Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Core Mathematics C2

Advanced Subsidiary

Wednesday 25 May 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6664/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

P46716A

1. A geometric series has first term a and common ratio $r = \frac{3}{4}$.

The sum of the first 4 terms of this series is 175.

(*a*) Show that $a = 64$.

(2)

(*b*) Find the sum to infinity of the series.

(2)

(*c*) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

(3)

(Total 7 marks)

---

2. The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leq x \leq 4.$$

(*a*) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 (8 - 2^{x-1})\ dx$.

(3)

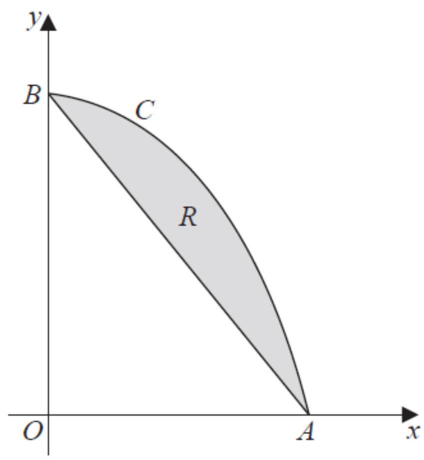

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}$, $0 \leq x \leq 4$.

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(*c*) Use your answer to part (*b*) to find an approximate value for the area of R.

(2)

(Total 6 marks)

3.

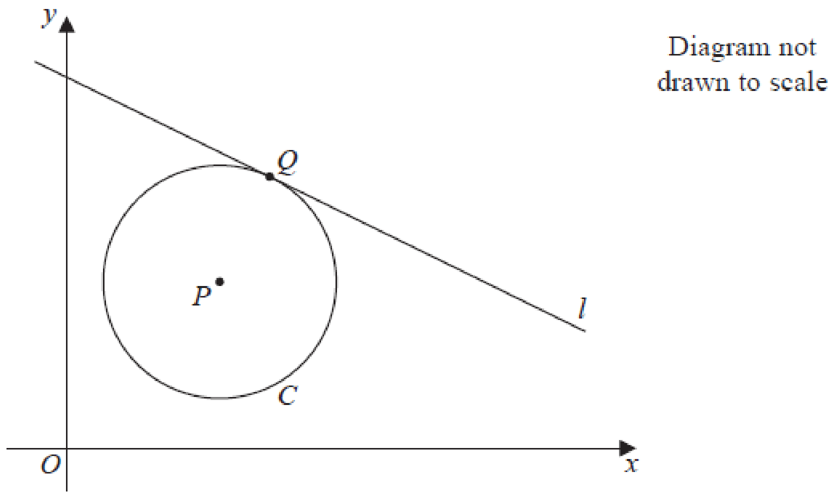

Figure 2

The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$, as shown in Figure 2.

(a) Find the length PQ, giving your answer as an exact value.

(2)

(b) Hence write down an equation for C.

(2)

The line l is a tangent to C at the point Q, as shown in Figure 2.

(c) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(Total 8 marks)

4.

$$f(x) = 6x^3 + 13x^2 - 4$$

(a) Use the remainder theorem to find the remainder when $f(x)$ is divided by $(2x + 3)$.

(2)

(b) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(c) Factorise $f(x)$ completely.

(4)

(Total 8 marks)

---

5. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4,$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \quad \text{where } k \text{ is a constant.}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2,$$

where A and B are constants.

(*b*) Write down the value of A.

(1)

(*c*) Find the value of k.

(2)

(*d*) Hence find the value of B.

(2)

(Total 9 marks)

---

6. (i) Solve, for $-\pi < \theta \leq \pi$,

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0,$$

giving your answers in terms of π.

(3)

(ii) Solve, for $0 \leq x < 360°$,

$$4\cos^2 x + 7\sin x - 2 = 0,$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total 9 marks)

7.

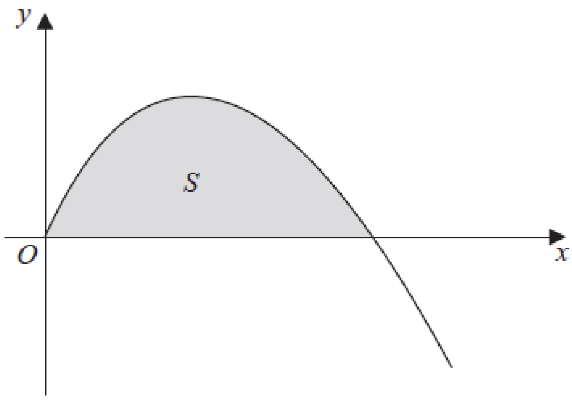

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}} \qquad x \geq 0\,.$$

The finite region *S*, bounded by the *x*-axis and the curve, is shown shaded in Figure 3.

(*a*) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x\,.$$

(3)

(*b*) Hence find the area of *S*.

(3)

(Total 6 marks)

---

8 (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2,$$

express *b* in terms of *a*.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0,$$

giving your answer to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total 7 marks)

9.

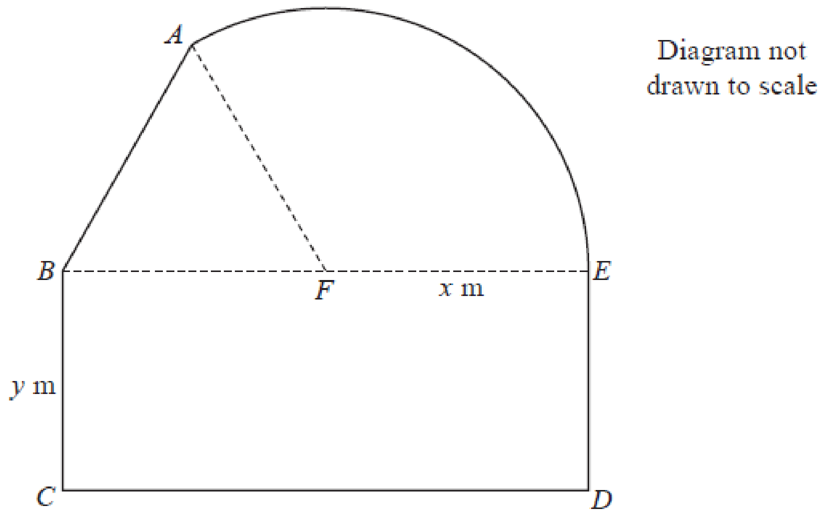

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leq x \leq 25$.

(*a*) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.

(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2,

(*b*) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right).$$

(3)

(*c*) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right).$$

(3)

(*d*) Use calculus to find the minimum value of P, giving your answer to the nearest metre.

(5)

(*e*) Justify, by further differentiation, that the value of P you have found is a minimum.

(2)

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

BLANK PAGE

Question Number	Scheme		Marks
1.	$r = \frac{3}{4},\ S_4 = 175$		
(a) **Way 1**	$\frac{a\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}$ or $\frac{a\left(1-\frac{3}{4}^4\right)}{1-\frac{3}{4}}$ or $\frac{a\left(1-0.75^4\right)}{1-0.75}$	Substituting $r = \frac{3}{4}$ or 0.75 and $n = 4$ into the formula for S_n	M1
	$175 = \frac{a\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}} \Rightarrow a = \frac{175\left(1-\frac{3}{4}\right)}{\left(1-\left(\frac{3}{4}\right)^4\right)}\ \left\{\Rightarrow a = \frac{\left(\frac{175}{4}\right)}{\left(\frac{175}{256}\right)} \Rightarrow\right\}\ \underline{a = 64}$ *	Correct proof	A1*
			[2]
(a) **Way 2**	$a + a\left(\frac{3}{4}\right) + a\left(\frac{3}{4}\right)^2 + a\left(\frac{3}{4}\right)^3$	$a + a\left(\frac{3}{4}\right) + a\left(\frac{3}{4}\right)^2 + a\left(\frac{3}{4}\right)^3$	M1
	$\frac{175}{64}a = 175\ \left(\Rightarrow a = \frac{175}{\left(\frac{175}{64}\right)}\right) \Rightarrow \underline{a = 64}$ * or 2.734375a=175 $\Rightarrow \underline{a = 64}$	Correct proof	A1*
			[2]
(a) **Way 3**	$\{S_4 =\}\ \frac{64\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}$ or $\frac{64\left(1-\frac{3}{4}^4\right)}{1-\frac{3}{4}}$ or $\frac{64\left(1-0.75^4\right)}{1-0.75}$	Applying the formula for S_n with $r = \frac{3}{4}$, $n = 4$ and a as 64.	M1
	$= 175$ so $a = 64$*	Obtains 175 with no errors seen and concludes $a = 64$*.	A1*
			[2]
(b)	$\{S_\infty\} = \frac{64}{\left(1-\frac{3}{4}\right)}$; $= 256$	$S_\infty = \frac{(\text{their } a)}{1-\frac{3}{4}}$ or $\frac{64}{1-\frac{3}{4}}$	M1;
		256	A1cao
			[2]
(c)	$\{D = T_9 - T_{10} = \}\ 64\left(\frac{3}{4}\right)^8 - 64\left(\frac{3}{4}\right)^9$	Writes down either $"64"\left(\frac{3}{4}\right)^8$ or awrt 6.4 or $"64"\left(\frac{3}{4}\right)^9$ or awrt 4.8, using $a = 64$ or their a	M1
		A correct expression for the difference (i.e. $\pm(T_9 - T_{10})$) using $a = 64$ or their a.	dM1
	$\left\{= 64\left(\frac{3}{4}\right)^8\left(\frac{1}{4}\right) = 1.6018066...\right\} = \underline{1.602}$ (3dp)	1.602 or -1.602	A1 cao
			[3]
			7

		Question 1 Notes
1. (a)		**Allow invisible brackets around fractions throughout all parts of this question**.
	M1	There are three possible methods as described above.
	A1	Note that this is a "show that" question with a printed answer. In **Way 1** this mark **usually** requires $a = p/q$ where p and q may be unsimplified brackets from the formula (or could be 11200/175 for example) as an intermediate step before the conclusion $a = 64$. Exceptions include $a = 175/4 * 256/175$ i.e. multiplication by reciprocal rather than division or $175 = 175a/64$ followed by the obvious $a = 64$ These also get A1 In "reverse" methods such as **Way 3** we need a conclusion "so $a = 64$" or some implication that their argument is reversible. Also a conclusion can be implied from a preamble, eg: "If I assume $a = 64$ then find $S = 175$ as given this implies $a = 64$ as required" This is a show that question and there should be no loss of accuracy. In all the methods **if** decimals are used there should **not be rounding**. If 0.68359375 appears this is correct. If it is rounded it would not give the exact answer. $64(1-0.31640625)$ or 43.75 are each correct – if they are rounded then treat this as incorrect e.g. **Way 3: "**43.75/0.25 = 175 so $a = 64$ is A1" but **"**43/0.25 = 175 so $a = 64$ is A0" and "44/0.25 = 175 so a = 64 is A0" Yet another **variant on Way 3**: take a=64 then find the next 3 terms as 48, 36, 27 then add 64+48+36+27 to get 175. Again need conclusion that $a = 64$ or some implication that their argument is reversible. Otherwise M1 A0
(b)	**M1**	$S_\infty = \frac{64}{1-\frac{3}{4}}$ or $\frac{(\text{their } a \text{ found in part } (a))}{1-\frac{3}{4}}$
	A1	256 cao
(c)	**NB**	Using **Sum of 10 terms** minus **Sum of 9 terms** is NOT a misread Scores **M0M0A0**
	M1	Can be **implied.** Writes down either $64\left(\frac{3}{4}\right)^8$ **or** $64\left(\frac{3}{4}\right)^9$, using $a = 64$ (or their a found in part (a)).
	Note	Ignore candidate's labelling of terms.
	Note	$64\left(\frac{3}{4}\right)^8 = 6.407226563...$ and $64\left(\frac{3}{4}\right)^9 = 4.805419922\ldots$
	dM1	**This is dependent on previous M mark and can be implied.** Either $64\left(\frac{3}{4}\right)^8 - 64\left(\frac{3}{4}\right)^9$ or $64\left(\frac{3}{4}\right)^9 - 64\left(\frac{3}{4}\right)^8$ or awrt 6.4 − awrt 4.8, using $a = 64$ (or their a from part (a)) .
	Note	1st M1 and 2nd M1 can be implied by the value of their $\text{difference} = \text{"their } a \text{ found in part (a)"} \times \frac{3^8}{4^9} \approx \frac{\text{"their } a \text{ found in part (a)"}}{40}$
	Note	Either $64\left(\frac{3}{4}\right)^9 - 64\left(\frac{3}{4}\right)^{10}$ or $64\left(\frac{3}{4}\right)^{10} - 64\left(\frac{3}{4}\right)^9$ is 1st M1, 2nd M0.
	A1	1.602 or −1.602 **cao (This answer with no working is M1M1A1) But 1.6 with no working is M0M0A0**
	Note	$\left\{D = \frac{1}{4}T_9 \Rightarrow\right\} D = \frac{1}{4}(64)\left(\frac{3}{4}\right)^8$ is 1st M1, 2nd M1
	Special case	Obtains awrt 6.4, then obtains awrt 4.8 but rounds to 6 – 5 when subtracting – award M1M1A0

Question Number	Scheme		Marks
	$y = 8 - 2^{x-1},\ 0 \leqslant x \leqslant 4$		
2. (a)	7	7	B1 **cao**
			[1]
(b)	$\left(\int_0^4 \left(8 - 2^{x-1}\right)\mathrm{d}x \approx\right) \frac{1}{2}\times 1;\times\left\{7.5 + 2\left(\text{"their 7"} + 6 + 4\right) + 0\right\}$	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$	B1;
		For structure of trapezium rule $\{............\}$ for a candidate's y-ordinates.	M1
	$\left\{= \frac{1}{2}\times 41.5\right\} = 20.75$ o.e.	20.75	A1 **cao**
			[3]
(c)	Area $(R) = \text{"20.75"} - \frac{1}{2}(7.5)(4)$		M1
	$= 5.75$	5.75	A1 **cao**
			[2]
			6

Question 2 Notes

(a)	**B1**	For 7 only
(b)	**B1**	For using $\frac{1}{2}\times 1$ or $\frac{1}{2}$ or equivalent.
	M1	Requires the correct $\{......\}$ bracket structure. It needs the 7.5 stated but the 0 may be omitted. The inner bracket needs to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however (unless it is 0)). M0 is awarded if values used in brackets are x values instead of y values
	A1	For 20.75 or fraction equivalent e.g. $20\frac{3}{4}$ *or* $\frac{83}{4}$
	Note	**NB: Separate trapezia may be used** : B1 for 0.5, M1 for 1/2 $h(a + b)$ used 3 or 4 times Then A1 as before.
	Special case:	Bracketing mistake $0.5\times(7.5 + 0) + 2(\text{ their } 7 + 6 + 4)$ scores B1 M1 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 37.75 usually indicates this error.
	Common error:	Many candidates use $\frac{1}{2}\times\frac{4}{5}$ and score B0 Then they proceed with $\left\{7.5 + 2\left(\text{"their 7"} + 6 + 4\right) + 0\right\}$ and score M1 This usually gives 16.6 for B0M1A0
(c)	**M1**	their answer to (b) $-$ area of triangle with base 4 and height 7.5 or alternative correct method e.g. their answer to (b) $-\int_0^4\left(7.5 - \frac{7.5}{4}x\right)\mathrm{d}x$ (Even if this leads to a negative answer) This may be implied by a correct answer or by an answer where they have subtracted 15 from their answer to part (b). Must use answer to part (b).
	A1	5.75 or fraction equivalent e.g. $5\frac{3}{4}$ *or* $\frac{23}{4}$

Question Number	Scheme		Marks
3.	$P(7, 8)$ and $Q(10, 13)$		
(a)	$\{PQ =\}\ \sqrt{(7-10)^2 + (8-13)^2}$ or $\sqrt{(10-7)^2 + (13-8)^2}$	Applies distance formula. Can be implied.	M1
	$\{PQ\} = \sqrt{34}$	$\sqrt{34}$ or $\sqrt{17}.\sqrt{2}$	A1
			[2]
(b) **Way 1**	$(x-7)^2 + (y-8)^2 = 34 \ \left(\text{or } \left(\sqrt{34}\right)^2\right)$	$(x \pm 7)^2 + (y \pm 8)^2 = k$, where k is a positive <u>value</u>.	M1
		$(x-7)^2 + (y-8)^2 = 34$	A1 oe
			[2]
(b) **Way 2**	$x^2 + y^2 - 14x - 16y + 79 = 0$	$x^2 + y^2 \pm 14x \pm 16y + c = 0$, where c is any <u>value</u> < 113.	M1
		$x^2 + y^2 - 14x - 16y + 79 = 0$	A1 oe
			[2]
(c) **Way 1**	$\{\text{Gradient of radius}\} = \dfrac{13-8}{10-7}$ or $\dfrac{5}{3}$	This must be seen or implied in part (c).	B1
	Gradient of tangent $= -\dfrac{1}{m}\left(= -\dfrac{3}{5}\right)$	Using a perpendicular gradient method on their gradient. So Gradient of tangent $= -\dfrac{1}{\text{gradient of radius}}$	M1
	$y - 13 = -\dfrac{3}{5}(x-10)$	$y - 13 = (\text{their changed gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
(c) **Way 2**	$2(x-7) + 2(y-8)\dfrac{dy}{dx} = 0$	Correct differentiation (or equivalent). Seen or implied	B1
	$2(10-7) + 2(13-8)\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{3}{5}$	Substituting **both** $x = 10$ and $y = 13$ into a valid differentiation to find a value for $\dfrac{dy}{dx}$	M1
	$y - 13 = -\dfrac{3}{5}(x-10)$	$y - 13 = (\text{their gradient})(x-10)$	M1
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
(c) **Way 3**	$10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$	$10x + 13y - 7(x+10) - 8(y+13) + 79 = 0$	B1
		$10x + 13y - 7(x+10) - 8(y+13) + c = 0$ where c is any <u>value</u> < 113	M2
	$3x + 5y - 95 = 0$	$3x + 5y - 95 = 0$ o.e.	A1
			[4]
			8

		Question 3 Notes
(a)	M1	Allow for $\{PQ=\}\ \sqrt{(7-10)^2+(8-13)^2}$ or for $\{PQ=\}\ \sqrt{3^2+5^2}$. Can be implied by answer.
	A1	Need to see $\sqrt{34}$. You can ignore subsequent work so $\sqrt{34}$ followed by 5.83 earns M1 A1, but $\{PQ=\}\ \sqrt{3^2+5^2}=5.83$, with no exact value for the answer given, earns M1A0. Allow $\pm\sqrt{34}$ this time. NB Some use equation of circle to find this distance Achieving $\sqrt{34}$ gets M1A1 Others find half of their $\pm\sqrt{34}$. Do not isw here as it is an error – confusing *d* with diameter. Give M1A0
(b)	M1	Either of the correct approaches for equation of circle (as shown on scheme)
	A1	Correct equation (two are shown and any correct equivalent is acceptable)
(c)		
		A correct start to finding the gradient of the tangent (see each scheme)
	B1	Complete method for finding the gradient of the tangent (see each scheme) Where implicit differentiation has been used the only slips allowed here should be sign slips.
	1st M1	Correct attempt at line equation for tangent at correct point (10, 13) with **their tangent** gradient. If the $y = mx + c$ method is used to find the equation, this M1 is earned at the point where the x- and y-values are substituted to find c e.g. $13 = -3/5 \times 10 + c$
	2nd M1	
	A1	Accept any correct answer of the required format; so integer multiple of $3x+5y-95=0$ or $3x-95+5y=0$ or $-3x-5y+95=0$ (must include "=0") e.g. $6x + 10y - 190 = 0$ earns A1 Also allow $5y + 3x - 95=0$ etc
	Common error	$\frac{dy}{dx}=2(x-7)+2(y-8)=6+10=16$ so $(y-13)=16(x-10)$ is marked B0 M0 M1 A0 (Way 2)

Question Number	Scheme		Marks
4.	$f(x) = 6x^3 + 13x^2 - 4$		
(a)	$f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 13\left(-\frac{3}{2}\right)^2 - 4 = 5$	Attempting $f\left(-\frac{3}{2}\right)$ or $f\left(\frac{3}{2}\right)$	M1
		5	A1 cao
			[2]
(b)	$f(-2) = 6(-2)^3 + 13(-2)^2 - 4$	Attempts $f(-2)$.	M1
	$= 0$, **and** so $(x+2)$ is a factor.	$f(-2) = 0$ with no sign or substitution errors **and for conclusion.**	A1
			[2]
(c)	$f(x) = \{(x+2)\}(6x^2 + x - 2)$		M1 A1
	$= (x+2)(2x-1)(3x+2)$		M1 A1
			[4]
			8

Question 4 Notes

	Note	**Long division scores no marks in part (a). The remainder theorem is required.**
(a)	**M1**	Attempting $f\left(-\frac{3}{2}\right)$ or $f\left(\frac{3}{2}\right)$. $6\left(-\frac{3}{2}\right)^3 + 13\left(-\frac{3}{2}\right)^2 - 4$ or $6\left(\frac{3}{2}\right)^3 + 13\left(\frac{3}{2}\right)^2 - 4$ is sufficient
	A1	5 cao
(b)	**M1**	Attempting $f(-2)$. (This is **not** given for f(2))
	A1	Must correctly show $f(-2) = 0$ **and** give a conclusion ***in part (b) only***. No simplification of terms is required here.
	Note	Stating "hence factor" or "it is a factor" or a "tick" or "QED" are possible conclusions. Also a conclusion can be implied from a preamble, eg: "If $f(-2) = 0$, $(x+2)$ is a factor…." **Long division scores no marks in part (b). The factor theorem is required.**
(c)	**1st M1**	Attempting to divide by $(x+2)$ leading to a quotient which is quadratic with at least two terms beginning with first term of $\pm 6x^2 +$ linear or constant term. Or $f(x) = (x+2)(\pm 6x^2 + \text{linear and/or constant term})$ (This may be seen in part (b) where candidates did not use factor theorem and might be referred to here)
	1st A1	$(6x^2 + x - 2)$ seen as quotient or as factor. If there is an error in the division resulting in a remainder give A0, but allow recovery to gain next two marks if $(6x^2 + x - 2)$ is used
	2nd M1	For a ***valid*** attempt to factorise **their** three term quadratic.
	A1	$(x+2)(2x-1)(3x+2)$ and needs all three factors on the same line. Ignore subsequent work (such as a **solution** to a quadratic equation).
	Special cases	**Calculator methods:** Award M1A1M1A1 for correct answer $(x+2)(2x-1)(3x+2)$ with no working. Award M1A0M1A0 for either $(x+2)(2x+1)(3x+2)$ or $(x+2)(2x+1)(3x-2)$ or $(x+2)(2x-1)(3x-2)$ with no working. (At least one bracket incorrect) Award M1A1M1A1 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x+2)(2x-1)(3x+2)$. Award M0A0M0A0 for a candidate who writes down $x = -2, \frac{1}{2}, -\frac{2}{3}$ giving no factors. Award M1A1M1A1 for $6(x+2)(x-\frac{1}{2})(x+\frac{2}{3})$ or $2(x+2)(x-\frac{1}{2})(3x+2)$ or equivalent. Award SC: M1A0M1A0 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x+2)(x-\frac{1}{2})(x+\frac{2}{3})$.

Question Number	Scheme		Marks
5.	(a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x)=(1+kx)(2-9x)^4 = A - 232x + Bx^2$		
(a) **Way 1**	First term of 16 in their final series		B1
	At least one of $({}^4C_1 \times ... \times x)$ or $({}^4C_2 \times ... \times x^2)$		M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 2**	$(2-9x)^4 = (4-36x+81x^2)(4-36x+81x^2)$		
		First term of 16 in their final series	B1
	$= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$	Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in x or at least 2 terms in x^2.	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 3**	$\{(2-9x)^4 =\}\ 2^4\left(1-\frac{9}{2}x\right)^4$	First term of 16 in final series	B1
	$= 2^4\left(1 + \underline{4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2} + ...\right)$	At least one of $\underline{(4 \times ... \times x) \text{ or } \left(\frac{4(3)}{2} \times ... \times x^2\right)}$	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
	Parts (b), (c) and (d) may be marked together		
(b)	$A = "16"$	Follow through their value from (a)	B1ft
			[1]
(c)	$\{(1+kx)(2-9x)^4\} = (1+kx)(16 - 288x + \{1944x^2 + ...\})$	*May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).*	M1
	x terms: $-288x + 16kx = -232x$		
	giving, $16k = 56 \Rightarrow \underline{k = \frac{7}{2}}$	$\underline{k = \frac{7}{2}}$	A1
			[2]
(d)	x^2 terms: $1944x^2 - 288kx^2$		
	So, $B = 1944 - 288\left(\frac{7}{2}\right)$; $= 1944 - 1008 = 936$	See notes	M1
		936	A1
			[2]
			9

	Question 5 Notes	
(a) Ways 1 and 3	**B1 cao**	16
	M1	Correct binomial coefficient associated with correct power of x *i.e* $\left({}^4C_1 \times ... \times x\right)$ or $\left({}^4C_2 \times ... \times x^2\right)$ They may have 4 and 6 or 4 and $\frac{4(3)}{2}$ or even $\binom{4}{1}$ and $\binom{4}{2}$ as their coefficients. Allow missing signs and brackets for the M marks.
	1st A1	At least one of $-288x$ or $+1944x^2$ (allow +- 288x)
	2nd A1	Both $-288x$ **and** $+1944x^2$ (May list terms separated by commas) Also full marks for correct answer with no working here. Again allow +- 288x
	Note	If the candidate then divides their final correct answer through by 8 or any other common factor then isw and mark correct series when first seen. So (a) B1M1A1A1 .It is likely that this approach will be followed by (b) B0, (c) M1A0, (d) M1A0 if they continue with their new series e.g. $2-36x+283x^2+...$ (Do not ft the value 2 as a mark was awarded for 16)
Way 2b	**Special Case**	**Slight Variation on the solution given in the scheme**
		$(2-9x)^4 = (2-9x)(2-9x)(4-36x+81x^2)$
		$= (2-9x)(8-108x+486x^2+...)$
		$= 16 - 216x + 972x^2 - 72x + 972x^2$ — First term of 16: **B1**; Multiplies out to give either 2 terms in x or 2 terms in x^2.: **M1**
		$= (16) - 288x + 1944x^2 + ...$ — At least one of $-288x$ or $+1944x^2$: **A1**; Both $-288x$ **and** $+1944x^2$: **A1**
		Parts (b), (c) and (d) may be marked together.
(b)	**B1ft**	Must **identify** $A = 16$ or $A =$ *their* constant term found in part (a). Or may write just 16 if this is clearly their answer to part (b). If they expand their series and have 16 as first term of a series it is not sufficient for this mark.
(c)	**M1**	Candidate shows intention to multiply (1+kx) by part of their series from (a) e.g. Just $(1+kx)(16-288x+...)$ or $(1+kx)(16-288x+1944x^2+...)$ are fine for M1.
	Note	This mark can also be implied by candidate multiplying out to find **two terms** (or coefficients) in x. i.e. f.t. their $-288x+16kx$ N.B. $-288kx = -232x$ with no evidence of brackets is M0 – allow copying slips, or use of factored series, as this is a method mark
	A1	$k = \frac{7}{2}$ o.e. so 3.5 is acceptable
(d)	**M1**	Multiplies out their $(1+kx)(16-288x+1944x^2+...)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find B using **these two** terms and a numerical value of k.
	A1	936
	Note	Award A0 for $B = 936x^2$ But allow A1 for $B = 936x^2$ **followed by** $B = 936$ and treat this as a correction Correct answers in parts (c) and (d) with no method shown may be awarded full credit.

Question Number	Scheme	Marks
6.	$1-2\cos\left(\theta-\frac{\pi}{5}\right)=0\,;\ -\pi<\theta\leqslant\pi$	
(i)	$\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$ — Rearranges to give $\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$ or $-\frac{1}{2}$	M1
	$\theta=\left\{-\frac{2\pi}{15},\ \frac{8\pi}{15}\right\}$ — At least one of $-\frac{2\pi}{15}$ or $\frac{8\pi}{15}$ or -24° or 96° or awrt 1.68 or awrt -0.419	A1
	Both $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$	A1
		[3]
NB Misread	**Misreading** $\frac{\pi}{5}$ as $\frac{\pi}{6}$ or $\frac{\pi}{3}$ (or anything else)– treat as misread so M1 A0 A0 is maximum mark	
	$4\cos^2 x+7\sin x-2=0,\ 0\leqslant x<360^{\circ}$	
(ii)	$4(1-\sin^2 x)+7\sin x-2=0$ — Applies $\cos^2 x=1-\sin^2 x$	M1
	$4-4\sin^2 x+7\sin x-2=0$	
	$4\sin^2 x-7\sin x-2=0$ — Correct 3 term, $4\sin^2 x-7\sin x-2\ \{=0\}$	A1 oe
	$(4\sin x+1)(\sin x-2)\ \{=0\}$, $\sin x=\ldots$ — Valid attempt at solving and $\sin x=\ldots$	M1
	$\sin x=-\frac{1}{4},\ \ \{\sin x=2\}$ — $\sin x=-\frac{1}{4}$ (See notes.)	A1 cso
	$x=\text{awrt}\{194.5,\ 345.5\}$ — At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0	A1ft
	awrt 194.5 **and** awrt 345.5	A1
		[6]
		9
NB Misread	**Writing equation as** $4\cos^2 x-7\sin x-2=0$ with a sign error should be marked by applying the scheme as **it simplifies** the solution (do not treat as misread) Max mark is 3/6	
	$4(1-\sin^2 x)-7\sin x-2=0$	**M1**
	$4\sin^2 x+7\sin x-2=0$	**A0**
	$(4\sin x-1)(\sin x+2)\ \{=0\}$, $\sin x=\ldots$ — Valid attempt at solving and $\sin x=\ldots$	**M1**
	$\sin x=+\frac{1}{4},\ \ \{\sin x=-2\}$ — $\sin x=\frac{1}{4}$ (See notes.)	**A0**
	$x=\text{awrt}165.5$	**A1ft**
	Incorrect answers	**A0**

Question 6 Notes		
(i)	**M1**	Rearranges to give $\cos\left(\theta - \frac{\pi}{5}\right) = \pm\frac{1}{2}$
	Note	M1 can be implied by seeing either $\frac{\pi}{3}$ or 60° as a result of taking $\cos^{-1}(...)$.
	A1	Answers **may be in degrees or radians** for this mark and may have just one correct answer Ignore mixed units in working if correct answers follow (recovery)
	A1	Both answers correct and in radians as multiples of π $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$ Ignore EXTRA solutions outside the range $-\pi < \theta \le \pi$ but lose this mark for extra solutions in this range.
(ii)	**1st M1**	Using $\cos^2 x = 1 - \sin^2 x$ on the given equation. [Applying $\cos^2 x = \sin^2 x - 1$, scores M0.]
	1st A1	Obtaining a correct three term equation eg. either $4\sin^2 x - 7\sin x - 2 \{= 0\}$ or $-4\sin^2 x + 7\sin x + 2 \{= 0\}$ or $4\sin^2 x - 7\sin x = 2$ or $4\sin^2 x = 7\sin x + 2$, etc.
	2nd M1	For a valid attempt at solving a 3TQ quadratic in sine. Methods include factorization, quadratic formula, completion of the square (unlikely here) and calculator. (See notes on page 6 for general principles on awarding this mark) Can use any variable here, s, y, x or $\sin x$, and an attempt to find at least one of the solutions for sin$x$. This solution may be outside the range for sin$x$
	2nd A1	$\sin x = -\frac{1}{4}$ BY A CORRECT SOLUTION ONLY UP TO THIS POINT**.** Ignore extra answer of $\sin x = 2$, but penalise if candidate states an incorrect result. e.g. $\sin x = -2$.
	Note	$\sin x = -\frac{1}{4}$ can be implied by later correct working if no errors are seen.
	3rd A1ft	At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0. This is a limited follow through. Only follow through on the error $\sin x = \frac{1}{4}$ and allow for 165.5 special case (as this is equivalent work) This error is likely to earn M1A1M1A0A1A0 so 4/6 or M1A0M1A0A1A0 if the quadratic had a sign slip.
	4thA1 **Note**	awrt 194.5 **and** awrt 345.5 If there are any EXTRA solutions inside the range $0 \leqslant x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final A1 mark. Ignore EXTRA solutions outside the range $0 \leqslant x < 360^\circ$.
	Special Cases	Rounding error Allow M1A1M1A1A1A0 for those who give two correct answers but wrong accuracy e.g. awrt 194, 346 (Remove final A1 for this error) Answers in radians:– **lose final** mark so either or both of 3.4, 6.0 gets A1ftA0 It is possible to earn M1A0A1A1 on the final 4 marks if an error results fortuitously in sin$x$ = -1/4 then correct work follows.

Question Number	Scheme		Marks
7. (a)	$\left\{\int\left(3x - x^{\frac{3}{2}}\right)\mathrm{d}x\right\} = \dfrac{3x^2}{2} - \dfrac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)}\ \{+c\}$	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$	M1
		At least one term correctly integrated	A1
		Both terms correctly integrated	A1
			[3]
(b)	$0 = 3x - x^{\frac{3}{2}} \Rightarrow 0 = 3 - x^{\frac{1}{2}}$ or $0 = x\left(3 - x^{\frac{1}{2}}\right) \Rightarrow x = ...$	Sets $y = 0$, in order to find the correct $x^{\frac{1}{2}} = 3$ or $x = 9$	M1
	$\left\{\text{Area}(S) = \left[\dfrac{3x^2}{2} - \dfrac{2}{5}x^{\frac{5}{2}}\right]_0^9\right\}$		
	$= \left(\dfrac{3(9)^2}{2} - \left(\dfrac{2}{5}\right)(9)^{\frac{5}{2}}\right) - \{0\}$	Applies the limit 9 on an integrated function with **no wrong lower limit** .	ddM1
	$\left\{= \left(\dfrac{243}{2} - \dfrac{486}{5}\right) - \{0\}\right\} = \dfrac{243}{10}$ or 24.3	$\dfrac{243}{10}$ or 24.3	A1 oe
			[3]
			6

Question 7 Notes

(a)	**M1**	Either $3x \to \pm\lambda x^2$ or $x^{\frac{3}{2}} \to \pm\mu x^{\frac{5}{2}}$, $\lambda, \mu \neq 0$
	1st A1	At least one term correctly integrated. Can be simplified or un-simplified but power must be simplified. Then isw.
	2nd A1	Both terms correctly integrated. Can be un-simplified (as in the scheme) but the n+1 in each denominator and power should be a single number. (e.g. 2 – not 1+1) Ignore subsequent work if there are errors simplifying. Ignore the omission of "$+c$". Ignore integral signs in their answer.
(b)	**1st M1**	Sets $y = 0$, and reaches the **correct** $x^{\frac{1}{2}} = 3$ or $x = 9$ (isw if $x^{\frac{1}{2}} = 3$ is followed by $x = \sqrt{3}$) Just seeing $x = \sqrt{3}$ without the correct $x^{\frac{1}{2}} = 3$ gains M0. May just see $x = 9$. Use of trapezium rule to find area is M0A0 as hence implies integration needed.
	ddM1	This mark is dependent on the two previous method marks and needs both to have been awarded. Sees the limit **9** substituted in an integrated function. (Do not follow through their value of x) Do not need to see MINUS 0 but if another value is used as lower limit – this is M0. This mark may be implied by 9 in the limit and a correct answer.
	A1	$\dfrac{243}{10}$ or 24.3
	Common Error	**Common Error** $0 = 3x - x^{\frac{3}{2}} \Rightarrow x^{\frac{1}{2}} = 3$ *so* $x = \sqrt{3}$ **Then** uses limit $\sqrt{3}$ etc gains M1 M0 A0 so 1/3

Question Number	Scheme	Marks
8(i)	Two Ways of answering the question are given in part (i)	
Way 1	$\log_3\left(\frac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\frac{a-2}{3b+1}\right) = 1$ — Applying the subtraction law of logarithms	M1
	$\frac{3b+1}{a-2} = 3^{-1}\left\{=\frac{1}{3}\right\}$ or $\left(\frac{a-2}{3b+1}\right) = 3$ — Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3 = a-2 \Rightarrow\}\ b = \frac{1}{9}a - \frac{5}{9}$ — $b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$	A1 oe
		[3]
	In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs	
(i) **Way 2**	Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$	2nd M1
	$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\frac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$	1st M1
	$\{3b+1 = \frac{a-2}{3}\}\ b = \frac{1}{9}a - \frac{5}{9}$	A1
		[3]
	Five Ways of answering the question are given in part (ii)	
(ii)	$32(2^{2x}) - 7(2^x) = 0$ — Deals with power 5 correctly giving ×32	M1
Way 1 **See also common approach below in notes**	So, $2^x = \frac{7}{32}$ — $2^x = \frac{7}{32}$ or $y = \frac{7}{32}$ or awrt 0.219	A1 oe
	$x\log 2 = \log\left(\frac{7}{32}\right)$ or $x = \frac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x = \log_2\left(\frac{7}{32}\right)$ — A valid method for solving $2^x = \frac{7}{32}$ Or $2^x = k$ to achieve $x = \ldots$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
	Begins with $2^{2x+5} = 7(2^x)$ (for Way 2 and Way 3) (see notes below)	
(ii) **Way 2**	$(2x+5)\log 2 = \log 7 + x\log 2$ — Correct application of **either** the power law **or** addition law of logarithms	M1
	Correct result after applying the power **and** addition laws of logarithms.	A1
	$2x\log 2 + 5\log 2 = \log 7 + x\log 2$ $\Rightarrow\ x = \frac{\log 7 - 5\log 2}{\log 2}$ — Multiplies out, collects x terms to achieve $x = \ldots$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
(ii) **Way 3**	$2x + 5 = \log_2 7 + x$ — Evidence of $\log_2$ and either $2^{2x+5} \to 2x+5$ or $7(2^x) \to \log_2 7 + \log_2(2^x)$	M1
	$2x + 5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow\ x = \log_2 7 - 5$ — Collects x terms to achieve $x = \ldots$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]

(ii) **Way 4**	$2^{2x+5} = 7(2^x) \Rightarrow 2^{x+5} = 7$	
	$x + 5 = \log_2 7$ or $\dfrac{\log 7}{\log 2}$ — Evidence of $\log_2$ and either $2^{x+5} \to x+5$ or $7 \to \log_2 7$	M1
	$x + 5 = \log_2 7$ oe.	A1
	$x = \log_2 7 - 5$ — Rearranges to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
Way 5 **(similar to Way 3)**	$2^{2x+5} = 2^{\log_2 7}(2^x)$ — 7 is replaced by $2^{\log_2 7}$	M1
	$2x + 5 = \log_2 7 + x$ — $2x + 5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$ — Collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
		7

	Question 8 Notes	
(i)	**1st M1**	Applying either the addition or subtraction law of logarithms correctly to combine any **two** log terms into **one** log term.
	2nd M1	For making a correct connection between log base 3 and 3 to a power.
	A1	$b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$ o.e. e.g. Accept $b = \frac{1}{3}\left(\frac{a}{3} - \frac{5}{3}\right)$ but not $b = \frac{a-2}{9} - \frac{3}{9}$ nor $b = \frac{\left(\frac{a}{3} - \frac{5}{3}\right)}{3}$
(ii)	**1st M1**	First step towards solution – an equation with one side or other correct or one term dealt with correctly (see five* possible methods above)
	1st A1	Completely correct first step – giving a correct equation as shown above
	dM1	Correct complete method (all log work correct) and working to reach $x =$ in terms of logs reaching a correct expression or one where the only errors are slips solving linear equations
	2nd A1	Accept answers which round to -2.19 If a second answer is also given this becomes A0
	Special Case in (i)	Writes $\dfrac{\log_3(3b+1)}{\log_3(a-2)} = -1$ and proceeds to $\dfrac{3b+1}{a-2} = 3^{-1}\left\{= \dfrac{1}{3}\right\}$ and to correct answer- Give M0M1A1 (special case)
	Common approach to part (ii)	Let $2^x = y$ Treat this as **Way 1** They get $32y^2 - 7y = 0$ for M1 and need to reach $y = \dfrac{7}{32}$ for A1 Then back to **Way 1** as before. Any letter may be used for the new variable which I have called y. If they use x and obtain $x = \dfrac{7}{32}$, this may be awarded M1A0M0A0 Those who get $y^2 - 7y + 32 = 0$ or $y^7 - 7y = 0$ will be awarded M0,A0,M0,A0
	Common Present-ation of Work in ii	**Many begin with** $\log\left(2^{2x+5}\right) - \log\left(7(2^x)\right) = 0$. It is possible to reach this in two stages correctly so do not penalise this and award the full marks if they continue correctly as in **Way 2**. If however the solution continues with $(2x+5)\log 2 - x\log 14 = 0$ or with $(2x+5)\log 2 - 7x\log 2 = 0$ (both incorrect) then they are awarded M1A0M0A0 just getting credit for the $(2x + 5)$ log2 term.
	Note	N.B. The answer (+)2.19 results from "algebraic errors solving linear equations" leading to $2^x = \frac{32}{7}$ and gets M1A0M1A0

Question Number	Scheme		Marks
9. (a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$	$\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360}\times \pi x^2$ simplified or un-simplified	M1
		$\frac{\pi x^2}{3}$	A1
			[2]
	Parts (b) and (c) may be marked together		
(b)	$\{A =\}\ \frac{1}{2}x^2\sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	Attempt to sum 3 areas (at least one correct)	M1
		Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ *	Correct proof.	A1 *
			[3]
(c)	$\{P =\}\ x + x\theta + y + 2x + y\ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	Correct expression in x and y for their θ measured in rads	B1ft
	$...2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$	Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$		
	$\Rightarrow P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$ *	Correct proof.	A1 *
			[3]
	Parts (d) and (e) should be marked together		
(d)	$\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$	$\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$	M1
		Correct differentiation (need not be simplified).	A1;
		Their $P' = 0$	M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ $(= 16.63392808...)$	$\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..$ (m)	awrt 120	A1
			[5]
(e)	$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow$ Minimum	Finds P'' **and** considers sign.	M1
		$\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
			[2]
			15

		Question 9 Notes
(a)	M1	Attempts to use $\text{Area}(FEA) = \frac{1}{2}x^2 \times \frac{2\pi}{3}$ (using radian angle) or $\frac{120}{360} \times \pi x^2$ (using angle in degrees)
	A1	$\frac{\pi x^2}{3}$ cao (Must be simplified and be their answer in part (a)) Answer only implies M1A1. N.B. $\text{Area}(FEA) = \frac{1}{2}x^2 \times 120$ is awarded M0A0
(b)	M1	An attempt to sum 3 " areas" consisting of rectangle, triangle and sector (allow slips even in dimensions) but **one area** should be correct
	1st A1	Correct expression for **two** of the **three** areas listed above. Accept any correct equivalents e.g. two correct from $\frac{1}{2}x^2 \sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{4}x^2\sqrt{3}$, $\frac{1}{2} \times \frac{2}{3}\pi x^2$, $2xy$
	2nd A1*	This is a given answer which should be stated and should be achieved without error so all three areas must have been correct and their sum put equal to 1000 and an intermediate step of rearrangement should be present.
(c)	B1ft	Correct expression for P from arc length, length AB and three sides of rectangle in terms of both x and y with $2y$ (or $y + y$), $3x$ (or $x + 2x$) (or $x + x + x$), and $x\theta$ clearly listed . Allow addition after substitution of y. NB $\theta = \frac{2\pi}{3}$ but allow use of their consistent θ in radians (usually $\theta = \frac{\pi}{3}$) from parts (a) and (b) for this mark. $120x$ or $60x$ do not get this mark.
	M1	Substitutes $y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ or their unsimplified attempt at y from earlier (allow slips e.g. sign slips) into $2y$ term.
	A1*	This is a given answer which should be stated and should be achieved without error
(d)	1st M1	Need to see at least $\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$
	1st A1	Correct differentiation of both terms (need not be simplified) Not follow through. Allow any correct equivalent. e.g. $\frac{dP}{dx} = -1000x^{-2} + \frac{\pi}{3} + 3 - \frac{\sqrt{3}}{4}$ Also allow $\frac{dP}{dx} = -1000x^{-2} + awrt\ 3.61$ Check carefully as there are many correct equivalents and some have two terms in $x\pi$ to differentiate obtaining for example $\frac{2\pi}{3} - \frac{8\pi}{24}$ instead of $\frac{\pi}{3}$
	2nd M1	Setting their $\frac{dP}{dx} = 0$. Do not need to find x, but if inequalities are used this mark cannot be gained until candidate states or uses a value of x without inequalities. May not be explicit but may be implied by correct working and value or expression for x . May result in $x^2 < 0$ so M1A0
	2nd A1	There is no requirement to write down a value for x, so this mark may be implied by a correct value for P. It may be given for a correct expression or value for x of 16.6, 16.7 or 17
	3rd A1	Allow answers wrt 120 but not 121
(e)	M1	Finds P'' **and** considers sign. Follow through **correct** differentiation of their P' (not just reduction of power)
	A1ft	Need $\frac{2000}{x^3}$ and > 0 (or positive value) and conclusion. Only follow through on a correct P'' and a value for x in the range $10 < x < 25$ (need not see x substituted but an x should have been found) If P is substituted then this is awarded M1 A0

	Special case	(d) Some candidates multiply P by 12 to "simplify" If they write $\frac{dP}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$; $= 0$ then solve they will get the correct x and P They should be awarded M1A0M1A1A1 in part (d). If they then do part (e) writing $\frac{d^2P}{dx^2} = \frac{24000}{x^3} > 0 \Rightarrow$ Minimum They should be awarded M1A0 (so lose 2 marks in all) If they wrote $\frac{d(12P)}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$; $= 0$ etc they could get full marks.