# Monday 14 January 2013 - Morning 

## Time: 1 hour 30 minutes

## Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 9 questions in this question paper. The total mark for this paper is 75.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Find the first 3 terms, in ascending powers of $x$, in the binomial expansion of

$$
(2-5 x)^{6} .
$$

Give each term in its simplest form.
2.

$$
\mathrm{f}(x)=a x^{3}+b x^{2}-4 x-3 \text {, where } a \text { and } b \text { are constants. }
$$

Given that $(x-1)$ is a factor of $\mathrm{f}(x)$,
(a) show that $a+b=7$.
(2)

Given also that, when $\mathrm{f}(x)$ is divided by $(x+2)$, the remainder is 9 ,
(b) find the value of $a$ and the value of $b$, showing each step in your working.
3. A company predicts a yearly profit of $£ 120000$ in the year 2013. The company predicts that the yearly profit will rise each year by $5 \%$. The predicted yearly profit forms a geometric sequence with common ratio 1.05.
(a) Show that the predicted profit in the year 2016 is $£ 138915$.
(1)
(b) Find the first year in which the yearly predicted profit exceeds $£ 200000$.
(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.
4. Solve, for $0 \leq x<180^{\circ}$,

$$
\cos \left(3 x-10^{\circ}\right)=-0.4,
$$

giving your answers to 1 decimal place. You should show each step in your working.
5. The circle $C$ has equation

$$
x^{2}+y^{2}-20 x-24 y+195=0 .
$$

The centre of $C$ is at the point $M$.
(a) Find
(i) the coordinates of the point $M$,
(ii) the radius of the circle $C$.
$N$ is the point with coordinates $(25,32)$.
(b) Find the length of the line $M N$.

The tangent to $C$ at a point $P$ on the circle passes through point $N$.
(c) Find the length of the line $N P$.
6. Given that $2 \log _{2}(x+15)-\log _{2} x=6$,
(a) show that $x^{2}-34 x+225=0$.
(b) Hence, or otherwise, solve the equation $2 \log _{2}(x+15)-\log _{2} x=6$.
7.


Figure 2
The triangle $X Y Z$ in Figure 1 has $X Y=6 \mathrm{~cm}, Y Z=9 \mathrm{~cm}, Z X=4 \mathrm{~cm}$ and angle $Z X Y=\alpha$.
The point $W$ lies on the line $X Y$.
The circular arc $Z W$, in Figure 1 is a major arc of the circle with centre $X$ and radius 4 cm .
(a) Show that, to 3 significant figures, $\alpha=2.22$ radians.
(2)
(b) Find the area, in $\mathrm{cm}^{2}$, of the major sector $X Z W X$.
(3)

The region enclosed by the major arc $Z W$ of the circle and the lines $W Y$ and $Y Z$ is shown shaded in Figure 1.

Calculate
(c) the area of this shaded region,
(d) the perimeter $Z W Y Z$ of this shaded region.
8. The curve $C$ has equation $y=6-3 x-\frac{4}{x^{3}}, x \neq 0$.
(a) Use calculus to show that the curve has a turning point $P$ when $x=\sqrt{ }$.
(b) Find the $x$-coordinate of the other turning point $Q$ on the curve.
(1)
(c) Find $\frac{\mathrm{d}^{2} y}{\mathrm{dx} x^{2}}$.
(d) Hence or otherwise, state with justification, the nature of each of these turning points $P$ and $Q$.
9.


Figure 2
The finite region $R$, as shown in Figure 2, is bounded by the $x$-axis and the curve with equation

$$
y=27-2 x-9 \sqrt{ } x-\frac{16}{x^{2}}, \quad x>0
$$

The curve crosses the $x$-axis at the points $(1,0)$ and $(4,0)$.
(a) Copy and complete the table below, by giving your values of $y$ to 3 decimal places.

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0 | 5.866 |  | 5.210 |  | 1.856 | 0 |

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of $R$, giving your answer to 2 decimal places.
(c) Use integration to find the exact value for the area of $R$.

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Mark Scheme

| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 1. | $(2-5 x)^{6}$ |  |  |
|  | $\left(2^{6}=\right) 64$ | Award this when first seen (not 64x ${ }^{0}$ ) | B1 |
|  | $+6 \times(2)^{5}(-5 x)+\frac{6 \times 5}{2}(2)^{4}(-5 x)^{2}$ | Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $\binom{6}{p} \times(2)^{6-p}(-5 x)^{p} \text { with } p=1 \text { or } p=2$ <br> consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients e.g. ${ }^{6} C_{1} \text { or }\binom{6}{1} \text { or even }\left(\frac{6}{1}\right)$ | M1 |
|  | -960x | Do not allow +-960x | A1 (first) |
|  | (+)6000x ${ }^{2}$ | Allow this to come from ( $5 x)^{2}$ | A1 (Second) |
|  | Ignore any extra terms and isw e.g. divides all terms by 2 <br> The terms do not have to form a sum i.e. they can be listed with commas or given on separate lines. |  |  |
|  | Special Case - decreasing powers can score M1 with the conditions as above for the second and third terms. |  |  |
|  | $(2-5 x)^{6}=64+\binom{6}{1}\left(2^{5}-5 x\right)+\binom{6}{2}\left(2^{4}+(-5 x)^{2}\right)$ scores B1 only as the powers of 2 and $(-5 x)$ are being added not multiplied. |  |  |
|  Fully correct answer with no working can score full marks. If either the second or third <br> term is correct, the M1 can be implied and the A1 scored for that term. |  |  |  |
|  |  |  | (4) |
| Way 2 |  | 64 and ( $1 \pm \ldots .$. - Award when first seen. | B1 |
|  | $\left(1-\frac{5 x}{2}\right)^{6}=1-6 \times \frac{5 x}{2}+\frac{6 \times 5}{2}\left(-\frac{5 x}{2}\right)^{2}$ | Correct structure for at least one of the underlined terms. E.g. a term of the form: $\binom{6}{p} \times(k x)^{p} \text { with } p=1 \text { or } p=2$ <br> consistently and $k \neq \pm 5$ <br> Condone sign errors. Condoned missing brackets if later work implies correct structure but it must be an expansion of $(1-k x)^{6}$ where $k \neq \pm 5$ | M1 |
|  | $-960 x$ | Do not allow +-960x | A1 |
|  | (+)6000x ${ }^{2}$ | Allow this to come from $\left(\frac{5 x}{2}\right)^{2}$ | A1 |
|  |  |  | (4) |




| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 4. |  |  |  |
|  | $\cos ^{-1}(-0.4)=113.58(\alpha)$ | Awrt 114 | B1 |
|  | $3 x-10=\alpha \Rightarrow x=\frac{\alpha+10}{3}$ | Uses their $\alpha$ to find $x$. <br> Allow $x=\frac{\alpha \pm 10}{3} \operatorname{not} \frac{\alpha}{3} \pm 10$ | M1 |
|  | Note: If $x=\frac{\alpha \pm 10}{3}$ is not clearly applied from their first angle it may be recovered if applied to their second or third angle. |  |  |
|  | $x=41.2$ | Awrt | A1 |
|  | $(3 x-10=) 360-\alpha(246.4 . \ldots$. | $360-\alpha$ (can be implied by 246.4...) | M1 |
|  | $x=85.5$ | Awrt | A1 |
|  | $(3 x-10=) 360+\alpha(=473.57 \ldots$. | $360+\alpha$ (Can be implied by 473.57...) | M1 |
|  | $x=161.2$ | Awrt | A1 |
|  | Note 1: Do not penalise incorrect accuracy more than once and penalise it the first time it occurs. E.g if answers are only given to the nearest integer $(41,85,161)$ only the first A mark that would otherwise be scored is lost. |  |  |
|  | Note 2: Ignore any answers outside the range. For extra answers in range in an otherwise fully correct solution lose final A1 |  |  |
|  | Note 3: Lack of working means that it is sometimes not clear where their intermediate angles are coming from. In these cases, if the final answers are incorrect score M0. |  |  |
|  | Note 4: Candidates are unlikely to be working in radians deliberately but may have their calculator in radian mode ( gives $\alpha=1.98$ ). In such cases the main scheme should be applied and the method marks are available. If you suspect that the candidate is working in radians correctly then please use the review mechanism and/or consult your team leader. |  |  |
| Way 2 | $\cos ^{-1}(0.4)=66.42(\alpha)$ |  |  |
|  | $180-66.42=113.58$ | Awrt 114 | B1 |
|  | $3 x-10=113.58 \Rightarrow x=\frac{113.58+10}{3}$ | Uses their 113.58 to find $x$ | M1 |
|  | $x=41.2$ | Awrt | A1 |
|  | $3 x-10=180+\alpha$ (246.4...) | $180+\alpha$ | M1 |
|  | to give $x=85.5$ |  | A1 |
|  | $3 x-10=540-\alpha$ (473.57....) | 540- $\alpha$ | M1 |
|  | to give $x=161.2$ |  | A1 |
|  |  |  |  |
|  | Special case - takes 0.4 as -0.4$\cos ^{-1}(0.4)=66.42(\alpha)$ |  | B0 |
|  | $3 x-10=66.4 \Rightarrow x=\frac{66.4 \pm 10}{3}$ |  | M1 |
|  | $x=41.2$ |  | A0 |
|  | $3 x-10=360-\alpha$ (293.6...) |  | M1 |
|  | $x=101.2$ |  | A0 |
|  | $3 x-10=360+\alpha$ (426.4....) |  | M1 |
|  | $x=145.5$ |  | A0 |
|  |  |  | (3/7) |







## Appendix

| 3(b) $\text { Way } 2$ | $120000 \times(1.05)^{n-1}>200000$ | Allow $n$ or $n-1$ and ">", "<", or "=" etc. | M1 |
| :---: | :---: | :---: | :---: |
|  | $\log _{1.05} 1.05^{n-1}>\log _{1.05}\left(\frac{5}{3}\right)$ | Takes logs correctly Allow $n$ or $n-1$ and ">", "<", or "=" etc. <br> This may be implied by $n-1>\log _{1.05}\left(\frac{5}{3}\right)$ and effectively gets the next A1 | M1 |
|  | e.g. $\log _{1.05}\left(120000 \times(1.05)^{n-1}\right)=(n-1) \log _{1.05}(120000 \times(1.05))$ would be M0 |  |  |
|  | $(n-1>) \log _{1.05} " \frac{5}{3} "$ | Allow $n$ or $n-1$ and ">", "<", or "=" etc. | A1 |
|  | 2024 | M1: Identifies a calendar year using their value of $n$ or $n-1$ | M1A1 |
|  |  | A1: 2024 only cso |  |
|  |  |  | (5) |


| $\begin{aligned} & \text { 3(b) } \\ & \text { MR? } \end{aligned}$ | $\frac{120000 \times\left(1-1.05^{n}\right)}{1-1.05}>200000$ |  | M0 |
| :---: | :---: | :---: | :---: |
|  | $1.05^{n}>\frac{13}{12}$ |  |  |
|  | $\log 1.05^{n}>\log \left(\frac{13}{12}\right)$ | Takes logs correctly | M1 |
|  | $n>\frac{\log \left(\frac{13}{12}\right)}{\log 1.05}$ |  | A0 |
|  | 2014 | M1: Identifies a calendar year using their value of $n$ or $n-1$ | M1A0 |
|  |  | A1: 2024 only |  |
|  | Trial \& Improvement for this MR is 0/5 |  |  |
|  |  |  | (2/5) |


| 4. <br> Way 3 | General Solution |  |  |
| :--- | :--- | :--- | :--- |
|  | $\cos ^{-1}(-0.4)=113.58(\alpha)$ | Awrt 114 | B1 |
|  | $3 x-10=360 n+113.58$ | $360 n+\alpha$ | M1 |
|  | $3 x-10=360 n-113.58$ | $360 n-\alpha$ | M1 |
|  | $3 x-10=\alpha \Rightarrow 3 x=\alpha+10$ |  |  |
|  | $x=\frac{360 n+123.58}{3}$ or $\frac{360 n-103.58}{3}$ | $x=\frac{360 n \pm 113.58 \pm 10}{3}$ | M1 |
|  | $x=41.2$ | Awrt | A1 |
|  | $x=85.5$ | Awrt | A1 |
|  | $x=161.2$ | Awrt | A1 |
|  |  |  | $\mathbf{( 7 )}$ |


| 4. | Special Case 1 |  |  |
| :--- | :--- | :--- | :--- |
|  | $\cos (3 x-10)=\cos (3 x)-\cos (10)$ |  |  |
|  | $\cos (3 x)=-0.4+\cos (10)$ |  |  |
|  | $\cos (3 x)=0.5848 \ldots$ |  |  |
|  | $3 x=54.2=\alpha$ |  |  |
|  | $x=18.1$ |  | M1 |
|  | $3 x=360-\alpha$ | $x=101.9$ | B0M0A0 so far |
|  | $3 x=360+\alpha$ | A60 | A |
|  | $x=138.1$ | $360+\alpha$ | M1 |
|  |  | Awrt | A0 |
|  |  |  | $\mathbf{( 2 / 7 )}$ |


| 4. Special Case 2-Quite common |  |  |  |
| :--- | :--- | :--- | :--- |
|  | $\cos ^{-1}(-0.4)=113.58(\alpha)$ | Awrt 114 | B1 |
|  | $3 x-10=\alpha \Rightarrow x=\frac{\alpha+10}{3}$ | Uses their $\alpha$ to find $x$. <br> Allow $x=\frac{\alpha \pm 10}{3}$ not $\frac{\alpha}{3} \pm 10$ | M1 |
|  | $x=41.2$ | Awrt | A1 |
|  | $3 x-10=\alpha \Rightarrow 3 x=\alpha+10$ |  | M0 |
|  | $3 x=360-(\alpha+10)$ |  | A0 |
|  | $x=78.8$ |  | M1 |
|  | $3 x=360+(\alpha+10)$ | $x=161.2$ | Awrt |
|  |  |  | A1 |


| 4. | Possible scenarios |  |  |
| :--- | :--- | :--- | :--- |
|  | Answers | Marks |  |
|  | $41.2,97.9$ | B1M1A1M0A0M0A0 |  |
|  | $41.2,97.9,142.7$ | B1M1A1M0A0M0A0 |  |
|  | $41.2,85.5,97.9$ | B1M1A1M1A1M0A0 |  |
|  | $41.2,97.9,161.2$ | B1M1A1M0A0M1A1 |  |
|  | $41.2,85.5,97.9,142.7$ | B1M1A1M1A1M0A0 |  |
|  | $41.2,85.5,97.9,161.2$ | B1M1A1M1A1M1A0 |  |
|  | $41.2,85.5,97.9,142.7,161.2$ | B1M1A1M1A1M1A0 |  |
|  |  |  |  |
|  |  |  |  |


| 6 <br> Way 2 | $2 \log (x+15)=\log (x+15)^{2}$ |  | B1 |
| :--- | :--- | :--- | :--- |
|  | $\log (x+15)^{2}=6+\log x$ |  |  |
|  | $2^{6}=64$ or $\log _{2} 64=6$ | 64 used in the correct context | B1 |
|  | $\log _{2} 64+\log _{2} x=\log _{2}(64 x)$ | Correct use of $\log a+\log b=\log a b$ | M1 |
|  | $(x+15)^{2}=64 x$ | Removes logs correctly | M1 |
|  | $\Rightarrow x^{2}+30 x+225=64 x$ | Must see expansion of $(x+15)^{2}$ to <br> score the final mark. |  |
|  | $\therefore x^{2}-34 x+225=0 *$ | Correct completion to printed answer | A1 |
|  |  |  | (5) |


| 6 <br> Way 3 | $2 \log (x+15)=\log (x+15)^{2}$ |  | B1 |
| :--- | :--- | :--- | :--- |
|  | $2^{6}=64$ or $\log _{2} 64=6$ | 64 used in the correct context | B1 |
|  | $(x+15)^{2}=64 x$ | $\log _{2}(x+15)^{2}-\log _{2} x=\log _{2} 64$ | Correct use <br> oflog $a+\log b=\log a b$ (implied) and <br> removes $\operatorname{logs~correctly.~}$ |
|  | $\Rightarrow x^{2}+30 x+225=64 x$ | Must see expansion of $(x+15)^{2}$ to <br> score the final mark. |  |
|  | $\therefore x^{2}-34 x+225=0 *$ | Correct completion to printed answer | A1 |
|  |  |  |  |


| $\mathbf{6}$ <br> Way 4 | $2 \log (x+15)=\log (x+15)^{2}$ |  | B1 |
| :---: | :--- | :--- | :--- |
|  | $\log (x+15)^{2}-\log x=\frac{\log (x+15)^{2}}{\log x}$ |  | M0 |
|  | $2^{6}=64$ or $\log _{2} 64=6$ | 64 used in the correct context | B1 |
|  | $\frac{\log _{2}(x+15)^{2}}{\log x}=6 \Rightarrow \frac{(x+15)^{2}}{x}=64$ |  | M0 |
|  | $\Rightarrow x^{2}+30 x+225=64 x$ |  | $A 0$ |
|  | $\therefore x^{2}-34 x+225=0 *$ |  | $(2 / 5)$ |


| 6 <br> Way 5 |  |  |  |
| :--- | :--- | :--- | :--- |
|  | $2 \log (x+15)-\log x=2 \log \left(\frac{x+15}{x}\right)$ |  | M0 |
|  | $\log _{2} \frac{(x+15)^{2}}{x}=6$ |  | B0 (first) |
|  | $2^{6}=64$ or $\log _{2} 64=6$ | 64 used in the correct context | B1 |
|  | $\frac{(x+15)^{2}}{x}=64$ |  | M1 |
|  | $\Rightarrow x^{2}+30 x+225=64 x$ | Incorrect solution | A0 |
|  | $\therefore x^{2}-34 x+225=0 *$ | $(2 / 5)$ |  |

