Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Monday 10 January 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35403A

1. $$f(x) = x^4 + x^3 + 2x^2 + ax + b,$$

where a and b are constants.

When f(x) is divided by (x – 1), the remainder is 7.

(a) Show that $a + b = 3$.

(2)

When f(x) is divided by (x + 2), the remainder is –8.

(b) Find the value of a and the value of b.

(5)

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2. In the triangle ABC, AB = 11 cm, BC = 7 cm and CA = 8 cm.

(a) Find the size of angle C, giving your answer in radians to 3 significant figures.

(3)

(b) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures.

(3)

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3. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(a) the common ratio of the series,

(3)

(b) the first term of the series,

(2)

(c) the sum to infinity of the series.

(2)

4.

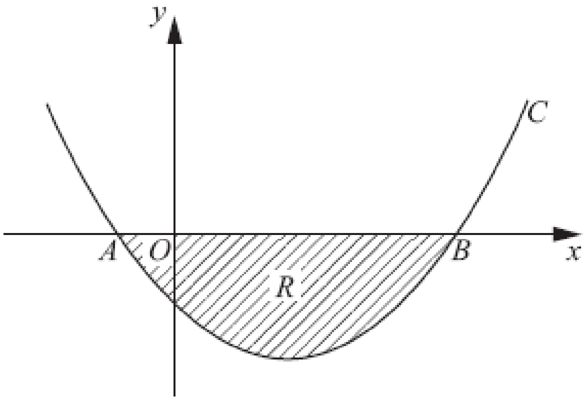

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5).$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

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5. Given that $\binom{40}{4} = \dfrac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\dfrac{q}{p}$.

(3)

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6. $$y = \frac{5}{3x^2 - 2}$$

(*a*) Copy and complete the table below, giving the values of *y* to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(*b*) Use the trapezium rule, with all the values of *y* from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

(4)

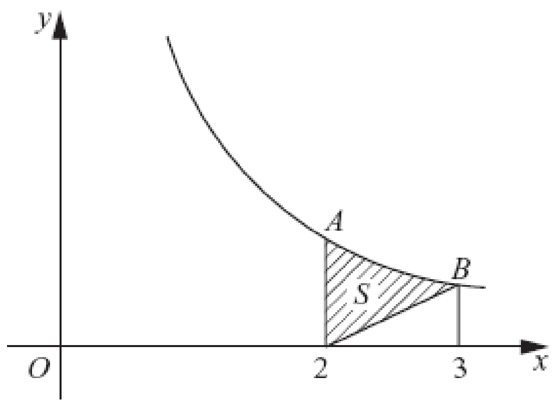

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points *A* and *B* on the curve, $x = 2$ and $x = 3$ respectively.

The region *S* is bounded by the curve, the straight line through *B* and (2, 0), and the line through *A* parallel to the *y*-axis. The region *S* is shown shaded in Figure 2.

(*c*) Use your answer to part (*b*) to find an approximate value for the area of *S*.

(3)

7. (*a*) Show that the equation

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

can be written in the form

$$4 \sin^2 x + 7 \sin x + 3 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$,

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

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8. (*a*) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

(2)

(*b*) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0,$$

giving your answers to 2 decimal places where appropriate.

(6)

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9. The points *A* and *B* have coordinates (–2, 11) and (8, 1) respectively.

Given that *AB* is a diameter of the circle *C*,

(*a*) show that the centre of *C* has coordinates (3, 6),

(1)

(*b*) find an equation for *C*.

(4)

(*c*) Verify that the point (10, 7) lies on *C*.

(1)

(*d*) Find an equation of the tangent to *C* at the point (10, 7), giving your answer in the form $y = mx + c$, where *m* and *c* are constants.

(4)

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10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5.$$

(a) Find $\dfrac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

TOTAL FOR PAPER: 75 MARKS

END

January 2011
Core Mathematics C2 6664
Mark Scheme

Question Number	Scheme	Marks
1. **(a)**	$\mathrm{f}(x) = x^4 + x^3 + 2x^2 + ax + b$	
	Attempting $\mathrm{f}(1)$ or $\mathrm{f}(-1)$.	M1
	$\mathrm{f}(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$ (as required) **AG**	A1 * **cso**
		(2)
(b)	Attempting $\mathrm{f}(-2)$ or $\mathrm{f}(2)$.	M1
	$\mathrm{f}(-2) = \underline{16 - 8 + 8 - 2a + b = -8}$ $\{\Rightarrow -2a + b = -24\}$	A1
	Solving both equations simultaneously to get as far as $a = ...$ or $b = ...$	dM1
	Any one of $a = 9$ or $b = -6$	A1
	Both $a = 9$ and $b = -6$	A1 **cso**
		(5) **[7]**
	Notes	
(a)	M1 for attempting either $\mathrm{f}(1)$ or $\mathrm{f}(-1)$. A1 for applying $\mathrm{f}(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a).	
(b)	M1: attempting either $\mathrm{f}(-2)$ or $\mathrm{f}(2)$. A1: correct underlined equation in *a* and *b*; eg $\underline{16 - 8 + 8 - 2a + b = -8}$ or equivalent, eg $-2a + b = -24$. dM1: an attempt to eliminate one variable from 2 linear simultaneous equations in a and b. Note that this mark is dependent upon the award of the first method mark. A1: any one of $a = 9$ or $b = -6$. A1: both $a = 9$ and $b = -6$ and a correct solution only.	
	Alternative Method of Long Division: (a) M1 for long division by $(x - 1)$ to give a remainder in *a* and *b* which is independent of *x*. A1 for $\{\text{Remainder} =\}$ $b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given.) (b) M1 for long division by $(x + 2)$ to give a remainder in *a* and *b* which is independent of *x*. A1 for $\{\text{Remainder} =\}$ $\underline{b - 2(a - 8) = -8}$ $\{\Rightarrow -2a + b = -24\}$. Then dM1A1A1 are applied in the same way as before.	

Question Number	Scheme	Marks
2.		
(a)	$11^2 = 8^2 + 7^2 - (2\times 8\times 7\cos C)$	M1
	$\cos C = \dfrac{8^2 + 7^2 - 11^2}{2\times 8\times 7}$ (or equivalent)	A1
	$\{\hat{C} = 1.64228...\} \Rightarrow \hat{C} = \text{awrt } 1.64$	A1 **cso** (3)
(b)	Use of Area $\Delta ABC = \frac{1}{2}ab\sin(\text{their } C)$, where a, b are any of 7, 8 or 11.	M1
	$= \frac{1}{2}(7\times 8)\sin C$ using the value of their C from part (a).	A1 ft
	$\{= 27.92848... \text{ or } 27.93297...\} = \text{awrt } 27.9$ (from angle of either 1.64^c or 94.1°)	A1 **cso** (3) **[6]**
	Notes	
(a)	M1 is also scored for $8^2 = 7^2 + 11^2 - (2\times 7\times 11\cos C)$ or $7^2 = 8^2 + 11^2 - (2\times 8\times 11\cos C)$ or $\cos C = \dfrac{7^2 + 11^2 - 8^2}{2\times 7\times 11}$ or $\cos C = \dfrac{8^2 + 11^2 - 7^2}{2\times 8\times 11}$ 1st A1: Rearranged correctly to make $\cos C = ...$ and numerically correct (possibly unsimplified). Award A1 for any of $\cos C = \dfrac{8^2 + 7^2 - 11^2}{2\times 8\times 7}$ or $\cos C = \dfrac{-8}{112}$ or $\cos C = -\dfrac{1}{14}$ or $\cos C = \text{awrt } -0.071$. SC: Also allow 1st A1 for $112\cos C = -8$ or equivalent. Also note that the 1st A1 can be implied for $\hat{C} = \text{awrt } 1.64$ or $\hat{C} = \text{awrt } 94.1^\circ$. **Special Case:** $\cos C = \dfrac{1}{14}$ or $\cos C = \dfrac{11^2 - 8^2 - 7^2}{2\times 8\times 7}$ scores a SC: M1A0A0. 2nd A1: for awrt 1.64 **cao** **Note that** $A = 0.6876...^c$ (or $39.401...^\circ$), $B = 0.8116...^c$ (or $46.503...^\circ$)	
(b)	M1: alternative methods must be fully correct to score the M1. For any (or both) of the M1 or the 1st A1; their C can either be in degrees or radians. Candidates who use $\cos C = \dfrac{1}{14}$ to give $C = 1.499...$, can achieve the correct answer of awrt 27.9 in part (b). These candidates will score M1A1A0cso, in part (b). Finding $C = 1.499...$ in part (a) and achieving awrt 27.9 with no working scores M1A1A0. Otherwise with no working in part (b), awrt 27.9 scores M1A1A1. **Special Case: If the candidate gives awrt 27.9 from any of the below then award M1A1A1.** $\frac{1}{2}(7\times 11)\sin(0.8116^c \text{ or } 46.503^\circ) = \text{awrt } 27.9$, $\frac{1}{2}(8\times 11)\sin(0.6876...^c \text{ or } 39.401...^\circ) = \text{awrt } 27.9$. **<u>Alternative: Hero's Formula:</u>** $A = \sqrt{13(13-11)(13-8)(13-7)} = \text{awrt } 27.9$, where M1 is attempt to apply $A = \sqrt{s(s-11)(s-8)(s-7)}$ and the first A1 is for the correct application of the formula.	

Question Number	Scheme	Marks
3. **(a)**	$ar = 750$ and $ar^4 = -6$ (could be implied from later working in either (a) or (b)).	B1
	$r^3 = \frac{-6}{750}$	M1
	$r = -\frac{1}{5}$ Correct answer from no working, except for special case below gains all three marks.	A1
		(3)
(b)	$a(-0.2) = 750$	M1
	$a\left\{= \frac{750}{-0.2}\right\} = -3750$	A1 ft
		(2)
(c)	Applies $\frac{a}{1-r}$ correctly using both their *a* and their $\lvert r \rvert < 1$. Eg. $\frac{-3750}{1--0.2}$	M1
	So, $S_\infty = -3125$	A1
		(2) **[7]**
	Notes	
(a)	B1: for both $ar = 750$ and $ar^4 = -6$ (may be implied from later working in either (a) or (b)). M1: **for eliminating *a*** by either dividing $ar^4 = -6$ by $ar = 750$ or dividing $ar = 750$ by $ar^4 = -6$, to achieve an equation in r^3 or $\frac{1}{r^3}$ Note that $r^4 - r = -\frac{6}{750}$ is M0. Note also that any of $r^3 = \frac{-6}{750}$ or $r^3 = \frac{750}{-6}\{= -125\}$ or $\frac{1}{r^3} = \frac{-6}{750}$ or $\frac{1}{r^3} = \frac{750}{-6}\{= -125\}$ are fine for the award of M1. SC: $ar^\alpha = 750$ and $ar^\beta = -6$ leading to $r^\delta = \frac{-6}{750}$ or $r^\delta = \frac{750}{-6}\{= -125\}$ or $\frac{1}{r^\delta} = \frac{-6}{750}$ or $\frac{1}{r^\delta} = \frac{750}{-6}\{= -125\}$ where $\delta = \beta - \alpha$ and $\delta \ge 2$ are fine for the award of M1. SC: $ar^2 = 750$ and $ar^5 = -6$ leading to $r = -\frac{1}{5}$ scores B0M1A1.	
(b)	M1 for inserting their *r* into either of their original correct equations of either $ar = 750$ or $\{a =\}\frac{750}{r}$ or $ar^4 = -6$ or $\{a =\}\frac{-6}{r^4}$ – in ***both a* and *r***. No slips allowed here for M1. A1 for either $a = -3750$ or *a* equal to the correct follow through result expressed either as an exact integer, or a fraction in the form $\frac{c}{d}$ where both *c* and *d* are integers, or correct to awrt 1 dp.	
(c)	M1 for applying $\frac{a}{1-r}$ correctly (only a slip in substituting *r* is allowed) using both their *a* and their $\lvert r \rvert < 1$. Eg. $\frac{-3750}{1--0.2}$. A1 for -3125 In parts (a) or (b) or (c), the correct answer with no working scores full marks.	

Question Number	Scheme		Marks
4. (a)	Seeing -1 and 5. (See note below.)		B1 (1)
(b)	$(x+1)(x-5) = \underline{x^2 - 4x - 5}$ or $\underline{x^2 - 5x + x - 5}$		B1
	$\int (x^2 - 4x - 5)\,dx = \frac{x^3}{3} - \frac{4x^2}{2} - 5x \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms correctly ft.	M1A1ft A1
	$\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x\right]_{-1}^{5} = (\ldots\ldots) - (\ldots\ldots)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(\frac{125}{3} - \frac{100}{2} - 25\right) - \left(-\frac{1}{3} - 2 + 5\right) = \left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) = -36\right\}$		
	Hence, Area $= 36$	Final answer must be 36, not -36	A1 (6) **[7]**

	Notes
(a)	B1: for -1 and 5. Note that $(-1, 0)$ and $(5, 0)$ are acceptable for B1. Also allow $(0, -1)$ and $(0, 5)$ generously for B1. Note that if a candidate writes down that $A: (5,0)$, $B: (-1,0)$, (ie A and B interchanged,) then B0. Also allow values inserted in the correct position on the x-axis of the graph.
(b)	B1 for $x^2 - 4x - 5$ or $x^2 - 5x + x - 5$. If you believe that the candidate is applying the Way 2 method then $-x^2 + 4x + 5$ or $-x^2 + 5x - x + 5$ would then be fine for B1. 1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $-5 \to 5x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft from their multiplied out brackets. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. Allow 2nd A1 also for $\frac{x^3}{3} - \frac{5x^2}{2} + \frac{x^2}{2} - 5x$. Note that $-\frac{5x^2}{2} + \frac{x^2}{2}$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -1 (and not 1 if the candidate has stated $x = -1$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. 3rd A1: For a final answer of 36, not -36. **Note:** An alternative method exists where the candidate states from the outset that Area $(R) = -\int_{-1}^{5} (x^2 - 4x + 5)\,dx$ is detailed in the Appendix.

Question Number	Scheme	Marks
5.		
(a)	$\binom{40}{4} = \frac{40!}{4!b!}$; $(1+x)^n$ coefficients of x^4 and x^5 are p and q respectively. $b = 36$ Candidates should usually "identify" two terms as their p and q respectively.	B1 (1)
(b)	Term 1: $\binom{40}{4}$ or ${}^{40}C_4$ or $\frac{40!}{4!36!}$ or $\frac{40(39)(38)(37)}{4!}$ or 91390 Term 2: $\binom{40}{5}$ or ${}^{40}C_5$ or $\frac{40!}{5!35!}$ or $\frac{40(39)(38)(37)(36)}{5!}$ or 658008 Any one of Term 1 or Term 2 correct. (Ignore the label of p and/or q.) Both of them correct. (Ignore the label of p and/or q.) Hence, $\frac{q}{p} = \frac{658008}{91390}$ $\left\{= \frac{36}{5} = 7.2\right\}$ for $\frac{658008}{91390}$ oe	M1 A1 A1 oe **cso** (3) **[4]**
	<u>Notes</u>	
(a)	B1: for only $b = 36$.	
(b)	The candidate may expand out their binomial series. At this stage no marks should be awarded until they start to identify either one or both of the terms that they want to focus on. Once they identify their terms then if one out of two of them (ignoring which one is p and which one is q) is correct then award M1. If both of the terms are identified correctly (ignoring which one is p and which one is q) then award the first A1. Term $1 = \binom{40}{4}x^4$ or ${}^{40}C_4(x^4)$ or $\frac{40!}{4!36!}x^4$ or $\frac{40(39)(38)(37)}{4!}x^4$ or $91390x^4$, Term $2 = \binom{40}{5}x^5$ or ${}^{40}C_5(x^5)$ or $\frac{40!}{5!35!}x^5$ or $\frac{40(39)(38)(37)(36)}{5!}x^5$ or $658008x^5$ are fine for any (or both) of the first two marks in part (b). 2nd A1 for stating $\frac{q}{p}$ as $\frac{658008}{91390}$ or equivalent. Note that $\frac{q}{p}$ must be independent of x. Also note that $\frac{36}{5}$ or 7.2 or any equivalent fraction is fine for the 2nd A1 mark. SC: If candidate states $\frac{p}{q} = \frac{5}{36}$, then award M1A1A0. Note that either $\frac{4!36!}{5!35!}$ or $\frac{5!35!}{4!36!}$ would be awarded M1A1.	

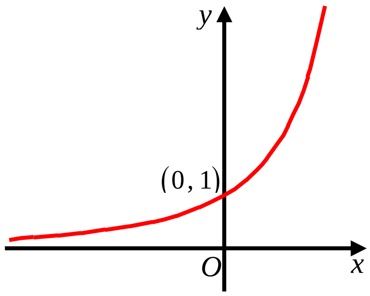
Question Number	Scheme		Marks
6. **(a)**	<table><tr><td>x</td><td>2</td><td>2.25</td><td>2.5</td><td>2.75</td><td>3</td></tr><tr><td>y</td><td>0.5</td><td>0.38</td><td>0.298507…</td><td>0.241691…</td><td>0.2</td></tr></table>		
	At $\{x = 2.5,\}$ $y = 0.30$ (only)	At least one y-ordinate correct.	B1
	At $\{x = 2.75,\}$ $y = 0.24$ (only)	Both y-ordinates correct.	B1
			(2)
(b)		Outside brackets $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$	B1 aef
	$\frac{1}{2}\times 0.25\,;\times\underline{\{0.5+0.2+2(0.38+\text{ their }0.30+\text{ their }0.24)\}}$	<u>For structure of</u> $\{\ldots\ldots\ldots\ldots\}$;	M1
		Correct expression <u>inside brackets</u> which all must be multiplied by their “outside constant”.	<u>A1</u> $\sqrt{\ }$
	$\{= \frac{1}{8}(2.54)\} = \text{awrt } 0.32$	awrt 0.32	A1
			(4)
(c)	Area of triangle $= \frac{1}{2}\times 1\times 0.2 = 0.1$		B1
	$\text{Area}(S) = \text{"}0.3175\text{"} - 0.1$		M1
	$= 0.2175$		A1 ft
			(3)
			[9]

Question Number	Scheme	Marks
	Notes	
(b)	B1 for using $\frac{1}{2}\times 0.25$ or $\frac{1}{8}$ or equivalent. M1 requires the correct $\{......\}$ bracket structure. This is for the first bracket to contain first *y*-ordinate plus last *y*-ordinate and the second bracket to be the summation of the remaining y-ordinates in the table. No errors (eg. an omission of a *y*-ordinate or an extra *y*-ordinate or a repeated *y*-ordinate) are allowed in the second bracket and the second bracket must be multiplied by 2. Only one copying error is allowed here in the $2(0.38+\text{ their }0.30+\text{ their }0.24)$ bracket. A1ft for the correct bracket $\{......\}$ following through candidate's *y*-ordinates found in part (a). A1 for answer of awrt 0.32 . <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly** then award M1A0A0 for either $\frac{1}{2}\times 0.25\times 0.5+2(0.38+\text{ their }0.30+\text{ their }0.24)+0.2$ (nb: yielding final answer of 2.1025) so that the 0.5 is only multiplied by $\frac{1}{2}\times 0.25$ or $\frac{1}{2}\times 0.25\times(0.5+0.2)+2(0.38+\text{ their }0.30+\text{ their }0.24)$ (nb: yielding final answer of 1.9275) so that the $(0.5+0.2)$ is multiplied by $\frac{1}{2}\times 0.25$. **Need to see trapezium rule – answer only (with no working) gains no marks.** **<u>Alternative:</u>** Separate trapezia may be used, and this can be marked equivalently. (See appendix.)	
(c)	B1 for the area of the triangle identified as either $\frac{1}{2}\times 1\times 0.2$ or 0.1. May be identified on the diagram. M1 for "part (b) answer" – "0.1 only" or "part (b) answer – their attempt at 0.1 only". (Strict attempt!) A1ft for correctly following through "part (b) answer" – 0.1. This is also dependent on the answer to (b) being greater than 0.1. Note: candidates may round answers here, so allow A1ft if they round their answer correct to 2 dp.	

Question Number	Scheme		Marks
7.			
(a)	$3\sin^2 x + 7\sin x = \cos^2 x - 4$; $0 \le x < 360^\circ$		
	$3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$		M1
	$4\sin^2 x + 7\sin x + 3 = 0$ **AG**		A1 * **cso**
			(2)
(b)	$(4\sin x + 3)(\sin x + 1)\ \{= 0\}$	Valid attempt at factorisation and $\sin x = ...$	M1
	$\sin x = -\frac{3}{4}$, $\sin x = -1$	Both $\sin x = -\frac{3}{4}$ and $\sin x = -1$.	A1
	$(\lvert\alpha\rvert = 48.59...)$		
	$x = 180 + 48.59$ or $x = 360 - 48.59$	Either $(180 + \lvert\alpha\rvert)$ or $(360 - \lvert\alpha\rvert)$	dM1
	$x = 228.59...$, $x = 311.41...$	Both awrt 228.6 and awrt 311.4	A1
	$\{\sin x = -1\} \Rightarrow x = 270$	270	B1
			(5)
			[7]

<u>**Notes**</u>

(a)	M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$). Note that applying $\cos^2 x = \sin^2 x - 1$, scores M0. A1 for obtaining the printed answer without error (except for implied use of zero.), although the equation at the end of the proof **must be = 0.** Solution **just** written only as above would score M1A1.
(b)	1st M1 for a valid attempt at factorisation, can use any variable here, *s*, *y*, *x* or $\sin x$, and an attempt to find at least one of the solutions. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 1st A1 for the two correct values of $\sin x$. If they have used a substitution, a correct value of their *s* or their *y* or their *x*. 2nd M1 for solving $\sin x = -k$, $0 < k < 1$ and realising a solution is either of the form $(180 + \lvert\alpha\rvert)$ or $(360 - \lvert\alpha\rvert)$ where $\alpha = \sin^{-1}(k)$. Note that you **cannot** access this mark from $\sin x = -1 \Rightarrow x = 270$. Note that this mark is dependent upon the 1st M1 mark awarded. 2nd A1 for both awrt 228.6 and awrt 311.4 B1 for 270. If there are any EXTRA solutions inside the range $0 \le x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 360^\circ$. **Working in Radians:** Note the answers in radians are $x = 3.9896...,\ 5.4351...,\ 4.7123...$ If a candidate works in radians then mark part (b) as above awarding the 2nd A1 for both awrt 4.0 and awrt 5.4 and the B1 for awrt 4.7 or $\frac{3\pi}{2}$. If the candidate would then score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question.) **No working:** Award B1 for 270 seen without any working. Award M0A0M1A1 for awrt 228.6 and awrt 311.4 seen without any working. Award M0A0M1A0 for any one of awrt 228.6 or awrt 311.4 seen without any working.

Question Number	Scheme		Marks
8.			
(a)	Graph of $y = 7^x,\ x \in \mathbb{R}$ and solving $7^{2x} - 4(7^x) + 3 = 0$		
		At least two of the three criteria correct. **(See notes below.)**	B1
		All three criteria correct. **(See notes below.)**	B1
			(2)
(b)	$y^2 - 4y + 3\ \{= 0\}$	Forming a quadratic {using $"y" = 7^x$ }.	M1
		$y^2 - 4y + 3\ \{= 0\}$	A1
	$\{\ (y-3)(y-1) = 0$ or $(7^x - 3)(7^x - 1) = 0\ \}$		
	$y = 3,\ \ y = 1$ or $7^x = 3,\ \ 7^x = 1$	Both $y = 3$ and $y = 1$.	A1
	$\{7^x = 3 \Rightarrow\}\ x\log 7 = \log 3$ or $x = \dfrac{\log 3}{\log 7}$ or $x = \log_7 3$	A valid method for solving $7^x = k$ where $k > 0,\ k \neq 1$	dM1
	$x = 0.5645....$	0.565 or awrt 0.56	A1
	$x = 0$	$x = 0$ stated as a solution.	B1
			(6) **[8]**

	<u>Notes</u>
(a)	B1B0: Any two of the following three criteria below correct. B1B1: All three criteria correct. Criteria number 1: Correct shape of curve for $x \geq 0$. Criteria number 2: Correct shape of curve for $x < 0$. Criteria number 3: $(0, 1)$ stated or 1 marked on the y-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the “correct” place on the y-axis.

Question Number	Scheme	Marks
(b)	1st M1 is an attempt to form a quadratic equation {using $"y" = 7^x$.} 1st A1 mark is for the correct quadratic equation of $y^2 - 4y + 3\{=0\}$. Can use any variable here, eg: y, x or 7^x. Allow M1A1 for $x^2 - 4x + 3\{=0\}$. Writing $\left(7^x\right)^2 - 4\left(7^x\right) + 3 = 0$ is also sufficient for M1A1. Award M0A0 for seeing $7^{x^2} - 4\left(7^x\right) + 3 = 0$ by itself without seeing $y^2 - 4y + 3\{=0\}$ or $\left(7^x\right)^2 - 4\left(7^x\right) + 3 = 0$. 1st A1 mark for both $y = 3$ and $y = 1$ or both $7^x = 3$ and $7^x = 1$. Do not give this accuracy mark for both $x = 3$ and $x = 1$, unless these are recovered in later working by candidate applying logarithms on these. Award M1A1A1 for $7^x = 3$ and $7^x = 1$ written down with no earlier working. 3rd dM1 for solving $7^x = k,\ k > 0,\ k \neq 1$ to give either $x\ln 7 = \ln k$ or $x = \frac{\ln k}{\ln 7}$ or $x = \log_7 k$. dM1 is dependent upon the award of M1. 2nd A1 for 0.565 or awrt 0.56. B1 is for the solution of $x = 0$, from ***any*** working.	

Question Number	Scheme		Marks
9.			
(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2 + (1-6)^2}$ or	Applies distance formula in order to find the radius.	M1
	$(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2 + (11-6)^2}$	Correct application of formula.	A1
	$(x-3)^2 + (y-6)^2 = 50 \left(\text{or } \left(\sqrt{50}\right)^2 \text{ or } \left(5\sqrt{2}\right)^2\right)$	$(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive <u>value</u>.	M1
		$(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$, } $\underline{(10-3)^2 + (7-6)^2 = 50}$, {so the point lies on C.}		<u>B1</u>
			(1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y - 7 = -7(x - 10)$	$y - 7 = (\text{their gradient})(x - 10)$	M1
	$y = -7x + 77$	$y = -7x + 77$ or $y = 77 - 7x$	A1 cao
			(4)
			[10]

	<u>Notes</u>
(a)	Alternative method: $C\left(-2 + \frac{8 - -2}{2}, 11 + \frac{1-11}{2}\right)$ or $C\left(8 + \frac{-2-8}{2}, 1 + \frac{11-1}{2}\right)$
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1st M1 generously for $\frac{(-2-8)^2 + (11-1)^2}{2}$ Award 1st M1A1 for $\frac{(-2-8)^2 + (11-1)^2}{4}$ or $\frac{\sqrt{(-2-8)^2 + (11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.
(c)	B1 awarded for correct verification of $\underline{(10-3)^2 + (7-6)^2 = 50}$ with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C. Also a candidate could either substitute $x = 10$ in C to find $y = 7$ or substitute $y = 7$ in C to find $x = 10$.

Question Number	Scheme	Marks
(d)	2^{nd} M1 mark also for the complete method of applying $7 = \text{(their gradient)}(10) + c$, finding c. **Note**: Award 2^{nd} M0 in (d) if their numerical gradient is either 0 or ∞. **Alternative:** For first two marks (differentiation): $2(x-3) + 2(y-6)\frac{dy}{dx} = 0$ (or equivalent) scores B1. 1^{st} M1 for substituting **both** $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$, which must contain both x and y. (This M mark can be awarded generously, even if the attempted "differentiation" is not "implicit".) **Alternative:** $(10-3)(x-3) + (7-6)(y-6) = 50$ scores B1M1M1 which leads to $y = -7x + 77$.	

Question Number	Scheme		Marks
10.			
(a)	$V = 4x(5-x)^2 = 4x(25-10x+x^2)$		
	So, $V = 100x - 40x^2 + 4x^3$	$\pm \alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
		$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	At least two of their expanded terms differentiated correctly.	M1
		$100 - 80x + 12x^2$	A1 cao
			(4)
(b)	$100 - 80x + 12x^2 = 0$	Sets their $\frac{dV}{dx}$ from part (a) = 0	M1
	$\left\{\Rightarrow 4\left(3x^2 - 20x + 25\right) = 0 \Rightarrow 4(3x-5)(x-5) = 0\right\}$		
	$\{\text{As } 0 < x < 5\}\ x = \frac{5}{3}$	$x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3},\quad V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$	Substitute candidate's value of x **where** $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074...$	Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
			(4)
(c)	$\frac{d^2V}{dx^2} = -80 + 24x$	Differentiates their $\frac{dV}{dx}$ **correctly** to give $\frac{d^2V}{dx^2}$.	M1
	When $x = \frac{5}{3},\ \frac{d^2V}{dx^2} = -80 + 24\left(\frac{5}{3}\right)$		
	$\frac{d^2V}{dx^2} = -40 < 0 \Rightarrow V$ is a maximum	$\underline{\frac{d^2V}{dx^2} = -40}$ and $\underline{< 0 \text{ or negative}}$ and $\underline{\text{maximum}}$.	A1 cso
			(2) **[10]**

	Notes
(a)	1st M1 for a three term cubic in the form $\pm \alpha x \pm \beta x^2 \pm \gamma x^3$. Note that an un-combined $\pm \alpha x \pm \lambda x^2 \pm \mu x^2 \pm \gamma x^3$, $\alpha, \lambda, \mu, \gamma \neq 0$ is fine for the 1st M1. 1st A1 for either $100x - 40x^2 + 4x^3$ or $100x - 20x^2 - 20x^2 + 4x^3$. 2nd M1 for any two of their expanded terms differentiated correctly. NB: If expanded expression is divided by a constant, then the 2nd M1 can be awarded for at least two terms are correct. Note for un-combined $\pm\lambda x^2 \pm \mu x^2$, $\pm 2\lambda x \pm 2\mu x$ counts as one term differentiated correctly. 2nd A1 for $100 - 80x + 12x^2$, **cao**. **Note:** See appendix for those candidates who apply the product rule of differentiation.

Question Number	Scheme	Marks
(b)	**Note you can mark parts (b) and (c) together.** Ignore the extra solution of $x = 5$ (and $V = 0$). Any extra solutions for *V* inside found for values inside the range of *x*, then award the final A0.	
(c)	M1 is for their $\frac{\mathrm{d}V}{\mathrm{d}x}$ differentiated **correctly (follow through)** to give $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$. A1 for all three of <u>$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = -40$</u> and <u>$< 0$ or negative</u> and <u>maximum</u>. Ignore any second derivative testing on $x = 5$ for the final accuracy mark. **<u>Alternative Method: Gradient Test:</u>** M1 for finding the gradient either side of their *x*-value from part (b) where $0 < x < 5$. A1 for <u>both gradients calculated correctly to the near integer</u>, <u>using > 0 and < 0 respectively or a correct sketch</u> and <u>maximum</u>. (See appendix for gradient values.)	

Question Number	Scheme		Marks
Aliter **4 (b)** Way 2	$(x+1)(x-5) = \underline{x^2 - 4x - 5}$ or $\underline{x^2 - 5x + x - 5}$	Can be implied by later working.	B1
	$-\int (x^2 - 4x - 5)\,dx = -\frac{x^3}{3} + \frac{4x^2}{2} + 5x \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 any two out of three terms correctly ft.	M1A1ft A1
	$\left[-\frac{x^3}{3} + \frac{4x^2}{2} + 5x\right]_{-1}^{5} = (......) - (......)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \left(-\frac{125}{3} + \frac{100}{2} + 25\right) - \left(\frac{1}{3} + 2 - 5\right) = \left(\frac{100}{3}\right) - \left(-\frac{8}{3}\right) \right\}$		
	Hence, Area $= 36$		A1
			(6)

Question Number	Scheme		Marks
Aliter **6 (b)** Way 2	$0.25\times\left\{\frac{0.5+0.38}{2}+\frac{0.38+0.30}{2}+\frac{0.30+0.24}{2}+\frac{0.24+0.2}{2}\right\}$	0.25 and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the denominator of 2.	M1
	$\frac{1}{2}\times 0.25\ ;\times\underline{\left\{(0.5+0.2)+2(0.38+\text{ their }0.30+\text{ their }0.24)\right\}}$	Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	A1 $\sqrt{}$
	$\left\{=\tfrac{1}{8}(2.54)\right\}=\text{awrt }0.32$	awrt 0.32	A1 **(4)**

Question Number	Scheme		Marks
Aliter **10 (a)** Way2	**Product Rule Method:** $\left\{\begin{array}{ll} u = 4x & v = (5-x)^2 \\ \frac{du}{dx} = 4 & \frac{dv}{dx} = 2(5-x)^1(-1) \end{array}\right\}$		
		$\pm(\text{their } u')(5-x)^2 \pm (4x)(\text{their } v')$	M1
	$\frac{dy}{dx} = 4(5-x)^2 + 4x(2)(5-x)^1(-1)$	A correct attempt at differentiating any one of either u or v correctly.	dM1
		Both $\frac{du}{dx}$ and $\frac{dv}{dx}$ correct	A1
	$\frac{dy}{dx} = 4(5-x)^2 - 8x(5-x)$	$4(5-x)^2 - 8x(5-x)$	A1
			(4)
Aliter **10 (a)** Way3	$\left\{\begin{array}{ll} u = 4x & v = 25 - 10x + x^2 \\ \frac{du}{dx} = 4 & \frac{dv}{dx} = -10 + 2x \end{array}\right\}$		
		$\pm(\text{their } u')\left(\text{their}(5-x)^2\right) \pm (4x)(\text{their } v')$	M1
	$\frac{dy}{dx} = 4(25 - 10x + x^2) + 4x(-10 + 2x)$	A correct attempt at differentiating any one of either u or **their** v correctly.	dM1
		Both $\frac{du}{dx}$ and $\frac{dv}{dx}$ correct	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	$100 - 80x + 12x^2$	A1
			(4)
	Note: The candidate needs to use a complete product rule method in order for you to award the the first M1 mark here. The second method mark is dependent on the first method mark awarded.		

Question Number	Scheme	Marks
Aliter **10 (c)** *Way 2*	**Gradient Test Method:** $\frac{dV}{dx} = 100 - 80x + 12x^2$ ***Helpful table!***	

x	$\frac{dV}{dx}$
0.8	43.68
0.9	37.72
1	32
1.1	26.52
1.2	21.28
1.3	16.28
1.4	11.52
1.429	10.204
1.5	7
1.6	2.72
1.7	-1.32
1.8	-5.12
1.9	-8.68
2	-12
2.1	-15.08
2.2	-17.92
2.3	-20.52
2.4	-22.88
2.5	-25

Question Number	Scheme	Marks
8 (b)	***Method of trial and improvement***	
	Helpful table:	

x	$y = 7^{2x} - 4\left(7^x\right) + 3$
0	0
0.1	-0.38348
0.2	-0.72519
0.3	-0.95706
0.4	-0.96835
0.5	-0.58301
0.51	-0.51316
0.52	-0.43638
0.53	-0.3523
0.54	-0.26055
0.55	-0.16074
0.56	-0.05247
0.561	-0.04116
0.562	-0.02976
0.563	-0.01828
0.564	-0.0067
0.565	0.00497
0.57	0.064688
0.58	0.19118
0.59	0.327466
0.6	0.474029
0.7	2.62723
0.8	6.525565
0.9	13.15414
1	24

Scheme	Marks
For a full method of trial and improvement by trialing $\text{f}(\text{value between } 0.1 \text{ and } 0.5645) = \text{value}$ and $\text{f}(\text{value between } 0.5645 \text{ and } 1) = \text{value}$	M1
Any one of these values correct to 1sf or truncated 1sf.	A1
Both of these values correct to 1sf or truncated 1sf.	A1
A method to confirm root to 2 dp by finding by trialing $\text{f}(\text{value between } 0.56 \text{ and } 0.5645) = \text{value}$ and $\text{f}(\text{value between } 0.5645 \text{ and } 0.565) = \text{value}$	M1
Both values correct to 1sf or truncated 1sf and the confirmation that the root is $x = 0.56$ (only)	A1
$x = 0$	B1
	(6)

Note: If a candidate goes from $7^x = 3$ with no working to $x = 0.5645...$ then give M1A1 implied.