## Core Mathematics C2

## Advanced Subsidiary

## Friday 13 January 2012 - Morning

## Time: 1 hour 30 minutes

Materials required for examination<br>Mathematical Formulae (Pink)

Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 9 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A geometric series has first term $a=360$ and common ratio $r=\frac{7}{8}$.

Giving your answers to 3 significant figures where appropriate, find
(a) the 20th term of the series,
(b) the sum of the first 20 terms of the series,
(c) the sum to infinity of the series.
(2)
2. A circle $C$ has centre $(-1,7)$ and passes through the point $(0,0)$. Find an equation for $C$.
3. (a) Find the first 4 terms of the binomial expansion, in ascending powers of $x$, of

$$
\left(1+\frac{x}{4}\right)^{8}
$$

giving each term in its simplest form.
(b) Use your expansion to estimate the value of $(1.025)^{8}$, giving your answer to 4 decimal places.
4. Given that $y=3 x^{2}$,
(a) show that $\log _{3} y=1+2 \log _{3} x$.
(b) Hence, or otherwise, solve the equation

$$
\begin{equation*}
1+2 \log _{3} x=\log _{3}(28 x-9) \tag{3}
\end{equation*}
$$

5. 

$$
\mathrm{f}(x)=x^{3}+a x^{2}+b x+3, \text { where } a \text { and } b \text { are constants. }
$$

Given that when $\mathrm{f}(x)$ is divided by $(x+2)$ the remainder is 7 ,
(a) show that $2 a-b=6$.

Given also that when $\mathrm{f}(x)$ is divided by $(x-1)$ the remainder is 4 ,
(b) find the value of $a$ and the value of $b$.
6.


Figure 1
Figure 1 shows the graph of the curve with equation

$$
y=\frac{16}{x^{2}}-\frac{x}{2}+1, \quad x>0 .
$$

The finite region $R$, bounded by the lines $x=1$, the $x$-axis and the curve, is shown shaded in Figure 1. The curve crosses the $x$-axis at the point $(4,0)$.
(a) Complete the table with the values of $y$ corresponding to $x=2$ and 2.5 .

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| $y$ | 16.5 | 7.361 |  |  | 1.278 | 0.556 | 0 |

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of $R$, giving your answer to 2 decimal places.
(c) Use integration to find the exact value for the area of $R$.
7.


Figure 2
Figure 2 shows $A B C$, a sector of a circle of radius 6 cm with centre $A$. Given that the size of angle $B A C$ is 0.95 radians, find
(a) the length of the arc $B C$,
(b) the area of the sector $A B C$.

The point $D$ lies on the line $A C$ and is such that $A D=B D$. The region $R$, shown shaded in Figure 2, is bounded by the lines $C D, D B$ and the arc $B C$.
(c) Show that the length of $A D$ is 5.16 cm to 3 significant figures.

Find
(d) the perimeter of $R$,
(e) the area of $R$, giving your answer to 2 significant figures.
8.


Figure 3
Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius $x$ metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to $x$ metres and width equal to $y$ metres.

Given that the area of the flowerbed is $4 \mathrm{~m}^{2}$,
(a) show that

$$
y=\frac{16-\pi x^{2}}{8 x}
$$

(b) Hence show that the perimeter $P$ metres of the flowerbed is given by the equation

$$
P=\frac{8}{x}+2 x .
$$

(c) Use calculus to find the minimum value of $P$.
(d) Find the width of each rectangle when the perimeter is a minimum.

Give your answer to the nearest centimetre.
9. (i) Find the solutions of the equation $\sin \left(3 x-15^{\circ}\right)=\frac{1}{2}$, for which $0 \leq x \leq 180^{\circ}$.
(ii)


Figure 4
Figure 4 shows part of the curve with equation

$$
y=\sin (a x-b), \text { where } a>0, \quad 0<b<\pi .
$$

The curve cuts the $x$-axis at the points $P, Q$ and $R$ as shown.
Given that the coordinates of $P, Q$ and $R$ are $\left(\frac{\pi}{10}, 0\right),\left(\frac{3 \pi}{5}, 0\right)$ and $\left(\frac{11 \pi}{10}, 0\right)$ respectively, find the values of $a$ and $b$.


| Question <br> number | Scheme | Marks |
| :---: | :--- | :--- |
| $\mathbf{2}$ | The equation of the circle is $(x+1)^{2}+(y-7)^{2}=\left(r^{2}\right)$ | M1 A1 |
|  | The radius of the circle is $\sqrt{(-1)^{2}+7^{2}}=\sqrt{50}$ or $5 \sqrt{2}$ or $r^{2}=50$ <br> So $(x+1)^{2}+(y-7)^{2}=50$ or equivalent | A1 |
| Notes | M1 is for this expression on left hand side- allow errors in sign of 1 and 7. <br> A1 correct signs (just LHS $)$ <br> M1 is for Pythagoras or substitution into equation of circle to give $r$ or $r^{2}$ <br> Giving this value as diameter is M0 <br> A1, cao for cartesian equation with numerical values but allow $(\sqrt{ } 50)^{2}$ or $(5 \sqrt{2})^{2}$ or any exact <br> equivalent <br> A correct answer implies a correct method - so answer given with no working earns all four <br> marks for this question. |  |
| Alternative <br> method | Equation of circle is $x^{2}+y^{2} \pm 2 x \pm 14 y+c=0$ <br> Equation of circle is $x^{2}+y^{2}+2 x-14 y+c=0$ <br> Uses $(0,0)$ to give $c=0$, or finds $r=\sqrt{(-1)^{2}+7^{2}}=\sqrt{50}$ or $5 \sqrt{2}$ or $r^{2}=50$ <br> So $x^{2}+y^{2}+2 x-14 y=0$ or equivalent | M1 |

\begin{tabular}{|c|c|}
\hline Question number \& Scheme Marks <br>
\hline 3 (a).

(b) \&  <br>

\hline Alternative for (b) Special case \& Starts again and expands $(1+0.025)^{8}$ to | $1+8 \times 0.025+\frac{8 \times 7}{2}(0.025)^{2}+\frac{8 \times 7 \times 6}{2 \times 3}(0.025)^{3},=1.2184$ |
| :--- | :--- |
| $($ Or $1+1 / 5+7 / 400+7 / 8000=1.2184)$ |$\quad$ B1,M1,A1 <br>


\hline Notes \& | (a) B1 must be simplified |
| :--- |
| The method mark (M1) is awarded for an attempt at Binomial to get the third and/or fourth term - need correct binomial coefficient combined with correct power of $x$. Ignore bracket errors or errors in powers of 4 . Accept any notation for ${ }^{8} C_{2}$ and ${ }^{8} C_{3}$, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs) |
| First A1 is for two completely correct unsimplified terms |
| A1 needs the fully simplified $\frac{7}{4} x^{2}$ and $\frac{7}{8} x^{3}$. |
| (b) B1 - states or uses $x=0.1$ or $\frac{x}{4}=\frac{1}{40}$ |
| M1 for substituting their value of $x(0<\mathrm{x}<1)$ into expansion (e.g. 0.1 (correct) or $0.01,0.00625$ or even 0.025 but not 1 nor 1.025 which would earn M0) A1 Should be answer printed cao (not answers which round to) and should follow correct work. Answer with no working at all is B0, M0, A0 |
| States 0.1 then just writes down answer is B1 M0A0 | <br>

\hline
\end{tabular}

\begin{tabular}{|c|c|c|}
\hline Question number \& Scheme \& Marks \\
\hline 4. (a) \& \begin{tabular}{l}
\(\log _{3} 3 x^{2}=\log _{3} 3+\log _{3} x^{2}\) or \(\log y-\log x^{2}=\log 3\) or \(\log y-\log 3=\log x^{2}\) \(\log _{3} x^{2}=2 \log _{3} x\) \\
Using \(\log _{3} 3=1\)
\[
3 x^{2}=28 x-9
\] \\
Solves \(3 x^{2}-28 x+9=0 \quad\) to give \(x=\frac{1}{3}\) or \(x=9\)
\end{tabular} \& \begin{tabular}{l}
B1 \\
B1 \\
B1 \\
(3) \\
M1 \\
M1 A1 \\
(3) \\
6
\end{tabular} \\
\hline Notes (a)

(b) \& \begin{tabular}{l}
B1 for correct use of addition rule (or correct use of subtraction rule) <br>
B1: replacing $\log x^{2}$ by $2 \log x \quad-\operatorname{not} \log 3 x^{2}$ by $2 \log 3 x$ this is $\mathbf{B 0}$ <br>
B1. for replacing $\log 3$ by $1 \quad$ (or use of $3^{1}=3$ ) <br>
If candidate has been awarded 3 marks and their proof includes an error or omis to $\log y$ withhold the last mark. <br>
So just B1 B1 B0 <br>
These marks must be awarded for work in part (a) only <br>
M1 for removing logs to get an equation in $x$-statement in scheme is sufficient. accurate without any errors seen in part (b). <br>
M1 for attempting to solve three term quadratic to give $x=$ (see notes on $m$ quadratics) <br>
A1 for the two correct answers - this depends on second M mark only. <br>
Candidates often begin again in part (b) and do not use part (a). <br>
If such candidates make errors in log work in part (b) they score first M0. The seco <br>
A are earned as before. It is possible to get M0M1A1 or M0M1A0.

 \& 

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\end{tabular} <br>

\hline Alternative to (b) using $y$ \& Eliminates $x$ to give $3 y^{2}-730 y+243=0$ with no errors is M1 Solves quadratic to find $y$, then uses values to find $x$ M1 A1 as before \& <br>
\hline
\end{tabular}






| Question number | Scheme | Marks |
| :---: | :---: | :---: |
| 9 (i) | $\sin (3 x-15)=\frac{1}{2}$ so $3 x-15=30 \quad(\alpha)$ and $x=15$ <br> Need $3 x-15=180-\alpha$ or $3 x-15=540-\alpha$ <br> Need $3 x-15=180-\alpha$ and $3 x-15=360+\alpha$ and $3 x-15=540-\alpha$ $x=55$ or 175 $x=55,135,175$ | M1 A1  <br> M1  <br> M1  <br> A1  <br> A1  <br>   <br>   |
| Notes | M1 Correct order of operation: inverse sine then linear algebra - not just $3 x-15=30$ (slips in linear algebra lose Accuracy mark) <br> A1 Obtains first solution 15 <br> M1 Uses either $180-\alpha$ or $540-\alpha$, <br> M1 uses all three $180-\alpha$ and $360+\alpha$ and $540-\alpha$ <br> A1, for one further correct solution 55 or 175, (depends only on second M1) <br> A1 - all 3 further correct solutions <br> If more than 4 solutions in range, lose last A1 <br> Common slips: Just obtains 15 and 55, or 15 and 175 - usually M1A1M1M0A1A0 <br> Just obtains 15 and 135 is usually M1A1M0M0A0A0 (It is easy to get this erroneously) <br> Obtains 5, 45, 125 and 165 - usually M1A0M1M1A0A0 <br> Obtains 25, 65, 145, (185) usually M1A0M1M1A0A0 <br> Working in radians - lose last A1 earned for $\frac{\pi}{12}, \frac{11 \pi}{36}, \frac{3 \pi}{4}$ and $\frac{35 \pi}{36}$ or numerical equivalents <br> Mixed radians and degrees is usually Method marks only <br> Methods involving no working should be sent to Review |  |
| 9 (ii) | At least one of $\begin{array}{lll} \text { of } & \left(\frac{a \pi}{10}-b\right)=0(\text { or } n \pi) & \\ & \left(\frac{a 3 \pi}{5}-b\right)=\pi & \{\text { or }(n+1) \pi\} \\ \text { or } & \left(\frac{a 11 \pi}{10}-b\right)=2 \pi & \{\text { or }(n+2) \pi\} \end{array}$ $\left(\frac{a 3 \pi}{5}-b\right)=\pi \quad\{\text { or } \quad(n+1) \pi\} \quad \text { or in degrees }$ <br> If two of above equations used eliminates $a$ or $b$ to find one or both of these or uses period property of curve to find $a$ or uses other valid method to find either $a$ or $b \quad$ (May see $\frac{5 \pi}{10} a=\pi$ so $a=$ ) Obtains $a=2$ <br> Obtains $b=\frac{\pi}{5}$ (must be in radians) | M1 <br> M1 <br> A1 <br> A1 |


| Notes | M1: Award for $\left(\frac{a \pi}{10}-b\right)=0$ or $\frac{a \pi}{10}=b$ BUT $\sin \left(\frac{a \pi}{10}-b\right)=0$ is M0 |
| :--- | :--- |
|  | M1: As described above but solving $\left(\frac{a \pi}{10}-b\right)=0 \quad$ with $\left(\frac{a 3 \pi}{5}-b\right)=0$ is M0 (It gives $\left.a=b=0\right)$ |
|  | Special cases: <br> Can obtain full marks here for both correct answers with no working M1M1A1A1 <br> For $a=2$ only, with no working, award M0M1A1A0 $\quad$ For $b=\frac{\pi}{5}$ only with no working <br> M1M0A0A1 |
| Alternative | Some use translations and stretches to give answers. <br> If they achieve $a=2$ they earn second method and first accuracy. If they achieve correct value for $b$ <br> they earn first method and second accuracy. <br> Common error is $a=2$ and $b=\frac{\pi}{10}$. This is usually M0M1A1A0 unless they have stated <br>  <br> $\left(\frac{a \pi}{10}-b\right)=0 \quad$ earlier in which case they earn first M1. |

