1.2721	+0.00167631
1.2722	+0.00332511
1.2723	+0.00497405
1.2724	+0.00662312
1.2725	+0.00827233

An acceptable answer to (c) using g(x) where g(x)= $\sqrt{\dfrac{4(3-x)}{(x+3)}}-x$

2nd M1 Calculates g(1.2715) and g(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
g(1.2715) = 0.0007559. Accept g(1.2715) = awrt (+)0.0008 1sf rounded or awrt 0.0007 truncated.
g(1.2725)=-0.00076105. Accept g(1.2725) = awrt -0.0008 1sf rounded or awrt -0.0007 truncated.

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx}=0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\tan 3x=-\sqrt{3}$	A1
	$3x=\frac{2\pi}{3}\Rightarrow x=\frac{2\pi}{9}$	M1A1
		(6)
	(b) At x=0 $\quad \frac{dy}{dx}=3$	B1
	Equation of normal is $-\frac{1}{3}=\frac{y-0}{x-0}$ or any equivalent $y=-\frac{1}{3}x$	M1A1
		(3)
		(9 marks)

(a) M1 Applies the product rule vu'+uv' to $e^{x\sqrt{3}}\sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form $\frac{dy}{dx}=Ae^{x\sqrt{3}}\sin 3x+e^{x\sqrt{3}}\times\pm B\cos 3x$

A1 Correct expression for $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$

M1 Sets **their** $\frac{dy}{dx}=0$, factorises out or divides by $e^{x\sqrt{3}}$ producing an equation in sin3x and cos3x

A1 Achieves either $\tan 3x=-\sqrt{3}$ or $\tan 3x=-\frac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by ÷3.

Accept $3x=\frac{5\pi}{3}\Rightarrow x=\frac{5\pi}{9}$ or $3x=\frac{-\pi}{3}\Rightarrow x=\frac{-\pi}{9}$ but not $x=\arctan(\frac{-\sqrt{3}}{3})$

A1 CS0 $x=\frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

(b) B1 Sight of **3** for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\frac{1}{m}$ and used together with (0, 0).

Eg $-\frac{1}{their\ 'm'}=\frac{y-0}{x-0}$ or equivalent is acceptable

A1 $y=-\frac{1}{3}x$ or any correct equivalent including $-\frac{1}{3}=\frac{y-0}{x-0}$.

Alternative in part (a) using the form $R\sin(3x+\alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$(\sqrt{12})\sin(3x+\frac{\pi}{3})=0$	A1
	$3x=\frac{2\pi}{3}\Rightarrow x=\frac{2\pi}{9}$	M1A1
		(6)

A1 Achieves either $(\sqrt{12})\sin(3x+\frac{\pi}{3})=0$ or $(\sqrt{12})\cos(3x-\frac{\pi}{6})=0$

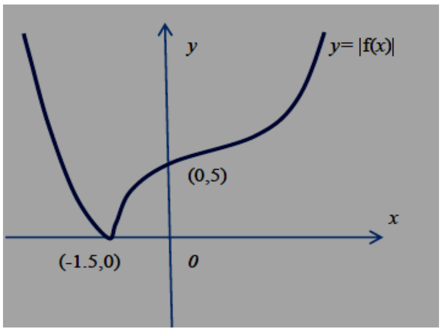
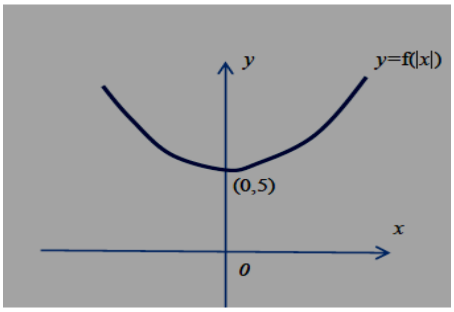
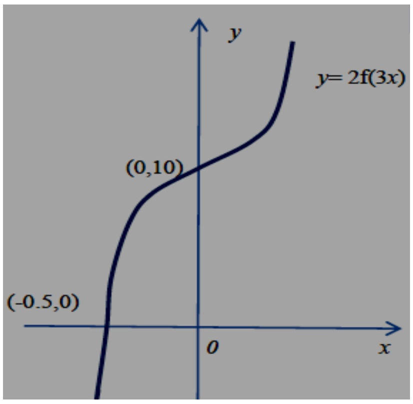
M1 Correct order of arcsin or arcos, etc to produce a value of x

Eg accept $3x+\frac{\pi}{3}=0$ or π or $2\pi\Rightarrow x=....$

A1 Cao $x=\frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\sqrt{3}\sin 3x=-3\cos 3x\Rightarrow\cos^2(3x)=\frac{1}{4}$ or $\sin^2(3x)=\frac{3}{4}$	A1
	$x=\frac{1}{3}\arccos(\pm\sqrt{\frac{1}{4}})$ oe	M1
	$x=\frac{2\pi}{9}$	A1

Question Number	Scheme		Marks
4.(a)		Shape including cusp	B1
		(-1.5, 0) **and** (0, 5)	B1
			(2)
(b)		Shape	B1
		(0,5)	B1
			(2)
(c)		Shape	B1
		(0,10)	B1
		(-0.5, 0)	B1
			(3)
			(7 marks)

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the x-axis at (-1.5, 0) **and** crossing the y-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the y- axis

B1 (0,5) lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to f(x)

B1 Curve **crosses** the y axis at (0, 10). The curve must appear in both quadrants 1 and 2

B1 Curve **crosses** the x axis at (-0.5, 0). The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using (0,3) as an example, accept both (3,0) or 3 written on the y axis (as long as the curve passes through the point)

Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1
Otherwise follow scheme

Question Number	Scheme	Marks
5.	(a) $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$ $= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$	B1 B1 (2)
	(b) $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta} = \dfrac{4}{4\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$ $= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1
	Using $1-\cos^2\theta = \sin^2\theta$ $\quad = \dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta}$	M1
	$= \dfrac{1}{\cos^2\theta} = \sec^2\theta$	M1A1* (4)
	(c) $\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\dfrac{1}{2}$	M1
	$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$	A1,A1 (3)
		(9 marks)

Note (a) and (b) can be scored together

(a) B1 One term correct. Eg. writes $4\operatorname{cosec}^2 2\theta$ as $\dfrac{4}{(2\sin\theta\cos\theta)^2}$ **or** $\operatorname{cosec}^2\theta$ as $\dfrac{1}{\sin^2\theta}$. Accept terms like $\operatorname{cosec}^2\theta = 1+\cot^2\theta = 1+\dfrac{\cos^2\theta}{\sin^2\theta}$. The question merely asks for an expression in sin θ and cos θ

B1 A fully correct expression in sin θ and cos θ . Eg. $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$ **Accept equivalents**

Allow a different variable say x's instead of θ's but do not allow mixed units.

b) M1 Attempts to combine their expression in $\sin\theta$ and $\cos\theta$ using a common denominator. The terms can be separate but the denominator must be correct and one of the numerators must have been adapted

M1 Attempts to form a 'single' term on the numerator by using the identity $1-\cos^2\theta = \sin^2\theta$

M1 Cancels correctly by $\sin^2\theta$ terms and replaces $\dfrac{1}{\cos^2\theta}$ with $\sec^2\theta$

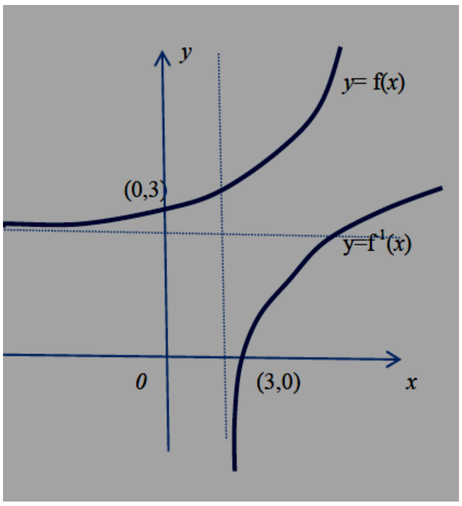
A1* Cso. This is a given answer. All aspects must be correct

IF IN ANY DOUBT SEND TO REVIEW OR CONSULT YOUR TEAM LEADER

c) M1 For $\sec^2\theta = 4$ leading to a solution of $\cos\theta$ by taking the root and inverting in either order . Similarly accept $\tan^2\theta = 3, \sin^2\theta = \dfrac{3}{4}$ leading to solutions of $\tan\theta, \sin\theta$. Also accept $\cos 2\theta = -\dfrac{1}{2}$

A1 Obtains one correct answer usually $\theta = \dfrac{\pi}{3}$ Do not accept decimal answers or degrees

A1 Obtains both correct answers. $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ Do not award if there are extra solutions inside the range. Ignore solutions outside the range.

Question Number	Scheme		Marks
6.	(a) f(x)>2		B1 (1)
	(b) $\text{fg}(x) = e^{\ln x} + 2, = x + 2$		M1,A1 (2)
	(c) $e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$		M1A1
	$\Rightarrow 2x + 3 = \ln 4$		
	$\Rightarrow x = \dfrac{\ln 4 - 3}{2}$ or $\ln 2 - \dfrac{3}{2}$		M1A1 (4)
	(d) Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y-2) = x$		M1
	$\text{f}^{-1}(x) = \ln(x-2), \quad x > 2.$		A1 , **B1ft** (3)
	(e) 	Shape for f(x)	B1
		(0, 3)	B1
		Shape for f^{-1}(x)	B1
		(3, 0)	B1 (4)
			(14 marks)

(a) B1 Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x>2

(b) M1 For applying the correct order of operations. Look for $e^{\ln x} + 2$. Note that $\ln e^x + 2$ is M0

A1 Simplifies $e^{\ln x} + 2$ to $x + 2$. Just the answer is acceptable for both marks

(c) M1 Starts with $e^{2x+3} + 2 = 6$ and proceeds to $e^{2x+3} = \ldots$

A1 $e^{2x+3} = 4$

M1 Takes ln's both sides, $2x + 3 = \ln ..$ and proceeds to x=….

A1 $x = \dfrac{\ln 4 - 3}{2}$ oe. eg $\ln 2 - \dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) **Note that this is marked M1A1A1 on EPEN**

M1 Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject.
All ln work must be correct. The 2 must be dealt with first.
Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0

A1 $f^{-1}(x) = \ln(x-2)$ or y=$\ln(x-2)$ or y=$\ln|x-2|$ There must be some form of bracket

B1ft Either x>2, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$
Do not accept y>2 or $f^{-1}(x)$>2.

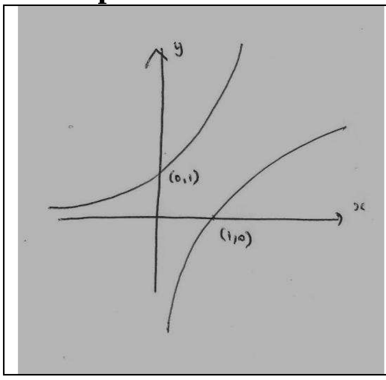
(e) B1 Shape for y=e^x. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the *x* axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.

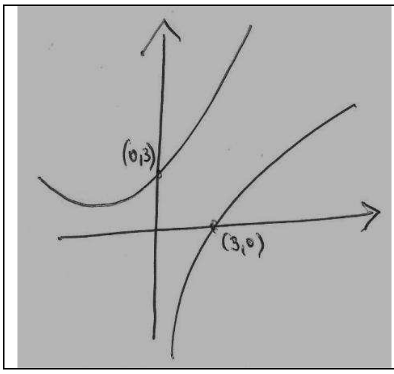
B1 (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

B1 Shape for y=ln*x*. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the *y* axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects y=e^x

B1 (3, 0) lies on the curve. Accept 3 written on the *x* axis as long as the point lies on the curve

Condone lack of labels in this part
Examples

	Scores **1,0,1,0.** Both shapes are fine, do not be concerned about asymptotes appearing at x=2, y=2. (See notes) Both co-ordinates are incorrect

	Scores **0,1,1,1** Shape for $y = e^x$ is incorrect, there is a minimum point on the graph. All other marks an be awarded

Question Number	Scheme	Marks
7.	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{\mathrm{d}}{\mathrm{d}x}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x}$	M1A1
		(3)
	(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{(2x-1)^{10}}$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{80x}{(2x-1)^6}$	A1
		(3)
	(b) $x = 3\tan 2y \quad \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = 6\sec^2 2y$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1+\tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$	
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1A1
		(5) **(11 marks)**

Note that this is marked B1M1A1 on EPEN

(a)(i) M1 Attempts to differentiate ln(3*x*) to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine.

M1 Attempts the product rule for $x^{\frac{1}{2}}\ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\ln(3x)\times Ax^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{B}{x}, \quad A, B > 0$$

A1 Any correct (un simplified) form of the answer. Remember to isw any incorrect further work

$$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x} = (\frac{\ln(3x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}) = x^{-\frac{1}{2}}(\frac{1}{2}\ln 3x + 1)$$

Note that this part does not require the answer to be in its simplest form

(ii) M1 Applies the quotient rule, a version of which appears in the formula booklet. If the formula is quoted it must be correct. There must have been an attempt to differentiate both terms. If the formula is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\frac{(2x-1)^5\times\pm10-(1-10x)\times C(2x-1)^4}{(2x-1)^{10\text{ or }7\text{ or }25}}$$

A1 Any un simplified form of the answer. Eg $\frac{dy}{dx}=\frac{(2x-1)^5\times-10-(1-10x)\times5(2x-1)^4\times2}{((2x-1)^5)^2}$

A1 Cao. It must be simplified as required in the question $\frac{dy}{dx}=\frac{80x}{(2x-1)^6}$

(b) M1 Knows that $3\tan2y$ differentiates to $C\sec^2 2y$. The lhs can be ignored for this mark. If they write $3\tan2y$ as $\frac{3\sin2y}{\cos2y}$ this mark is awarded for a correct attempt of the quotient rule.

A1 Writes down $\frac{dx}{dy}=6\sec^2 2y$ or implicitly to get $1=6\sec^2 2y\frac{dy}{dx}$

Accept from the quotient rule $\frac{6}{\cos^2 2y}$ or even $\frac{\cos2y\times6\cos2y-3\sin2y\times-2\sin2y}{\cos^2 2y}$

M1 An attempt to invert 'their' $\frac{dx}{dy}$ to reach $\frac{dy}{dx}=f(y)$, or changes the subject of their implicit

. differential to achieve a similar result $\frac{dy}{dx}=f(y)$

M1 Replaces an expression for $\sec^2 2y$ in their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with x by attempting to use $\sec^2 2y=1+\tan^2 2y$. Alternatively, replaces an expression for y in $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with $\frac{1}{2}\arctan(\frac{x}{3})$

A1 Any correct form of $\frac{dy}{dx}$ in terms of x. $\frac{dy}{dx}=\frac{1}{6(1+(\frac{x}{3})^2)}$ $\frac{dy}{dx}=\frac{3}{18+2x^2}$ *or* $\frac{1}{6\sec^2(\arctan(\frac{x}{3}))}$

Question Number	Scheme	Marks
7.	**(a)(ii) Alt using the product rule**	
	Writes $\frac{1-10x}{(2x-1)^5}$ as $(1-10x)(2x-1)^{-5}$ and applies vu'+uv'.	
	See (a)(i) for rules on how to apply	
	$(2x-1)^{-5}\times-10+(1-10x)\times-5(2x-1)^{-6}\times2$	M1A1
	Simplifies as main scheme to $80x(2x-1)^{-6}$ or equivalent	A1
		(3)
	(b) Alternative using arctan. They must attempt to differentiate to score any marks. Technically this is M1A1M1A2	
	Rearrange $x=3\tan2y$ to $y=\frac{1}{2}\arctan(\frac{x}{3})$ **and attempt to differentiate**	M1A1
	Differentiates to a form $\frac{A}{1+(\frac{x}{3})^2}$, $=\frac{1}{2}\times\frac{1}{(1+(\frac{x}{3})^2)}\times\frac{1}{3}$ or $\frac{1}{6(1+(\frac{x}{3})^2)}$ oe	M1, A2
		(5)

Question Number	Scheme	Marks
8.	(a) $R=25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = \text{(awrt)}73.7^0$	M1A1
		(3)
	(b) $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$	M1
	$2x + their\,'\alpha' = 60^0$	A1
	$2x + their\,'\alpha' = \text{their } 300^0 \text{ or their } 420^0 \Rightarrow x = ..$	M1
	$x = awrt\ 113.1^0, 173.1^0$	A1A1
		(5)
	(c) Attempts to use $\cos 2x = 2\cos^2 x - 1$ **AND** $\sin 2x = 2\sin x \cos x$ in the expression	M1
	$14\cos^2 x - 48\sin x\cos x = 7(\cos 2x + 1) - 24\sin 2x$	
	$= 7\cos 2x - 24\sin 2x + 7$	A1
		(2)
	(d) $14\cos^2 x - 48\sin x\cos x = R\cos(2x+\alpha) + 7$	
	Maximum value =’R’+’c’	M1
	= 32 cao	A1
		(2)
		(12 marks)

(a) B1 Accept 25, awrt 25.0, √625. Condone ± 25

M1 For $\tan\alpha = \pm\frac{24}{7}$ $\tan\alpha = \pm\frac{7}{24}$ $\sin\alpha = \pm\frac{24}{their\ R}$, $\cos\alpha = \pm\frac{7}{their\ R}$

A1 $\alpha = \text{(awrt)}73.7^0$. The answer 1.287 (radians) is A0

(b) M1 For using part (a) and dividing by their R to reach $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$

A1 Achieving $2x + their\ \alpha = 60^{(0)}$. This can be implied by $113.1^{(0)}/113.2^{(0)}$ or $173.1^{(0)}/173.2^{(0)}$ or $-6.8^{(0)}$ / $-6.85^{(0)}$/$-6.9^{(0)}$

M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark

Look for $\frac{360 \pm \text{'their' principal value}\pm\text{'their' }\alpha}{2}$

A1 $x = awrt\ 113.1^0 / 113.2^0$ OR $173.1^0 / 173.2^0$.

A1 $x = awrt\ 113.1^0$ AND 173.1^0. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

(c) M1 Attempts to use $\cos 2x = 2\cos^2 x - 1$ **and** $\sin 2x = 2\sin x \cos x$ in expression.
Allow slips in sign on the $\cos 2x$ term. So accept $2\cos^2 x = \pm\cos 2x \pm 1$
A1 Cao $= 7\cos 2x - 24\sin 2x + 7$. The order of terms is not important. Also accept a=7, b=-24, c=7

(d) M1 This mark is scored for adding their *R* to their *c*
A1 cao 32

Radian solutions- they will lose the first time it occurs (usually in a with 1.287 radians) Part b will then be marked as follows

(b) M1 For using part (a) and dividing by their *R* to reach $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$

A1 The correct principal value of $\frac{\pi}{3}$ or awrt 1.05 radians. Accept $60^{(0)}$
This can be implied by awrt – 0.12 radians or awrt or 1.97 radians or awrt 3.02 radians

M1 Finding a secondary value of *x* from their principal value. A correct answer will imply this mark
Look for $\frac{2\pi \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$ Do not allow mixed units.
A1 $x = awrt\ 1.97$ OR 3.02.
A1 $x = awrt\ 1.97$ AND 3.02. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range