Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 16 June 2011 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Differentiate with respect to x

(a) $\ln(x^2 + 3x + 5)$,

(2)

(b) $\dfrac{\cos x}{x^2}$.

(3)

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2. $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \leq x < 2\pi.$$

(a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$.

(2)

The equation $f(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}, \qquad x_0 = 0.8,$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

3.

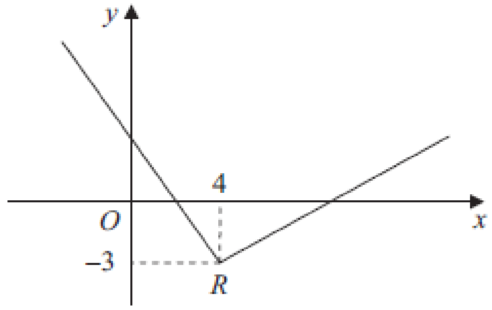

Figure 1

Figure 1 shows part of the graph of $y = \mathrm{f}(x),\ x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point R (4, – 3), as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2\mathrm{f}(x + 4)$,

(3)

(*b*) $y = |\mathrm{f}(-x)|$.

(3)

On each diagram, show the coordinates of the point corresponding to R.

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4. The function f is defined by

$$\mathrm{f}: x \mapsto 4 - \ln(x + 2), \quad x \in \mathbb{R}, \quad x \geq -1.$$

(*a*) Find $\mathrm{f}^{-1}(x)$.

(3)

(*b*) Find the domain of f^{-1}.

(1)

The function g is defined by

$$\mathrm{g}: x \mapsto \mathrm{e}^{x^2} - 2, \quad x \in \mathbb{R}.$$

(*c*) Find $\mathrm{fg}(x)$, giving your answer in its simplest form.

(3)

(*d*) Find the range of fg.

(1)

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5. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt},$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(*a*) Write down the value of p.

(1)

(*b*) Show that $k = \frac{1}{4} \ln 3$.

(4)

(*c*) Find the value of t when $\frac{dm}{dt} = -0.6 \ln 3$.

(6)

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6. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan \theta, \qquad \theta \neq 90n°, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1.$$

(5)

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7. $$\mathrm{f}(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}.$$

(*a*) Show that

$$\mathrm{f}(x) = \frac{5}{(2x+1)(x+3)}.$$

(5)

The curve C has equation $y = \mathrm{f}(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(*b*) Find an equation of the normal to C at P.

(8)

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8. (*a*) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$\mathrm{f}(x) = \mathrm{e}^{2x}\cos 3x.$$

(*b*) Show that $\mathrm{f}'(x)$ can be written in the form

$$\mathrm{f}'(x) = R\mathrm{e}^{2x}\cos(3x + \alpha),$$

where R and α are the constants found in part (*a*).

(5)

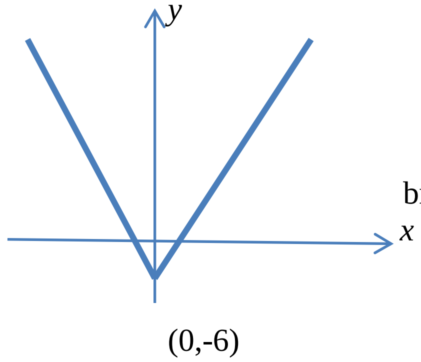
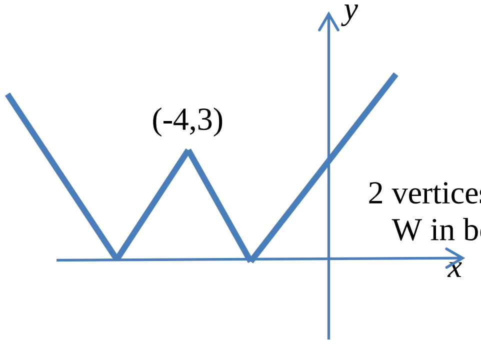
(*c*) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = \mathrm{f}(x)$ has a turning point.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1 (a)	$\frac{1}{(x^2+3x+5)} \times \ldots$, = $\frac{2x+3}{(x^2+3x+5)}$	M1,A1
		(2)
(b)	Applying $\frac{vu'-uv'}{v^2}$	M1,
	$\frac{x^2 \times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2 \sin x - 2x\cos x}{x^4} = \frac{-x\sin x - 2\cos x}{x^3}$ oe	A2,1,0
		(3)
		5 Marks
2 (a)	f(0.75)= -0.18…	
	f(0.85) = 0.17…..	M1
	Change of sign, hence root between x=0.75 and x=0.85	A1
		(2)
(b)	Sub x_0=0.8 into $x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}$ to obtain x_1	M1
	Awrt x_1=0.80219 and x_2=0.80133	A1
	Awrt x_3 = 0.80167	A1
		(3)
(c)	f(0.801565) = -2.7…..$\times 10^{-5}$	
	f(0.801575) = +8.6….$\times 10^{-6}$	M1A1
	Change of sign and conclusion	A1
		(3)
	See Notes for continued iteration method	
		8 Marks

Question Number	Scheme	Marks
3 (a)		
	V shape	B1
	vertex on y axis &both branches of graph cross x axis	B1
	‘y’ co-ordinate of R is -6	B1
		(3)
(b)		
	W shape	B1
	2 vertices on the negative x axis. W in both quad 1 & quad 2.	B1dep
	R’=(-4,3)	B1
		(3)
		6 Marks
4 (a)	$y = 4 - \ln(x+2)$	
	$\ln(x+2) = 4 - y$	
	$x + 2 = e^{4-y}$	
	$x = e^{4-y} - 2$	M1
	$f^{-1}(x) = e^{4-x} - 2$ oe	M1A1
		(3)
(b)	$x \leq 4$	B1
		(1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$	M1
	$fg(x) = 4 - x^2$	dM1A1
		(3)
(d)	$fg(x) \leq 4$	B1ft
		(1)
		8 Marks

Question Number	Scheme	Marks
5 (a)	p=7.5	B1
		(1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
	See notes for additional correct solutions and the last A1	
		(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

Question Number	Scheme	Marks
6 (a)	$\dfrac{1}{\sin 2\theta}-\dfrac{\cos 2\theta}{\sin 2\theta}=\dfrac{1-\cos 2\theta}{\sin 2\theta}$	M1
	$=\dfrac{2\sin^2\theta}{2\sin\theta\cos\theta}$	M1A1
	$=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$ cso	A1* (4)
(b)(i)	$\tan 15° = \dfrac{1}{\sin 30°}-\dfrac{\cos 30°}{sin 30°}$	M1
	$\tan 15° = \dfrac{1}{\frac{1}{2}}-\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=2-\sqrt{3}$ cso	dM1 A1*
		(3)
(b)(ii)	$\tan 2x = 1$	M1
	$2x = 45°$	A1
	$2x = 45° + 180°$	M1
	$x = 22.5°, 112.5°, 202.5°, 292.5°$	A1(any two) A1 (5)
		12 Marks
	Alt for (b)(i) $\tan 15° = \tan(60° - 45°)$ *or* $\tan(45° - 30°)$	
	$\tan 15° = \dfrac{\tan 60 - \tan 45}{1+\tan 60 tan 45}$ *or* $\dfrac{\tan 45 - \tan 30}{1+\tan 45 \tan 30}$	M1
	$\tan 15° = \dfrac{\sqrt{3}-1}{1+\sqrt{3}}$ *or* $\dfrac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan 15° = 2-\sqrt{3}$	A1*

Question Number	Scheme	Marks
7 (a)	$x^2 - 9 = (x+3)(x-3)$	B1
	$\frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x+3)(x-3)}$	
	$= \frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$= \frac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$= \frac{5\cancel{(x-3)}}{(2x+1)\cancel{(x-3)}(x+3)} = \frac{5}{(2x+1)(x+3)}$	A1*
		(5)
(b)	$f(x) = \frac{5}{2x^2+7x+3}$	
	$f'(x) = \frac{-5(\mathbf{4x+7})}{(2x^2+7x+3)^2}$	M1**M1**A1
	$f'(-1) = -\frac{15}{4}$	M1A1
	Uses $m_1m_2=-1$ to give gradient of normal$=\frac{4}{15}$	M1
	$\frac{y-(-\frac{5}{2})}{(x--1)} = their\ \frac{4}{15}$	M1
	$y + \frac{5}{2} = \frac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		13 Marks

Question Number	Scheme	Marks
8		
(a)	$R^2 = 2^2 + 3^2$	M1
	$R = \sqrt{13}$ or 3.61 …..	A1
	$\tan\alpha = \frac{3}{2}$	M1
	$\alpha = 0.983 \ldots$	A1
		(4)
(b)	$f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$	M1A1A1
	$= e^{2x}(2\cos 3x - 3\sin 3x)$	M1
	$= e^{2x}(R\cos(3\text{x} + \alpha)$	
	$= Re^{2x}\cos(3x + \alpha)$	A1* cso
		(5)
(c)	$f'(x) = 0 \Rightarrow \cos(3\text{x} + \alpha) = 0$	M1
	$3x + \alpha = \frac{\pi}{2}$	M1
	x=0.196… awrt 0.20	A1
		(3)
		12 Marks
	Alternative to part (c)$\Rightarrow$	
	$f'(x) = 0 \Rightarrow 2\cos 3x - 3\sin 3x = 0$	M1
	$\tan 3x = \frac{2}{3}$	M1
	x=0.196… awrt 0.20	A1
		(3)