Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Wednesday 20 May 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43178A

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10},$$

giving each term in its simplest form.

(4)

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2. A circle C with centre at the point $(2, –1)$ passes through the point A at $(4, –5)$.

(a) Find an equation for the circle C.

(3)

(b) Find an equation of the tangent to the circle C at the point A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

3.

$$f(x) = 6x^3 + 3x^2 + Ax + B, \text{ where } A \text{ and } B \text{ are constants.}$$

Given that when $f(x)$ is divided by $(x + 1)$ the remainder is 45,

(a) show that $B – A = 48$.

(2)

Given also that $(2x + 1)$ is a factor of $f(x)$,

(b) find the value of A and the value of B.

(4)

(c) Factorise $f(x)$ fully.

(3)

4.

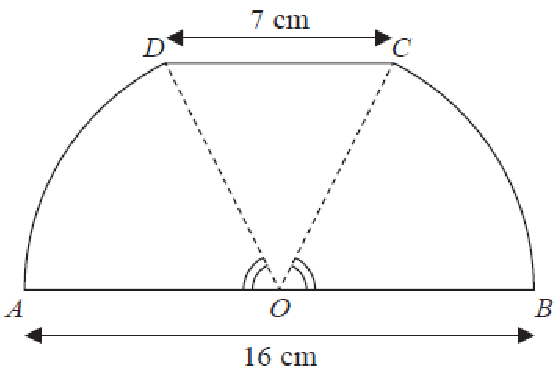

Figure 1

Figure 1 shows a sketch of a design for a scraper blade. The blade $AOBCDA$ consists of an isosceles triangle COD joined along its equal sides to sectors OBC and ODA of a circle with centre O and radius 8 cm. Angles AOD and BOC are equal. AOB is a straight line and is parallel to the line DC. DC has length 7 cm.

(*a*) Show that the angle COD is 0.906 radians, correct to 3 significant figures.

(2)

(*b*) Find the perimeter of $AOBCDA$, giving your answer to 3 significant figures.

(3)

(*c*) Find the area of $AOBCDA$, giving your answer to 3 significant figures.

(3)

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5. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162.

Find

(*a*) the common ratio,

(4)

(*b*) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290.

(4)

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6. (*a*) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\ \mathrm{d}x,$$

giving each term in its simplest form.

(4)

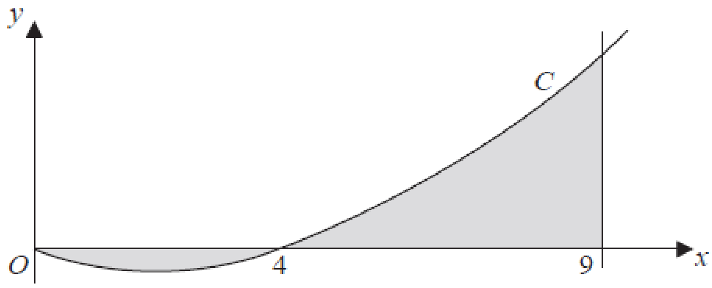

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geq 0.$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0).

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$.

(*b*) Use your answer from part (*a*) to find the total area of the shaded regions.

(5)

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7. (i) Use logarithms to solve the equation $8^{2x+1} = 24$, giving your answer to 3 decimal places.

(3)

(ii) Find the values of y such that

$$\log_2 (11y - 3) - \log_2 3 - 2\log_2 y = 1, \qquad y > \frac{3}{11}.$$

(6)

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8. (i) Solve, for $0 \leq \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0,$$

giving your answers in terms of π.

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \qquad 0 \leq k \leq 3,$$

(*a*) find $\cos x$ in terms of k.

(3)

(*b*) When $k = 3$, find the values of x in the range $0 \leq x < 360°$.

(3)

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9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75\pi\ \text{cm}^3$.

The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(*a*) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}.$$

(4)

(*b*) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(*c*) Justify that the answer that you have obtained in part (*b*) is a minimum.

(1)

TOTAL FOR PAPER: 75 MARKS

END

May 2015
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme		Marks
1.	$\left(2-\frac{x}{4}\right)^{10}$		
Way 1	$2^{10}+\binom{10}{1}2^9\left(-\frac{1}{4}x\right)+\binom{10}{2}2^8\left(-\frac{1}{4}x\right)^2+...$	For **either** the x term **or** the x^2 term including a correct binomial coefficient with a correct power of x	M1
		First term of 1024	B1
	$=1024-1280x+720x^2$	**Either** $-1280x$ **or** $720x^2$ (Allow +-1280x here)	A1
		Both $-1280x$ and $720x^2$ (Do not allow +-1280x here)	A1 **[4]**
Way 2	$\left(2-\frac{x}{4}\right)^{10}=2^k\left(1-10\times\frac{x}{8}+\frac{10\times9}{2}\left(-\frac{x}{8}\right)^2\right)$		M1
	$1024(1\pm.......)$		
	$=1024-1280x+720x^2$		B1A1 A1 **[4]**

Notes

M1: For either the x term or the x^2 term having correct structure i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. The powers of 2 or of ¼ may be wrong or missing.

B1: Award this for 1024 when first seen as a distinct constant term (not $1024x^0$) and not 1 + 1024

A1: For one correct term in x with coefficient simplified. Either -1280x or $720x^2$ (allow +-1280x here)

Allow $720x^2$ to come from $\left(\frac{x}{4}\right)^2$ with no negative sign. So use of + sign throughout could give M1 B1 A1 A0

A1: For both correct simplified terms i.e. -1280x and $720x^2$ (**Do not** allow +-1280x here)

Allow terms to be listed for full marks e.g. 1024, $-1280x$, $+720x^2$

N.B. If they follow a correct answer by a factor such as $512-640x+360x^2$ then isw

Terms may be listed. Ignore any extra terms.

Notes for Way 2

M1: Correct structure for at least one of the underlined terms. i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. k may even be 0 or 2^k may not be seen. Just consider the bracket for this mark.

B1: Needs 1024(1…. To become 1024

A1, A1: as before

Question Number	Scheme		Marks
	Way 1	Way 2	
2 (a)	$(x \mp 2)^2 + (y \pm 1)^2 = k$, $k > 0$	$x^2 + y^2 \mp 4x \pm 2y + c = 0$	M1
	Attempts to use $r^2 = (4-2)^2 + (-5+1)^2$	$4^2 + (-5)^2 - 4\times 4 + 2\times -5 + c = 0$	M1
	Obtains $(x-2)^2 + (y+1)^2 = 20$	$x^2 + y^2 - 4x + 2y - 15 = 0$	A1 **(3)**
	N.B. Special case: $(x-2)^2 - (y+1)^2 = 20$ is not a circle equation but earns M0M1A0		
(b) Way 1	Gradient of radius from centre to (4, -5) = -2 (must be correct)		B1
	Tangent gradient = $-\dfrac{1}{\text{their numerical gradient of radius}}$		M1
	Equation of tangent is $(y+5) = '\tfrac{1}{2}'(x-4)$		M1
	So equation is $x - 2y - 14 = 0$ (or $2y - x + 14 = 0$ or other integer multiples of this answer)		A1 **(4)**
b)Way 2	Quotes $xx' + yy' - 2(x + x') + (y + y') - 15 = 0$ and substitutes (4, -5)_		B1
	$4x - 5y - 2(x+4) + (y-5) - 15 = 0$ so $2x - 4y - 28 = 0$ (or alternatives as in Way 1)		M1,M1A1 **(4)**
b)Way 3	Use differentiation to find expression for gradient of circle		
	Either $2(x-2) + 2(y+1)\dfrac{dy}{dx} = 0$ **or** states $y = -1 - \sqrt{20-(x-2)^2}$ so $\dfrac{dy}{dx} = \dfrac{(x-2)}{\sqrt{20-(x-2)^2}}$		B1
	Substitute $x = 4$, $y = -5$ after valid differentiation to give gradient =		M1
	Then as Way 1 above $(y+5) = '\tfrac{1}{2}'(x-4)$ so $x - 2y - 14 = 0$		M1 A1 **(4)**
			[7]

Notes

(a) **M1:** Uses centre to write down equation of circle in one of these forms. There may be sign slips as shown.

M1: Attempts distance between two points **to establish** r^2 (independent of first M1)- allow one sign slip only using distance formula with -5 or -1, usually $(-5-1)$ in 2nd bracket. Must not identify this distance as diameter.

This mark may alternatively (e.g. way 2)be given for substituting (4, -5) into a **correct circle** equation with one unknown

Can be awarded for $r = \sqrt{20}$ or for $r^2 = 20$ stated or implied but not for $r^2 = \sqrt{20}$ or $r = 20$ or $r = \sqrt{5}$

A1: Either of the answers printed or correct equivalent e.g. $(x-2)^2 + (y+1)^2 = (2\sqrt{5})^2$ is A1 but $2\sqrt{5}^2$ (no bracket) is A0 unless there is recovery

Also $(x-2)^2 + (y-(-1))^2 = (2\sqrt{5})^2$ may be awarded M1M1A1as a correct equivalent.

N.B. $(x-2)^2 + (y+1)^2 = 40$ commonly arises from one sign error evaluating r and earns M1M1A0

(b) **Way 1:**

B1: Must be correct answer -2 if evaluated (otherwise may be implied by the following work)

M1: Uses negative reciprocal of their gradient

M1: Uses $y - y_1 = m(x - x_1)$ with (4,-5) and their **changed** gradient **or** uses $y = mx + c$ and (4, -5) with their changed gradient (not gradient of radius) to find c

A1: answers in scheme or multiples of these answers (must have " = 0"). NB Allow $1x - 2y - 14 = 0$

N.B. $(y+5) = '\tfrac{1}{2}'(x-4)$ following gradient of is ½ after errors leads to $x - 2y - 14 = 0$ but is worth B0M0M0A0

Way 2: Alternative method (b) is rare.

Way 3: Some may use implicit differentiation to differentiate- others may attempt to make y the subject and use chain rule

B1: the differentiation must be accurate and the algebra accurate too. Need to take (-) root not (+)root in the alternative

M1: Substitutes into their gradient function but must follow valid accurate differentiation

M1: Must use "their" tangent gradient and $y + 5 = m(x - 4)$ but allow over simplified attempts at differentiation for this mark.

A1: As in Way 1

Question Number	Scheme	Marks
3.	$\text{f}(x)=6x^3+3x^2+Ax+B$	
Way 1 (a)	Attempting $\text{f}(1)=45$ or $\text{f}(-1)=45$	M1
	$\text{f}(-1)=-6+3-A+B=45$ or $-3-A+B=45 \Rightarrow B-A=48$ * (allow 48 = $B-A$)	A1 * **cso**
		(2)
Way 1 (b)	Attempting $\text{f}(-\frac{1}{2})=0$	M1
	$6(-\frac{1}{2})^3+3(-\frac{1}{2})^2+A(-\frac{1}{2})+B=0$ or $-\frac{1}{2}A+B=0$ or $A=2B$	A1 o.e.
	Solve to obtain $B=-48$ and $A=-96$	M1 A1
		(4)
Way 2 (a)	Long Division	
	$(6x^3+3x^2+Ax+B)\div(x\pm1)=6x^2+px+q$ and sets remainder = 45	M1
	Quotient is $6x^2-3x+(A+3)$ and remainder is $B-A-3=45$ so $B-A=48$ *	A1*
Way 2 (b)	$(6x^3+3x^2+Ax+B)\div(2x+1)=3x^2+px+q$ and sets remainder = 0	M1
	Quotient is $3x^2+\dfrac{A}{2}$ and remainder is $B-\dfrac{A}{2}=0$	A1
	Then Solve to obtain $B=-48$ and $A=-96$ as in scheme above (Way 1)	M1 A1
(c)	Obtain $(3x^2-48)$, (x^2-16), $(6x^2-96)$, $\left(3x^2+\dfrac{A}{2}\right)$, $(3x^2+B)$, $\left(x^2+\dfrac{A}{6}\right)$ or $\left(x^2+\dfrac{B}{3}\right)$ as	B1ft
	factor or as quotient after division by $(2x+1)$. Division by $(x+4)$ or $(x-4)$ see below	
	Factorises $(3x^2-48)$, (x^2-16), $(48-3x^2)$, $(16-x^2)$ or $(6x^2-96)$	M1
	$=3(2x+1)(x+4)(x-4)$ (if this answer follows from a wrong A or B then award A0)	A1cso
	isw if they go on to solve to give $x = 4$, -4 and -1/2	**(3) [9]**

Notes

(a) Way 1: **M1:** 1 or –1 substituted into f(x) and expression put equal to ±45

A1*: Answer is given. Must have substituted –1 and put expression equal to +45. Correct equation with powers of –1 evaluated and conclusion with no errors seen.

Way 2: **M1**: Long division as far as a remainder which is set equal to ±45

A1*: See correct quotient and correct remainder and printed answer obtained with no errors

(b) Way 1: **M1**: Must see $\text{f}(-\frac{1}{2})$ and "= 0" unless subsequent work implies this.

A1: Give credit for a correct equation **even unsimplified** when first seen, then isw. A correct equation implies M1A1.

M1: Attempts to solve the **given equation from part (a)** and their simplified or unsimplified linear equation in A and B from part (b) as far as A =... or B = ...(must eliminate one of the constants but algebra need not be correct for this mark). May just write down the correct answers.

A1: **Both** A and B correct

Way 2: **M1**: Long division as far as a remainder which is set equal to 0

A1: See correct quotient and correct remainder put equal to 0

M1A1: As in Way 1

There may be a mixture of Way 1 for (a) and Way 2 for (b) or vice versa.

(c) **B1**: May be written straight down or from long division, inspection, comparing coefficients or pairing terms

M1: Valid attempt to factorise a **listed** quadratic (see general notes) so $(3x-16)(x+3)$ could get M1A0

A1cso: (Cannot be awarded if A or B is wrong) Needs the answer in the scheme or $-3(2x+1)(4+x)(4-x)$ or equivalent but factor 3 must be shown and there must be all the terms together with brackets.

Way 2: A minority might divide by $(x-4)$ or $(x+4)$ obtaining $(6x^2+27x+12)$ or $(6x^2-21x-12)$ for B1

They then need to factorise $(6x^2+27x+12)$ or $(6x^2-21x-12)$ for M1

Then A1cso as before

Special cases:

If they write down $\text{f}(x)=3(2x+1)(x+4)(x-4)$ with no working, this is B1 M1 A1

But if they give $\text{f}(x)=(2x+1)(x+4)(x-4)$ with no working (from calculator?) give B1M0A0

And $\text{f}(x)=(2x+1)(3x+12)(x-4)$ or $\text{f}(x)=(6x+3)(x+4)(x-4)$ or $\text{f}(x)=(2x+1)(x+4)(3x-12)$ is B1M1A0

Question Number	Scheme	Marks
4.(a)	In triangle *OCD* **complete method** used to find angle *COD* so: Either $\cos COD = \frac{8^2+8^2-7^2}{2\times8\times8}$ or uses $\angle COD = 2\times\arcsin\frac{3.5}{8}$ oe **so** $\angle COD =$	M1
	($\angle COD = 0.9056(331894)$) = 0.906 (3sf) * accept awrt 0.906	A1 * **(2)**
(b)	Uses $s = 8\theta$ for any θ in radians or $\frac{\theta}{360}\times 2\pi\times 8$ for any θ in degrees	M1
	$\theta = \frac{\pi - "COD"}{2}$ $(= awrt\ 1.12)$ or $2\theta(= awrt\ 2.24)$ and Perimeter = 23+(16 $\times\theta$)	M1
	accept awrt 40.9 (cm)	A1 **(3)**
(c)	Either Way 1: (Use of Area of two sectors + area of triangle) Area of triangle = $\frac{1}{2}\times8\times8\times\sin 0.906$ (or 25.1781155 accept awrt 25.2)or $\frac{1}{2}\times8\times7\times\sin 1.118$ or $\frac{1}{2}\times7\times h$ after *h* calculated from correct Pythagoras or trig.	M1
	Area of sector = $\frac{1}{2}8^2\times"1.117979732"$ (or 35.77535142 accept awrt 35.8)	M1
	Total Area = Area of two sectors + area of triangle =awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 **(3)**
	Or Way 2: (Use of area of semicircle – area of segment) Area of semi-circle = $\frac{1}{2}\times\pi\times8\times8$ (or 100.5)	M1
	Area of segment = $\frac{1}{2}8^2\times("0.906"-\sin"0.906")$ (or 3.807)	M1
	So area required = awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 **(3)** **[8]**

Notes

(a) **M1**: Either use correctly quoted cosine rule – may quote as $7^2 = 8^2+8^2-2\times8\times8\cos\alpha \Rightarrow \alpha = ...$
Or split isosceles triangle into two right angled triangles and use arcsin or longer methods using Pythagoras and arcos (i.e. $\pi - 2\times\arccos\frac{3.5}{8}$). **There are many ways of showing this result.**

Must conclude that $\angle COD =$

A1*: (NB this is a given answer) If any errors or over-approximation is seen this is A0. It needs correct work **leading to stated answer** of 0.906 or awrt 0.906 for A1. The cosine of *COD* is equal to 79/128 or awrt 0.617. Use of 0.62 (2sf) does not lead to printed answer. They may give 51.9 in degrees then convert to radians. This is fine.

The minimal solution $7^2 = 8^2+8^2-2\times8\times8\cos\alpha \Rightarrow \alpha = ...$ 0.906 (with no errors seen) can have M1A1 but errors rearranging result in M1A0

(b) **M1**: Uses formula for arc length with *r* = 8 and any angle i.e. $s = 8\theta$ if working in rads or $s = \frac{\theta}{360}\times 2\pi\times 8$ in degrees

(If the formula is quoted with *r* the 8 may be implied by the value of their $r\theta$)

M1: Uses angles on straight line (or other geometry) to find angle *BOC* or *AOD* and uses Perimeter = 23 + arc lengths *BC* and *AD* (may make a slip – in calculation or miscopying)

A1: correct work leading to awrt 40.9 not 40.8 (do not need to see cm) This answer implies M1M1A1

(c) Way 1: **M1**: Mark is given for **correct** statement of area of triangle $\frac{1}{2}\times8\times8\times\sin 0.906$ (must use correct angle) or for correct answer (awrt 25.2) Accept alternative correct methods using Pythagoras and ½ base×height

M1: Mark is given for formula for area of sector $\frac{1}{2}8^2\times"1.117979732"$ with *r* = 8 and their angle *BOC* or *AOD* or (*BOC* + *AOD)* **not** *COD* . May use $A = \frac{\theta}{360}\times\pi\times8^2$ **if working in degrees**

A1: Correct work leading to awrt 96.7, 96.8 or 96.9 (This answer implies M1M1A1)

NB. Solution may combine the two sectors for part (b) and (c) and so might use $2\times\angle BOC$ rather than $\angle BOC$

Way 2: **M1**: Mark is given for **correct** statement of area of semicircle $\frac{1}{2}\times\pi\times8\times8$ or for correct answer 100.5

M1: Mark is given for formula for area of segment $\frac{1}{2}8^2\times("0.906"-\sin"0.906")$ with *r* = 8 or 3.81 **A1**: As in Way 1

Question Number	Scheme	Marks
5.(i)	**Mark (a) and (b) together**	
(a)	$a+ar=34$ or $\dfrac{a(1-r^2)}{(1-r)}=34$ or $\dfrac{a(r^2-1)}{(r-1)}=34$; $\dfrac{a}{1-r}=162$	B1; B1
(Way 1)	Eliminate a to give $(1+r)(1-r)=\dfrac{17}{81}$ or $1-r^2=\dfrac{34}{162}$ .. (not a cubic)	aM1
	(and so $r^2=\frac{64}{81}$ and) $r=\frac{8}{9}$ only	aA1 **(4)**
(b)	Substitute their $r=\frac{8}{9}$ ($0<r<1$) to give $a=$ $a=18$	bM1 bA1 **(2)**
(Way 2) Part (b) first	Eliminate r to give $\dfrac{34-a}{a}=1-\dfrac{a}{162}$	bM1
	gives $a=18$ or 306 and rejects 306 to give $a=18$	bA1
Then part (a) again	Substitute a = 18 to give $r=$	aM1
	$r=\frac{8}{9}$	aA1
(ii)	$\dfrac{42(1-\frac{6}{7}^n)}{1-\frac{6}{7}}>290$ (For trial and improvement approach see notes below)	M1
	to obtain So $(\frac{6}{7})^n<(\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n>(\frac{294}{4})$ or $(\frac{6}{7})^n<(\frac{2}{147})$	A1
	So $n>\dfrac{\log"(\frac{4}{294})"}{\log(\frac{6}{7})}$ or $\log_{\frac{6}{7}}"(\frac{4}{294})"$ or equivalent but must be log of positive quantity	M1
	(i.e. $n>$ 27.9) so $n=28$	A1 **(4)**

Notes

(i) (a) **B1**: Writes **a correct** equation connecting a and r and 34 (allow equivalent equations – may be implied)

B1: Writes a **correct** equation connecting a and r and 162 (allow equivalent equation – may be implied)

Way 1: a**M1**: Eliminates a correctly for these two equations to give $(1+r)(1-r)=\dfrac{17}{81}$ or $(1+r)(1-r)=\dfrac{34}{162}$ or equivalent – **not a cubic** – should have factorized (1 - r) to give a correct quadratic

a**A1:** Correct value for r. Accept 0.8 recurring or 8/9 (not 0.889) Must only have positive value.

b**M1**: Substitutes their r ($0<r<1$) into a correct formula to give value for a. Can be implied by $a=18$

b**A1**: must be 18 (not answers which round to 18)

Way 2: Finds *a* first - B1, B1: As before then award the (b) M and A marks before the (a) M and A marks

b**M1**: Eliminates r correctly to give $\dfrac{34-a}{a}=1-\dfrac{a}{162}$ or $a^2-324a+5508=0$ or equivalent

b**A1:** Correct value for a so $a=18$ only. (Only award after 306 has been rejected)

a**M1**: Substitutes their 18 to give $r=$

a**A1**: $r=\frac{8}{9}$ only

(ii) **M1:** Allow n **or** $n-1$ and any symbols from ">", "<", or "=" etc **A1 :** Must be power n (**not** $n-1$) with any symbol

M1: Uses logs correctly on $(\frac{6}{7})^n$ or $(\frac{7}{6})^n$ **not on** $(36)^n$ to get as far as n Allow any symbol

A1: $n=28$ cso (any errors with inequalities earlier e.g. failure to reverse the inequality when dividing by the negative $\log(\frac{6}{7})$ or any contradictory statements must be penalised here) Those with equals throughout may gain this mark if they follow 27.9 by n=28. Just $n=28$ without mention of 27.9 is only allowed following correct inequality work.

Special case: Trial and improvement: Gives $n=28$ as S = awrt 290.1 (M1A1)and when $n=27$ S = (awrt) 289 so $n=28$ (M1A1) – $n=28$ with no working is M1A0M0A0 and insufficient accuracy is M1A0M1A0

Uses nth term instead of sum of n terms – over simplified – do not treat as misread – award 0/4

Question Number	Scheme	Marks
	May mark (a) and (b) together	
6. (a)	Expands to give $10x^{\frac{3}{2}} - 20x$	B1
	Integrates to give $\frac{10}{"\frac{5}{2}"}x^{"\frac{5}{2}"} + \frac{-"20"x^2}{2}$ (+ c)	M1 A1ft
	Simplifies to $4x^{\frac{5}{2}} - 10x^2 (+ c)$	A1cao **(4)**
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted)	M1
	Use limits 4 and 9 either way round on their integrated function	dM1
	Obtains either $\pm$ –32 or $\pm$ 194 needs at least one of the previous M marks for this to be awarded	A1
	(So area = $\left\lvert\int_0^4 ydx\right\rvert + \int_4^9 ydx$) i.e. 32 + 194, = 226	ddM1,A1 **(5)** **[9]**

Notes

(a) **B1**: Expands the bracket correctly

M1: Correct integration process **on at least one term** after attempt at multiplication. (Follow correct expansion or one slip resulting in $10x^k - 20x$ where k may be $\frac{1}{2}$ or $\frac{5}{2}$ or resulting in $10x^{\frac{3}{2}} - Bx$, where B may be 2 or 5)

So $x^{\frac{3}{2}} \to \frac{x^{\frac{5}{2}}}{5/2}$ or $x^{\frac{1}{2}} \to \frac{x^{\frac{3}{2}}}{3/2}$ or $x^{\frac{5}{2}} \to \frac{x^{\frac{7}{2}}}{7/2}$ and/or $x \to \frac{x^2}{2}$.

A1: Correct unsimplified follow through for both terms of their integration. Does not need (+ c)

A1: Must be simplified and correct– allow answer in scheme or $4x^{2\frac{1}{2}} - 10x^2$. Does not need (+ c)

(b) **M1**: (does not depend on first method mark) Attempt to substitute 4 into their integral (however obtained but must not be differentiated) or seeing their evaluated number (usually 32) is enough – do not need to see minus zero.

d**M1:** (depends on first method mark in (a)) Attempt to subtract either way round using the limits 4 and 9 $A \times 9^{\frac{5}{2}} - B \times 9^2$ with $A \times 4^{\frac{5}{2}} - B \times 4^2$ is enough – or seeing 162 –(-32) {but not 162 – 32 }

A1: At least one of the values (32 and 194) correct (needs just one of the two previous M marks in (b)) or may see 162 + 32 + 32 or 162 + 64 or may be implied by correct final answer if not evaluated until last line of working

ddM1: Adds 32 and 194 (may see 162 + 32 + 32 or may be implied by correct final answer if not evaluated until last line of working). This depends on everything being correct to this point.

A1cao: Final answer of 226 not (- 226)

Common errors: $4 \times 4^{\frac{5}{2}} - 10 \times 4^2 + 4 \times 9^{\frac{5}{2}} - 10 \times 9^2 - \ 4 \times 4^{\frac{5}{2}} - 10 \times 4^2 = \pm$ 162 obtains M1 M1 A0 (neither 32 nor 194 seen and final answer incorrect) then M0 A0 so 2/5

Uses correct limits to obtain -32 +162 +32 = +/-162 is M1 M1 A1 (32 seen) M0 A0 so 3/5

Special case: In part (b) Uses limits 9 and 0 = 972 – 810 - 0 = 162 M0 M1 A0 M0A0 scores 1/5
This also applies if 4 never seen.

Question Number	Scheme	Marks
7. (i)	$8^{2x+1}=24$	
	$(2x+1)\log 8 = \log 24$ or $(2x+1)=\log_8 24$	or $8^{2x}=3$ and so $(2x)\log 8 = \log 3$ or $(2x)=\log_8 3$	M1
	$x=\frac{1}{2}\left(\frac{\log 24}{\log 8}-1\right)$ or $x=\frac{1}{2}\left(\log_8 24-1\right)$	$x=\frac{1}{2}\left(\frac{\log 3}{\log 8}\right)$ or $x=\frac{1}{2}\left(\log_8 3\right)$ o.e.	dM1
	=0.264	A1 **(3)**
(ii)	$\log_2(11y-3)-\log_2 3-2\log_2 y=1$	
	$\log_2(11y-3)-\log_2 3-\log_2 y^2=1$.	M1
	$\log_2\frac{(11y-3)}{3y^2}=1$ or $\log_2\frac{(11y-3)}{y^2}=1+\log_2 3=2.58496501$	dM1
	$\log_2\frac{(11y-3)}{3y^2}=\log_2 2$ or $\log_2\frac{(11y-3)}{y^2}=\log_2 6$ (allow awrt 6 if replaced by 6 later)	B1
	Obtains $6y^2-11y+3=0$ o.e. i.e. $6y^2=11y-3$ for example	A1
	Solves quadratic to give $y=$	ddM1
	$y=\frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	A1 **(6)** **[9]**
Notes (i)	**M1**: Takes logs and uses law of powers correctly. (Any log base may be used) Allow lack of brackets. **dM1**: Make *x* subject of their formula correctly (may evaluate the log before subtracting 1 and calculate e.g. (1.528 -1)/2) **A1**: Allow answers which round to 0.264	
(ii)	**M1**: Applies power law of logarithms replacing $2\log_2 y$ by $\log_2 y^2$ **dM1**: Applies quotient or product law of logarithms correctly to the three log terms including term in y^2. (dependent on first M mark) or applies quotient rule to two terms and collects constants (allow "triple" fractions) $1+\log_2 3$ on RHS is not sufficient – need $\log_2 6$ or 2.58… e.g. $\log_2(11y-3)=\log_2 3+\log_2 y^2+\log_2 2$ becoming $\log_2(11y-3)=\log_2 6y^2$ **B1**: States or uses $\log_2 2=1$ or $2^1=2$ at any point in the answer so may be given for $\log_2(11y-3)-\log_2 3-2\log_2 y=\log_2 2$ or for $\frac{(11y-3)}{3y^2}=2$, for example (Sometimes this mark will be awarded before the second M mark, and it is possible to score M1M0B1in some cases) Or may be given for $\log_2 6=2.584962501..$ or $2^{2.584962501..}=6$ **A1**: This or equivalent quadratic equation (does not need to be in this form but should be equation) **ddM1:** (dependent on the two previous M marks) Solves their quadratic equation following reasonable log work using factorising, completion of square, formula or implied by both answers correct. **A1:** Any equivalent correct form – need both answers- allow awrt 0.333 for the answer 1/3 *NB: If "=0" is missing from the equation but candidate continues correctly and obtains correct answers then allow the penultimate A1 to be implied (Allow use of *x* or other varable instead of *y* throughout)	

Question Number	Scheme	Marks
8. (i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = \frac{\pi}{3}$ — Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = \frac{\pi}{3}$	M1
	Adds π or 2π to previous value of angle (to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$)	M1
	So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range)	A1 **(3)**
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$ Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$	dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent	A1 **(3)**
(b)	$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)	M1
	Obtains two solutions from 0 , 139 , 221 (0 or 2.42 or 3.86 in radians)	dM1
	x = 0 and 139 and 221 (allow awrt 139 and 221) must be in degrees	A1 **(3)**
		[9]

Notes

(i) **M1**: Obtains $\frac{\pi}{3}$. Allow $x = \frac{\pi}{3}$ or even $\theta = \frac{\pi}{3}$. Need not see working here. May be implied by $\theta = \frac{\pi}{9}$ in final answer (allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark)

Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm\frac{1}{\sqrt{3}}$

M1: Adding π or 2π to a previous value however obtained. It is not dependent on the previous mark.

(May be implied by final answer of $\theta = \frac{4\pi}{9}$ **or** $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420).

A1: Need all three correct answers in terms of π and **no extras in range**.

Three correct answers implies M1M1A1

NB : $\theta = 20°, 80°, 140°$ earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0

(ii) (a) **M1:** Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$).

This must be awarded in (ii) (a) for an expression with k not after k = 3 is substituted.

dM1: Uses formula or completion of square to obtain cos$x$ = expression in k

(Factorisation attempt is M0) **A1:** cao **-** award for their final simplified expression

(b) **M1**: **Either** attempts to substitute k = 3 into their answer to obtain two values for cos$x$

Or restarts with k = 3 to find two values for cos$x$ (They cannot earn marks in ii(a) for this)

In both cases they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) **and** correct method for solving their quadratic (usual rules – see notes) The values for cos$x$ may be >1 or < -1

dM1: Obtains **two correct** values for x

A1: Obtains **all three correct values** in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.

Question Number	Scheme	Marks
9. (a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ **or (**Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\frac{75\pi}{\pi r^2}$ or $(h=)\frac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\frac{75}{r^2}\right)$ — Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\frac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\frac{dC}{dr}=\right\}$ $12\pi r-\frac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\frac{300\pi}{r^2}=0$ **so** r^k = value where $k=\pm 2,\pm 3,\pm 4$	dM1
	Use **cube** root to obtain $r=\left(their\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow r = 3, and thus C =	ddM1
	Then C = awrt 483 or 484	A1cao (5)
(c)	$\left\{\frac{d^2C}{dr^2}=\right\}$ $12\pi+\frac{600\pi}{r^3}>0$ **so minimum**	B1ft (1) **[10]**

Notes

(a) **B1:** States $3\times 2\pi r^2$ or states $2\times 2\pi rh$

B1ft: Obtains a **correct** expression for h in terms of r **(**ft only follows misread of V)

M1: Substitutes their expression for h into **area or cost** expression of form Ar^2+Brh

A1*: Had correct expression for C and achieves **given** answer in part (a) including "C =" or "Cost=" and **no errors seen** such as C = area expression without multiples of (£)3 and (£)2 at any point. Cost and area must be perfectly distinguished at all stages for this A mark.

N.B. Candidates using Curved Surface Area = $\frac{2V}{r}$ - please send to review

(b) **M1:** Attempts to differentiate as evidenced by at least one term differentiated correctly

A1ft: Correct derivative – allow $12\pi r-300\pi r^{-2}$ then isw if the power is misinterpreted (ft only for misread)

dM1: Sets their $\frac{dC}{dr}$ to 0, and obtains r^k = value where k = 2, 3 or 4 (needs correct collection of powers of r from their original derivative expression – allow errors dividing by 12π)

ddM1: Uses **cube** root to find r **or** see r = awrt 3 as evidence of cube root and substitutes into correct expression for C to obtain value for C

A1: Accept awrt 483 or 484

(c) **B1ft: Finds** correct expression for $\frac{d^2C}{dr^2}$ and deduces value of $\frac{d^2C}{dr^2}$ > 0 so minimum (r may have been wrong)

OR checks gradient to left and right of 2.92 and shows gradient goes from negative to zero to positive so minimum

OR checks value of C to left and right of 2.92 and shows that C > 483 so deduces minimum (i.e. uses shape of graph) Only ft on misread of V for each ft mark (see below)

N..B. Some candidates have **misread** the volume as 75 instead of 75π. PTO for marking instruction.

Following this misread candidates cannot legitimately obtain the printed answer in part (a). Either they obtain $C = 6\pi r^2 + \dfrac{300}{r}$ or they "fudge" their working to appear to give the printed answer.

The policy for a misread is **to subtract 2 marks from A or B marks**. In this case the A mark is to be subtracted from part (a) and the final A mark is to be subtracted from part (b)
The maximum mark for part (a) following this misread is 3 marks. The award is B1 B1 M1 A0 as a maximum.

(a) B1: as before

B1: Uses volume to give $(h =)\dfrac{75}{\pi r^2}$

M1: $(C) = 6\pi r^2 + 4\pi r\left(\dfrac{75}{\pi r^2}\right)$

A0: Printed answer is not obtained without error

Most Candidates may then adopt the printed answer and gain up to full marks for the rest of the question so 9 of the 10 marks maximum in all.

Any candidate who proceeds with **their** answer $C = 6\pi r^2 + \dfrac{300}{r}$ may be awarded up to 4 marks in part (b). These are M1A1dM1ddM1A0 and then the candidate may also be awarded the B1 mark in part (c). So 8 of the 10 marks maximum in all.

(b) M1 A1: $\left\{\dfrac{dC}{dr} =\right\}\ 12\pi r - \dfrac{300}{r^2}$ or $12\pi r - 300r^{-2}$ (then isw)

dM1: $12\pi r - \dfrac{300}{r^2} = 0$ **so** r^k = value where k = 2, 3 or 4 or $12\pi r - \dfrac{300}{r^2} = 0$ **so** r^k = value

ddM1: Use **cube** root to obtain $r = \left(\textit{their}\ \frac{300}{12\pi}\right)^{\frac{1}{3}}$ $(=1.996)$ - allow r = 2, and thus C = … must use $C = 6\pi r^2 + \dfrac{300}{r}$

A0: Cannot obtain C = 483 or 484

(c) B1: $\left\{\dfrac{d^2C}{dr^2} =\right\}\ 12\pi + \dfrac{600}{r^3} > 0$ **so minimum** OR checks gradient to left and right of 1.966 and shows gradient goes from negative to zero to positive so minimum
OR checks value of C to left and right of 1.966 and shows that C > 225.4 so deduces minimum (i.e. uses shape of graph)
There is an example in Practice of this misread.