Paper Reference(s) 66664/01 Edexcel GCE

Core Mathematics C2 Advanced Subsidiary

Thursday 24 May 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination Mathematical Formulae (Pink) Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

P40685A This publication may only be reproduced in accordance with Edexcel Limited copyright policy. ©2012 Edexcel Limited. **1.** Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of

 $(2-3x)^5$,

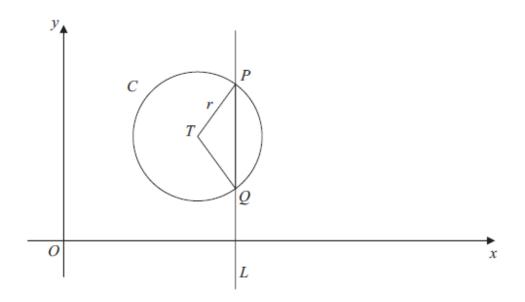
giving each term in its simplest form.

2. Find the values of *x* such that

$$2\log_3 x - \log_3(x - 2) = 2$$

(5)

(4)





The circle C with centre T and radius r has equation

 $x^2 + y^2 - 20x - 16y + 139 = 0.$

- (a) Find the coordinates of the centre of C.
- (b) Show that r = 5 (2)

The line *L* has equation x = 13 and crosses *C* at the points *P* and *Q* as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q. (3)

Given that, to 3 decimal places, the angle *PTQ* is 1.855 radians,

(d) find the perimeter of the sector *PTQ*.

(3)

(2)

(3)

 $f(x) = 2x^3 - 7x^2 - 10x + 24.$

- (a) Use the factor theorem to show that (x + 2) is a factor of f(x).
- (b) Factorise f(x) completely. (4)

P40685A

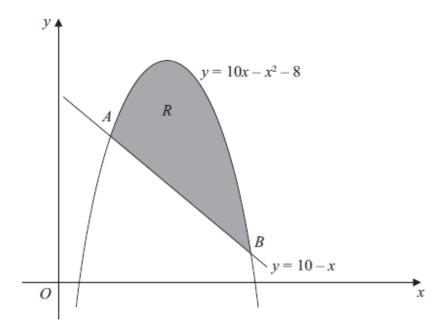




Figure 2 shows the line with equation y = 10 - x and the curve with equation $y = 10x - x^2 - 8$. The line and the curve intersect at the points *A* and *B*, and *O* is the origin.

(*a*) Calculate the coordinates of *A* and the coordinates of *B*.

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

6. (*a*) Show that the equation

 $\tan 2x = 5 \sin 2x$

can be written in the form

$$(1-5\cos 2x)\sin 2x = 0.$$

(*b*) Hence solve, for $0 \le x \le 180^\circ$,

 $\tan 2x = 5 \sin 2x,$

giving your answers to 1 decimal place where appropriate. You must show clearly how you obtained your answers.

(5)

(2)

P40685A

5.

4

(5)

(7)

$$y = \sqrt{3^x + x}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

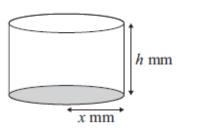
x	0	0.25	0.5	0.75	1	
У	1	1.251			2	
						(2

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of

$$\int_0^1 \sqrt{(3^x+x)} \, \mathrm{d}x.$$

You must show clearly how you obtained your answer.

(4)





A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm³,

(a) express h in terms of x,

(b) show that the surface area, A mm², of a tablet is given by A = $2\pi x^2 + \frac{120}{x}$.

(1)

(3)

The manufacturer needs to minimise the surface area $A \text{ mm}^2$, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum. (5)

- (d) Calculate the minimum value of A, giving your answer to the nearest integer. (2)
- (e) Show that this value of A is a minimum. (2)

5

P40685A

7.

8.

- 9. A geometric series is $a + ar + ar^2 + ...$
 - (*a*) Prove that the sum of the first *n* terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r} \tag{4}$$

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(<i>b</i>)	the common ratio,	(2)
(<i>c</i>)	the first term,	(2)
(<i>d</i>)	the sum to infinity.	
		(3)

TOTAL FOR PAPER: 75 MARKS

END

Summer 2012 6664 Core Mathematics 2 Mark Scheme

Question number	Scheme	Marks	
1	$\left[(2-3x)^5 \right] = \dots + {\binom{5}{1}} 2^4 (-3x) + {\binom{5}{2}} 2^3 (-3x)^2 + \dots \dots$	M1	
	$=32, -240x, +720x^{2}$	B1, A1, A1	
		Total 4	
Notes	M1: The method mark is awarded for an attempt at Binomial to get the second term – need correct binomial coefficient combined with correct power of <i>x</i> . Igomissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for e.g. $\binom{5}{1}$ and $\binom{5}{2}$ (unsimplified) or 5 and 10 from Pascal's triangle This mark	and/or third gnore errors (or ${}^{5}C_{1}$ and ${}^{5}C_{2}$,	
	if no working is shown, but either or both of the terms including x is corr		
	B1: must be simplified to 32 (writing just 2^5 is B0). 32 must be the only of in the final answer- so $32 + 80 - 3x + 80 + 9x^2$ is B0 but may be eligible for	constant term	
	A1: is cao and is for $-240 x$. (not $+-240x$) The x is required for this mark A1: is c.a.o and is for $720x^2$ (can follow omission of negative sign in working)		
	A list of correct terms may be given credit i.e. series appearing on different lines		
	Ignore extra terms in x^3 and/or x^4 (isw)		
Special Case	Special Case: <i>Descending powers</i> of x would be $(-3x)^5 + 2 \times 5 \times (-3x)^4 + 2^2 \times {5 \choose 3} \times (-3x)^3 + i.e243x^5 + 810x^4 - 1080x^4$	3 + This is a	
	(-5x) + 2x + 3x(-5x) + 2x + 3x + 310x - 1080x	\pm This is a	
	misread but award as s.c. M1B1A0A0 if completely "correct" or M1 B0A	A0A0 for	
Alternative	<u>correct</u> binomial coefficient in any form with the correct power of x 5 - 3r = (5) - 3r		
Method	Method 1: $\left[(2-3x)^5 \right] = 2^5 (1 + {5 \choose 1} (-\frac{3x}{2}) + {5 \choose 2} (\frac{-3x}{2})^2 +)$ is M1B0A0A	0 { The M1 is	
	for the expression in the bracket and as in first method– need correct bin coefficient combined with correct power of <i>x</i> . Ignore bracket errors or errors (or powers of 2 or 3 or sign or bracket errors}		
	– answers must be simplified to $= 32, -240x, +720x^2$ for full marks (awar		
	$\left[(2-3x)^5 \right] = 2(1 + \binom{5}{1}(-\frac{3x}{2}) + \binom{5}{2}(\frac{-3x}{2})^2 + \dots) \text{ would also be awarded}$	M1B0A0A0	
	Method 2: Multiplying out : B1 for 32 and M1A1A1 for other terms with M1 x^2 term is correct. Completely correct is $4/4$	awarded if <i>x</i> or	

Question number	Scheme	Marks	
2	$2\log x = \log x^2$	B1	
	$\log_3 x^2 - \log_3 (x - 2) = \log_3 \frac{x^2}{x - 2}$	M1	
	$\frac{x^2}{x-2} = 9$	A1 o.e.	
	Solves $x^2 - 9x + 18 = 0$ to give $x =$	M1	
	x = 3, $x = 6$	A1	
		Total 5	
Notes	B1 for this correct use of power rule (may be implied)M1: for correct use of subtraction rule (or addition rule) for logs		
	N.B. $2\log_3 x - \log_3(x-2) = 2\log_3 \frac{x}{x-2}$ is M0		
	A1. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.)		
	M1 for attempting to solve $x^2 - 9x + 18 = 0$ to give $x = ($ see notes on marking quadratics) A1 for these two correct answers		
Alternative Method	$\log_3 x^2 = 2 + \log_3(x-2)$ is B1,		
	so $x^2 = 3^{2 + \log_3(x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1		
	Here M1 is for complete method i.e.correct use of powers after logs are used correctly		
Common Slips	$2\log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but the statement is M0 and so leads to no further marks		
	$2\log_3 x - \log_3(x-2) = 2$ so $\log_3 x - \log_3(x-2) = 1$ and $\log_3 \frac{x}{x-2} = 1$ can earn M1 for		
	correct subtraction rule following error, but no other marks		
Special Case	$\frac{\log x^2}{\log(x-2)} = 2$ leading to $\frac{x^2}{x-2} = 9$ and then to $x = 3$, $x = 6$, usually earns B1M0A0, but may		
	then earn M1A1 (special case) so 3/5 [This <i>recovery</i> after uncorrected error is very common]		
	Trial and error, Use of a table or just stating answer with both $x=3$ and $x=6$ sh B0M0A0 then final M1A1 i.e. $2/5$	ould be awarded	

Question number	Scheme	Marks
3	Obtain $(x \pm 10)^2$ and $(y \pm 8)^2$	M1
(a)	Obtain $(x-10)^2$ and $(y-8)^2$	A1
	Centre is (10, 8). N.B. This may be indicated on diagram only as (10, 8)	A1 (3)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25 (= r^2)$ or $(r^2 =)$ "100"+"64"-139	M1
	r = 5 * (this is a printed answer so need one of the above two reasons)	A1 (2)
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$	(2) M1
	e.g $x=13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y=$ or $13^2 + y^2 - 20 \times 13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$	A1, A1
(d)	y = 4 or 12 (on EPEN mark one correct value as A1A0 and both correct as A1A1) Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275)	(3) M1
	Perimeter $PTQ = 2r$ + their arc PQ (Finding perimeter of triangle is M0 here)	M1
	= 19.275 or 19.28 or 19.3	A1 (3)
		11 marks
Alternatives	<i>Method 2:</i> From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$	M1
(a)	Centre is $(-g, -f)$, and so centre is $(10, 8)$.	A1, A1
OR	<i>Method 3:</i> Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" and Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem). (10,8) is M1A1A1	M1 A1 A1 (3)
(b)	Method 2: Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)$ "100"+"64"-139 r = 5 *	M1 A1
OR	Method 3: Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2 + (12-8)^2}$ r = 5 *	M1 A1 cao (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate " $8 \pm h$ " - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	M1
Notes (a)	Mark (a) and (b) together M1 as in scheme and can be <u>implied</u> by $(\pm 10, \pm 8)$. Correct centre (10, 8) implies M1A	1A1
(b)	M1 for a correct method leading to $r =$, or $r^2 = "100" + "64" - 139$ (not 139 - "100" or for using equation of circle in $(x \pm 10)^2 + (y \pm 8)^2 = k^2$ form to identify $r =$	
	3^{rd} A1 $r = 5$ (NB This is a given answer so should follow $k^2 = 25$ or $r^2 = 100 + 64 - $ Special case: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow M1A1 for $r = 5$ work as $r^2 = 100 + 64 - 139$	
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and leading to perimeter of 32.14 for major sector	$\theta = 4.428$

Question number	Scheme	Marks
4 (a)	$f(-2) = 2.(-2)^{3} - 7.(-2)^{2} - 10.(-2) + 24$ = 0 so (x+2) is a factor	M1 A1 (2)
(b)	$f(x) = (x+2)(2x^{2}-11x+12)$ f(x) = (x+2)(2x-3)(x-4)	M1 A1 dM1 A1 (4)
		6 marks
Notes (a) (b)	M1 : Attempts $f(\pm 2)$ (Long division is M0) A1 : is for =0 and conclusion Note: Stating "hence factor" or "it is a factor" or a " $$ " (tick) or "QED" is fine for the conclusion. Note also that a conclusion can be implied from a <u>preamble</u> , eg: "If $f(-2) = 0$, $(x + 2)$ is a factor" (Not just $f(-2)=0$) 1 st M1: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen as could be done "by inspection." Or <i>Alternative Method</i> : 1 st M1: Use $(x+2)(ax^2 + bx + c) = 2x^3 - 7x^2 - 10x + 24$ with expansion and comparison of coefficients to obtain $a = 2$ and to obtain values for b and c 1 st A1: For seeing $(2x^2 - 11x + 12)$. [Can be seen here in (b) after work done in (a)] 2 nd M1: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and needs factors 2 nd A1: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.)	
	Note: Some candidates will go from $\{(x+2)\}(2x^2 - 11x + 12)$ to $\{x = -2\}, x = \frac{3}{2}, 4$, and not list all three factors. Award these responses M1A1M0A0. Finds $x = 4$ and $x = 1.5$ by factor theorem, formula or calculator and produces factors M1	
	f(x) = (x+2)(2x-3)(x-4) or $f(x) = 2(x+2)(x-1.5)(x-4)$ o.e. is full m f(x) = (x+2)(x-1.5)(x-4) loses last A1	

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Question number	Scl	heme	Marks
Method 1	Puts $10 - x = 10x - x^2 - 8$ and	Or puts $y = 10(10 - y) - (10 - y)^2 - 8$	M1
5 (a)	rearranges to give three term quadratic	and rearranges to give three term quadratic	2.61
	Solves their " $x^2 - 11x + 18 = 0$ " using acceptable method as in general principles	Solves their " $y^2 - 9y + 8 = 0$ " using	M1
	to give $x =$	acceptable method as in general principles to give $y =$	
	Obtains $x = 2$, $x = 9$ (may be on	Obtains $y = 8$, $y = 1$ (may be on diagram)	A1
	diagram or in part (b) in limits) Substitutes their <i>x</i> into a given equation	Substitutes their y into a given equation to	M1
	to give $y = (may be on diagram)$	give $x = (may be on diagram or in part (b))$	1011
	y = 8, y = 1	x = 2, x = 9	A1 (5)
(b)	$\int (10x - x^2 - 8) dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \left\{ + \frac{x^3}{3} - \frac{x^3}{$	2}	M1 A1
		-]	A1
	F (2) 3 7 ⁹		dM1
	$\left[\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x \right]_2^9 = (\dots) - (\dots)$		GIVII
	$=90 - \frac{4}{2} = 88\frac{2}{3}$ or $\frac{266}{3}$		
	5		
	Area of trapezium = $\frac{1}{2}(8+1)(9-2) = 31.5$		B1
	So area of <i>R</i> is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao
			(7)
			12 marks
Notes (a)	First M1: See scheme Second M1 : See	notes relating to solving quadratics	iiiai K5
	Third M1 : This may be awarded if one su		
		values, or from Graphical calculator are 5/5 to working or from table is M0M0A0M1A	0
(b)	M1 : $x^n \to x^{n+1}$ for any one term.		
	1^{st} A1: at least two out of three terms correct		
		part(a)) into an "integrated function" and sub	tracts,
	either way round		
	(NB: If candidate changes all signs to get $\int (-10x + x^2 + 8) dx = -\frac{10x^2}{2} + \frac{x^3}{3} + 8x \{+c\}$ This is M1 A1 A1		
	Then uses limits dM1 and trapezium is B1		
	Needs to <i>change sign of value obtained</i> from integration for final M1A1 so $-88\frac{2}{3} - 31.5$ is M0A0) B1 : Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus		
	triangle $\frac{1}{2} \times 8 \times 8 - \frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6]		
	M1: Their Area under curve – Their Area under line (if integrate both need same limits)		
	A1: Accept 57.16recurring but not 57.16 PTO for Alternative method		

Method 2			
for (b)	Area of R		
	$= \int_{2}^{9} (10x - x^{2} - 8) - (10 - x) \mathrm{d}x$	3 rd M1 (in (b)): Uses difference between two functions in integral.	
	$\int_{2}^{9} -x^{2} + 11x - 18 \mathrm{d}x$	M: $x^n \to x^{n+1}$ for any one term.	M1
	• 2	A1 at least two out of these three simplified terms	A1
	$= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+c\}$	Correct integration. (Ignore $+ c$).	A1
	$\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (\dots) - (\dots)$	Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	This mark is implied by final answer wh	ich rounds to 57.2	B1
	See above working(allow bracketing err mark for (b) here:		M1
	$40.5 - (-16\frac{2}{3})$	$=57\frac{1}{6}$ cao	A1
			(7)
Special case of above method	$\int_{2}^{9} x^{2} - 11x + 18 dx = \frac{x^{3}}{3} - \frac{11x^{2}}{2} + 18x$	{+ <i>c</i> }	M1A1A1
method	$\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (\dots) - (\dots)$		DM1
	This mark is implied by final answer	which rounds to 57.2 (not -57.2)	B1
	Difference of functions implied (see	above expression)	M1
	$40.5 - (-16\frac{2}{3})$	$=57\frac{1}{6}$ cao	A1
			(7)
Special	Integrates expression in y e.g. " y^2 –	9y+8=0": This can have first	
	Case 2 M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having u	sed Method 2 loses last two marks	
	Common Error:		
	Integrates $-x^2 + 9x - 18$ is likely to b	e M1A1A0dM1B0M1A0	
	Integrates $2-11x - x^2$ is likely to e N	I1A0A0dM1B0M1A0	
	Writing $\int_{2}^{9} (10x - x^2 - 8) - (10 - x) dx$	only earns final M mark	
L			

Question number	Scheme		Marks
6(a)	States or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$		M1
	$\frac{\sin 2x}{\cos 2x} = 5\sin 2x \Longrightarrow \sin 2x - 5\sin 2x \cos 2x = 0 \Longrightarrow \sin 2x \sin 2x \cos 2x = 0$	$n 2x(1-5\cos 2x) = 0 *$	A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
	$\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) or 2	2x = 281.54 (or 281.6)	M1
	<i>x</i> = 39.2 (or 39.3), 140.8 (or 141)		A1, A1 (5)
Notes	·		7 marks
	(a) M1: Statement that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or Replacement of statement but may involve θ instead of 2x. A1: the answer is given so all steps should be given. N.B. $\sin 2x - 5 \sin 2x \cos 2x = 0$ or $-5 \sin 2x \cos 2x + $		
	must be seen and be followed by printed answer for A1 mark sin $2x = 5 \sin 2x \cos 2x$ is not sufficient. (b) Statement of 0 and 180 with no working gets B1 B0 (M1 : This mark for one of the two statements given (must A1, A1 : first A1 for 39.2, second for 140.8 <i>Special case</i> solving $\cos 2x = -1/5$ giving $2x = 101.5$ of 140.8 omitted would give M1A1A0 Allow answers which round to 39.2 or 39.3 and which ro Answers in radians lose last A1 awarded (These are 0, 0.) Excess answers in range lose last A1 Ignore excess answ All 5 correct answers with no extras and no working gets the method here	bod) as it is two solutions t relate to $2x$ not just to x) or 258.5 is awarded M1A0A0 ound to 140.8 and allow 141 68, 1.57, 2.46 and 3.14) vers outside range.	

Question number	Scheme	Marks	
7 (a)	x 0 0.25 0.5 0.75 1 y 1 1.251 1.494 1.741 2	B1, B1 (2)	
(b)	$\frac{1}{2} \times 0.25$, $\{(1+2)+2(1.251+1.494+1.741)\}$ o.e.	B1, M1,A1 ft	
	=1.4965	A1 (4) 6 marks	
Notes	(a) first B1 for 1.494 and second B1 for 1.741 (1.740 is B 0) Wrong accuracy e.g. 1.49, 1.74 is B1B0	U Mar RS	
	 (b) B1: Need ½ of 0.25 or 0.125 o.e. M1: requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however) x values: M0 if values used in brackets are x values instead of y values 		
	A 1ft follows their answers to part (a) and is for {correct expression} Final A1: Accept 1.4965, 1.497. or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table) Separate trapezia may be used : B1 for 0.125, M1 for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and A1 ft if it is all correct) e.g 0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2) is M1 A0 equivalent to missing one term in { } in main scheme		
	Special Case: Bracketing mistake: i.e. 0.125(1+2) +2(1.251+1.494+1.741) scores B1 M1 A0 A0 for 9.347 If the final answer implies that the calculation has been done correctly i.e. 1.4965 (then full marks can be given). Need to see trapezium rule – answer only (with no working) is 0/4 any doubts send to review		
	Special Case; Uses 1.740 to give 1.49625 or 1.4963 or 1.496 or 1.50 gets, B1 B0 B1M1A1ft then A1 (lose 1 mark)		
	NB Bracket is 11.972		

Question			
number	Scheme	Marks	
8 (a)	$(h=)\frac{60}{\pi x^2}$ or equivalent exact (not decimal) expression e.g. $(h=)60 \div \pi x^2$	B1	(1)
(b)	$(A =)2\pi x^2 + 2\pi xh$ or $(A =)2\pi r^2 + 2\pi rh$ or $(A =)2\pi r^2 + \pi dh$ may not be simplified and may appear on separate lines	B1	
	Either $(A) = 2\pi x^2 + 2\pi x \left(\frac{60}{\pi x^2}\right)$ or As $\pi x h = \frac{60}{x}$ then $(A =)2\pi x^2 + 2\left(\frac{60}{x}\right)$	M1	
	$A = 2\pi x^2 + \left(\frac{120}{x}\right) \qquad \bigstar$	A1 cso	(3)
(c)	$\left(\frac{dA}{dx}\right) = 4\pi x - \frac{120}{x^2}$ or $= 4\pi x - 120x^{-2}$	M1 A1	
	$4\pi x - \frac{120}{x^2} = 0$ implies $x^3 =$ (Use of > 0 or < 0 is M0 then M0A0)	M1	
	$x = \sqrt[3]{\frac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1	(5)
(d)	$A = 2\pi (2.12)^2 + \frac{120}{2.12}, = 85 \qquad (\text{only ft } x = 2 \text{ or } 2.1 - \text{both give } 85)$	M1, A1	(3)
(e)	Either $\frac{d^2 A}{dx^2} = 4\pi + \frac{240}{x^3}$ and sign Or (<i>method 2</i>) considers gradient to left and right of their 2.12 (e.g at 2 and 2.5)	M1	
	considered (May appear in (c)) Or (method 3) considers value of A either side		
	Finds numerical values for gradients and observeswhich is > 0 and therefore minimumgradients go from negative to zero to positive so(most substitute 2.12 but it is not essentialconcludes minimumto see a substitution) (may appear in (c)) OR finds numerical values of A , observing	A1 13 mar	(2)
	greater than minimum value and draws conclusion	15 mai	KS
Notes	(a) B1 : This expression must be correct and in part (a) $\frac{60}{\pi r^2}$ is B0 (b) B1 : Accept any equivalent correct form – may be on two or more lines. M1 : substitute their expression for <i>h</i> in terms of <i>x</i> into Area formula of the form $kx^2 + cxh$ A1 : There should have been no errors in part (b) in obtaining this printed answer (c) M1 : At least one power of <i>x</i> decreased by 1 A1 accept any equivalent correct answer M1 : Setting $\frac{dA}{dx} = 0$ and finding a value for x^3 ($x^3 =$ may be implied by answer). Allow $\frac{dy}{dx} = 0$ dM1 : Using cube root to find <i>x</i> A1 : For any equivalent correct answer (need 3sf or more) Correct answer implies previous M n (d) M1 : Substitute the (+ve) <i>x</i> value found in (c) into equation for <i>A</i> and evaluate . A1 is for 8 (e) M1 : Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ attempted and sign considered A1 : Clear statements and conclusion. (numerical substitution of <i>x</i> is not necessary in first metho or calculation could be wrong but $A''(x)$ must be correct . Must not see 85 substituted)	nark 5 only) must be	d <i>x</i>

Question	Sche	Scheme	
9 (a)	$(S_n =) a + ar + (ar^2) + + ar^{n-1}$ and $rS_n = ar + ar^2 + (ar^3) + ar^n$		M1
	$S_n - rS_n = a - ar^n$		M1
	$S_n(1-r) = a(1-r^n)$		dM1
	And so result $S_n = \frac{a(1-r^n)}{(1-r)}$ *		A1 (4)
(b)	Divides one term by other (either way) to give $r^2 =$ then square roots to give $r =$	Or: (<i>Method 2</i>) Finds geometric mean i.e 3.24 and divides one term by 3.24 or 3.24 by one term	M1
	$r^2 = \frac{1.944}{5.4}$, $r = 0.6$ (ignore – 0.6)	r = 0.6 (ignore – 0.6)	A1 (2)
(c)	Uses $5.4 \div r^2$ or $1.944 \div r^4$, to give $a = a = 15$	=	M1, A1ft (2)
(d)	Uses $S = \frac{15}{1 - 0.6}$, to obtain 37.5		M1A1 ,A1 (3)
			11 marks
Notes	(a) M1: Lists both of these sums ($S_n =$) may b	e omitted, $r S_n$ (or rS) must be stated	<u> </u>
Special Case	 (a) M1: Lists both of these sums (S_n=) may be omitted, rS_n (or rS) must be stated 1st two terms must be correct in each series. Last term must be arⁿ⁻¹ or arⁿ in first series and the corresponding arⁿ or arⁿ⁺¹ in second series. Must be n and not a number. Reference made to other terms e.g. space or dots to indicate missing terms M1: Subtracts series for rS from series for S (or other way round) to give RHS = ±(a - arⁿ). This may have been obtained by following a pattern. If wrong power stated on line 1 M0 here. (Ignore LHS)M0M0M0A0 dM1: Factorises both sides correctly- must follow from a previous M1 (It is possible to obtain M0M1M1A0 or M1M0M1A0) A1: completes the proof with no errors seen No errors seen: First line absolutely correct, omission of second line, third and fourth lines correct: M1M0M1A1 See next sheet of common errors. Refer any attempts involving sigma notation, or any proofs by induction to team leader. Also attempts which begin with the answer and work backwards. (b) M1: Deduces r² by dividing either term by other and attempts square root A1: any correct equivalent for r e.g. 3/5 Answer only is 2/2 (<i>Method</i> 2) Those who find fourth term must use √ab and not ½(a + b) then must use it in a division with given term to obtain r = (c) M1: May be done in two steps or more e.g. 5.4 + r then divided by r again A1ft: follow through their value of r. Just a = 15 with no wrong working implies M1A1 (d) M1: States sum to infinity formula with values of a and r found earlier, provided r <1 		
	A1 : uses 15 and 0.6 (or 3/5) (This is not a ft ma		
Common errors	(i) Fraction inverted in (b) $r^2 = \frac{5.4}{1.944}$ and $r = 1\frac{2}{3}$, then correct ft gives M1A0 M1 A1ft M0A0A0 i.e. 3/7 (ii) Uses $r = 0.36$: (b)M0A0 (c)M1A1ft (d) M1A0A0 i.e. 3/7 (iii) Uses $ar^3 = 5.4$, $ar^5 = 1.944$ Likely to have (b)M1A1 (c)M0A0 (d) M1A0A0 i.e. 3/7		