Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Tuesday 15 June 2010 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35385A

1. (*a*) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan \theta.$$

(2)

(*b*) Hence find, for $-180° \leq \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)

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2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}.$$

The point P on C has x-coordinate 2.

Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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3. $$\mathrm{f}(x) = 4 \operatorname{cosec} x - 4x + 1, \qquad \text{where } x \text{ is in radians.}$$

(*a*) Show that there is a root α of $\mathrm{f}(x) = 0$ in the interval [1.2, 1.3].

(2)

(*b*) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \frac{1}{\sin x} + \frac{1}{4}$$

(2)

(*c*) Use the iterative formula

$$x_{n+1} = \frac{1}{\sin x_n} + \frac{1}{4}, \qquad x_0 = 1.25,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(*d*) By considering the change of sign of $\mathrm{f}(x)$ in a suitable interval, verify that $\alpha = 1.291$ correct to 3 decimal places.

(2)

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4. The function f is defined by

$$\mathrm{f} : x \mapsto |2x - 5|, \quad x \in \mathbb{R}.$$

(*a*) Sketch the graph with equation $y = \mathrm{f}(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(*b*) Solve $\mathrm{f}(x) = 15 + x$.

(3)

The function g is defined by

$$\mathrm{g} : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5.$$

(*c*) Find fg(2).

(2)

(*d*) Find the range of g.

(3)

5.

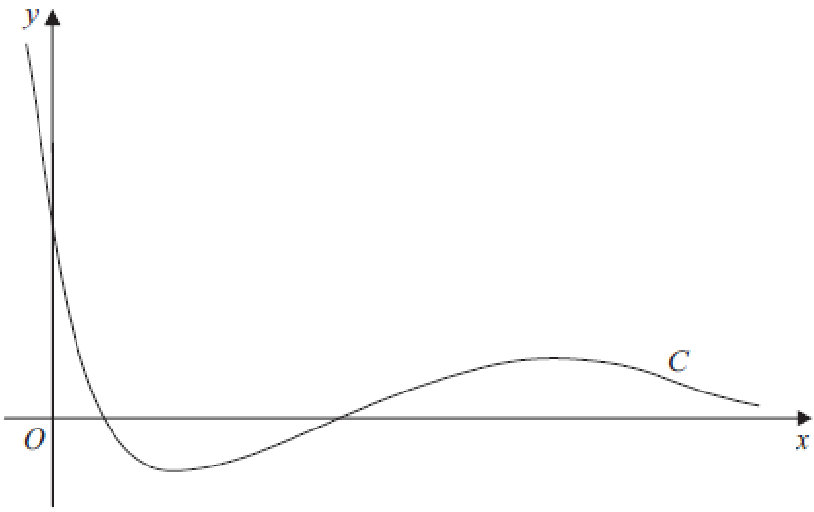

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\dfrac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)

6.

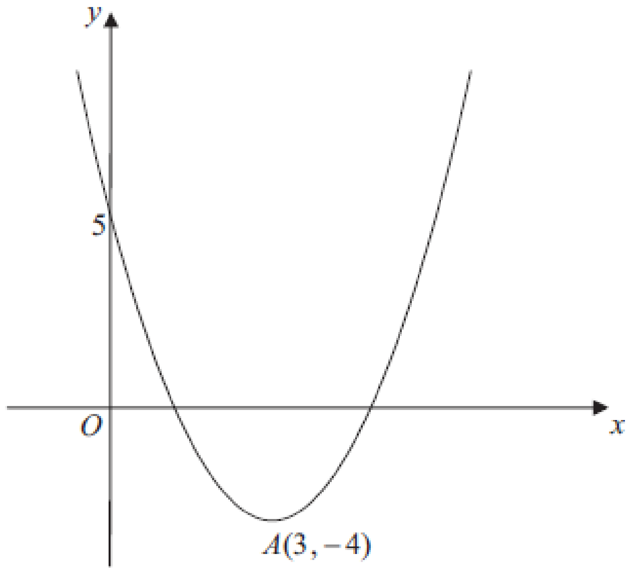

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(*a*) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

(*b*) Sketch the curve with equation $y = f(|x|)$.

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(*c*) Find $f(x)$.

(2)

(*d*) Explain why the function f does not have an inverse.

(1)

7. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(*b*) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin \left(\frac{4\pi t}{25}\right) - 1.5 \cos \left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

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8. (*a*) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}.$$

(3)

Given that

$$\ln (2x^2 + 9x - 5) = 1 + \ln (x^2 + 2x - 15), \quad x \neq -5,$$

(*b*) find x in terms of e.

(4)

TOTAL FOR PAPER: 75 MARKS

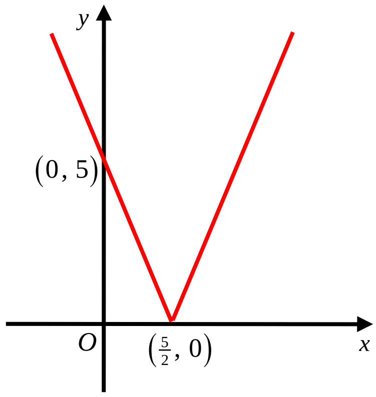
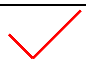
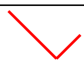
END

June 2010
6665 Core Mathematics C3
Mark Scheme

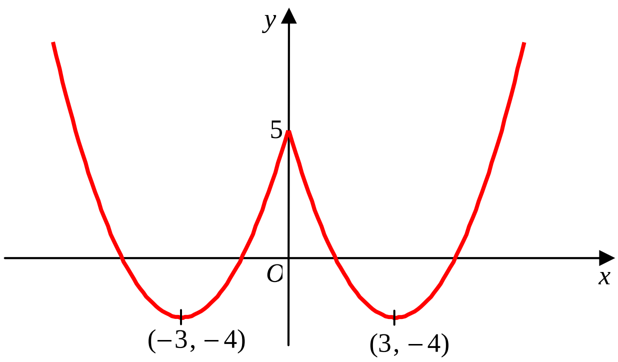
Question Number	Scheme	Marks
1. (a)	$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1
	$\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG**	A1 **cso**
		(2)
(b)	$2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$	M1
	$\theta_1 =$ awrt 26.6°	A1
	$\theta_2 =$ awrt -153.4°	A1 $\sqrt{}$
		(3) **[5]**
	(a) M1: Uses **both** a correct identity for $\sin 2\theta$ **and** a correct identity for $\cos 2\theta$. Also allow a candidate writing $1+\cos 2\theta = 2\cos^2\theta$ on the denominator. Also note that angles **must be consistent** in when candidates apply these identities. A1: Correct proof. No errors seen. (b) 1st M1 for either $2\tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$, seen or implied. A1: awrt 26.6 A1 $\sqrt{}$: awrt -153.4° or $\theta_2 = -180^\circ + \theta_1$ **Special Case**: For candidate solving, $\tan\theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). **Special Case:** Note that those candidates who writes $\tan\theta = 1$, and gives ONLY two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.	

Question Number	Scheme	Marks
2.	At P, $y = \underline{3}$	B1
	$\frac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$	M1$\underline{\text{A1}}$
	$\frac{dy}{dx} = \frac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
	$m(\mathbf{N}) = \frac{-1}{-18}$ or $\frac{1}{18}$	M1
	$\mathbf{N}$: $y - 3 = \frac{1}{18}(x-2)$	M1
	$\mathbf{N}$: $\underline{x - 18y + 52 = 0}$	A1 **[7]**
	1st M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2nd M1: Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$; 3rd M1: Uses $m(\mathbf{N}) = -\frac{1}{\text{their } m(\mathbf{T})}$. 4th M1: $y - y_1 = m(x-2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and $x = 2$. Note: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

Question Number	Scheme	Marks
3. (a)	$f(1.2) = 0.49166551\ldots$, $f(1.3) = -0.048719817\ldots$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1 (2)
(b)	$4\operatorname{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\operatorname{cosec}x + 1$ $\Rightarrow x = \operatorname{cosec}x + \frac{1}{4} \Rightarrow \underline{x = \frac{1}{\sin x} + \frac{1}{4}}$	M1 A1 * (2)
(c)	$x_1 = \frac{1}{\sin(1.25)} + \frac{1}{4}$ $x_1 = 1.303757858\ldots$, $x_2 = 1.286745793\ldots$ $x_3 = 1.291744613\ldots$	M1 A1 A1 (3)
(d)	$f(1.2905) = 0.00044566695\ldots$, $f(1.2915) = -0.00475017278\ldots$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	M1 A1 (2) **[9]**
	(a) M1: Attempts to evaluate both $f(1.2)$ and $f(1.3)$ and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $f(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by x_1 = awrt 1.3 or x_1 = awrt 46°. A1: Both x_1 = awrt 1.3038 and x_2 = awrt 1.2867 A1: x_3 = awrt 1.2917 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x). A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	

Question Number	Scheme	Marks
4. (a)	 	M1A1 (2)
(b)	$\underline{x = 20}$ $2x - 5 = -(15 + x)$; $\Rightarrow \underline{x = -\frac{10}{3}}$	B1 M1;A1 oe. (3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3) - 5 \rvert; = \lvert -11 \rvert = 11$	M1;A1 (2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\min} = -3$ Either $\text{g}_{\min} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$ $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	M1 B1 A1 (3) **[10]**
	(a) M1: V or  or  graph with vertex on the x-axis. A1: $(\frac{5}{2}, \{0\})$ and $(\{0\}, 5)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$ (c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \lvert 2x - 5 \rvert$ or for inserting $x = 2$ into $\lvert 2(x^2 - 4x + 1) - 5 \rvert$. There must be evidence of the modulus being applied. (d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $\text{f}(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$. A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0. **Note that:** $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that**: $-3 \geqslant \text{g}(x) \geqslant 6$ is A0. **Note that:** $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0. **Note that for the final Accuracy Mark**: If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0. If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.	

Question Number	Scheme	Marks
5. (a)	Either $y = 2$ or $(0, 2)$	B1
		(1)
(b)	When $x = 2$, $y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1
	$(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$	M1
	Either $x = 2$ (for possibly B1 above) or $\underline{x = \tfrac{1}{2}}$.	A1
		(3)
(c)	$\dfrac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1A1A1
		(3)
(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$	M1
	$2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$	M1
	$x = \tfrac{7}{2}, 1$	A1
	When $x = \tfrac{7}{2}$, $y = 9e^{-\frac{7}{2}}$, when $x = 1$, $y = -e^{-1}$	ddM1A1
		(5)
		[12]
	(b) If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark. (c) M1: (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v') A1: Any one term correct. A1: Both terms correct. (d) 1st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0. 2nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix. 3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part. Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf. Final A1: Both $\{x = 1\}$, $y = -e^{-1}$ and $\{x = \tfrac{7}{2}\}$, $y = 9e^{-\frac{7}{2}}$. **cao** Note that both exact values of y are required.	

Question Number	Scheme	Marks
6. (a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1
		(4)
(b)		B1 B1 B1
		(3)
(c)	$\text{f}(x) = (x-3)^2 - 4$ or $\text{f}(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1)
		[10]
	(b) B1: Correct shape for $x \geqslant 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$. **Note:** The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive y-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the y-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the y-axis. (c) M1: Either states $\text{f}(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $\text{f}(x) = x^2 + ax + b$, with $\text{f}(0) = 5$ and $\text{f}(3) = -4$ to find both a and b. A1: Either $(x-3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $\text{f}(0) = 5$ and also $\text{f}(6) = 5$. Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.	

Question Number	Scheme	Marks
7. (a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha = \text{awrt } 0.6435$	M1A1
		(3)
(b) (i)	Max Value = 2.5	B1 $\sqrt{}$
(ii)	$\underline{\sin(\theta - 0.6435) = 1}$ or $\underline{\theta - \text{their } \alpha = \frac{\pi}{2}}$; $\Rightarrow \theta = \text{awrt } 2.21$	<u>M1</u>;A1 $\sqrt{}$
		(3)
(c)	$H_{\text{Max}} = 8.5$ (m)	B1 $\sqrt{}$
	$\underline{\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1}$ or $\underline{\frac{4\pi t}{25} = \text{their (b) answer}}$; $\Rightarrow t = \text{awrt } 4.41$	M1;A1
		(3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$; $\Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1
	Either $t = \text{awrt } 2.1$ or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \left\{\pi - 0.411517\ldots \text{ or } 2.730076\ldots^c\right\}$	ddM1
	Times $= \{14:06,\ 18:43\}$	A1 (6)
		[15]
	(a) B1: $R = 2.5$ or $R = \sqrt{6.25}$. For $R = \pm 2.5$, award B0. M1: $\tan\alpha = \pm\frac{1.5}{2}$ or $\tan\alpha = \pm\frac{2}{1.5}$ A1: $\alpha = \text{awrt } 0.6435$ (b) B1 $\sqrt{}$: 2.5 or follow through the value of R in part (a). M1: For $\sin(\theta - \text{their } \alpha) = 1$ A1 $\sqrt{}$: awrt 2.21 or $\frac{\pi}{2}$ + their α rounding correctly to 3 sf. (c) B1 $\sqrt{}$: 8.5 or 6 + their R found in part (a) as long as the answer is greater than 6. M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 1$ or $\frac{4\pi t}{25} = \text{their (b) answer}$ A1: For $\sin^{-1}(0.4)$ This can be implied by awrt 4.41 or awrt 4.40. (d) M1: $6 + (\text{their } R)\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = 7$, M1: $\sin\left(\frac{4\pi t}{25} \pm \text{their } \alpha\right) = \frac{1}{\text{their } R}$ A1: For $\sin^{-1}(0.4)$. This can be implied by awrt 0.41 or awrt 2.73 or other values for different α's. Note this mark can be implied by seeing 1.055. A1: Either $t = \text{awrt } 2.1$ or $t = \text{awrt } 6.7$ ddM1: either π – their PV^c. Note that this mark is dependent upon the two M marks. This mark will usually be awarded for seeing either 2.730… or 3.373… A1: Both $t = 14:06$ and $t = 18:43$ or both 126 (min) and 403 (min) or both 2 hr 6 min and 6 hr 43 min.	

Question Number	Scheme	Marks
8. (a)	$\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$	M1 B1 A1 **aef**
		(3)
(b)	$\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$	M1
	$\dfrac{2x^2+9x-5}{x^2+2x-15}=\mathrm{e}$	dM1
	$\dfrac{2x-1}{x-3}=\mathrm{e} \Rightarrow \quad 3\mathrm{e}-1 = x(\mathrm{e}-2)$	M1
	$\Rightarrow \quad x = \dfrac{3\mathrm{e}-1}{\mathrm{e}-2}$	A1 **aef cso**
		(4) **[7]**
	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x+5)(x-3)$. Can be seen anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give $\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1.$ The product law of logarithms can be used to achieve $\ln\left(2x^2+9x-5\right)=\ln\left(\mathrm{e}\left(x^2+2x-15\right)\right).$ The product and quotient law could also be used to achieve $\ln\left(\dfrac{2x^2+9x-5}{\mathrm{e}(x^2+2x-15)}\right)=0.$ dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect x terms together and factorise. Note that this is not a dependent method mark. A1: $\dfrac{3\mathrm{e}-1}{\mathrm{e}-2}$ or $\dfrac{3\mathrm{e}^1-1}{\mathrm{e}^1-2}$ or $\dfrac{1-3\mathrm{e}}{2-\mathrm{e}}$. aef Note that the answer needs to be in terms of e. The decimal answer is 9.9610559… Note that the solution must be correct in order for you to award this final accuracy mark. **Note: See Appendix for an alternative method of long division.**	