Paper Reference(s) 66664/01 Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Wednesday 9 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination Mathematical Formulae (Pink) Items included with question papers Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

$$y = 3^x + 2x.$$

(a) Complete the table below, giving the values of y to 2 decimal places.

| x | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|------|-----|-----|-----|---|
| у | 1 | 1.65 | | | | 5 |

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_{0}^{1} (3^{x} + 2x) dx$.

(4)

(2)

(5)

(2)

| $f(x) = 3x^3 - 5x^2 - 58x + 40$ | • |
|---------------------------------|---|
|---------------------------------|---|

(a) Find the remainder when f(x) is divided by (x - 3).

Given that (x - 5) is a factor of f(x),

(b) find all the solutions of f(x) = 0.

3.

 $y = x^2 - k\sqrt{x}$, where k is a constant.

| (a) Find $\frac{dy}{dx}$. | |
|----------------------------|-----|
| | (2) |

(b) Given that y is decreasing at x = 4, find the set of possible values of k.

(2)

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of (1+ ax)⁷, where a is a constant. Give each term in its simplest form.
 (4)

Given that the coefficient of x^2 in this expansion is 525,

(*b*) find the possible values of *a*.

(2)

1.

5. (a) Given that $5 \sin \theta = 2 \cos \theta$, find the value of $\tan \theta$.

(b) Solve, for $0 \le x < 360^\circ$,

$$5\sin 2x = 2\cos 2x$$
,

giving your answers to 1 decimal place.

(5)

(1)



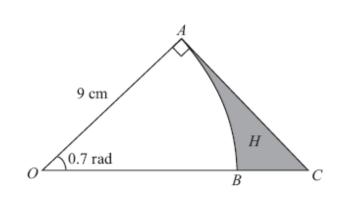




Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

| (<i>a</i>) Find the length of the arc <i>AB</i> . | (2) |
|---------------------------------------------------------------------------------------------|-----|
| (b) Find the area of the sector <i>OAB</i> . | (2) |
| The line AC shown in Figure 1 is perpendicular to OA, and OBC is a straight line. | |
| (c) Find the length of AC , giving your answer to 2 decimal places. | (2) |
| The region <i>H</i> is bounded by the arc <i>AB</i> and the lines <i>AC</i> and <i>CB</i> . | |
| (<i>d</i>) Find the area of <i>H</i> , giving your answer to 2 decimal places. | (3) |

7. (*a*) Given that

 $2 \log_3(x-5) - \log_3(2x-13) = 1,$

show that $x^2 - 16x + 64 = 0$.

(b) Hence, or otherwise, solve $2 \log_3 (x-5) - \log_3 (2x-13) = 1$.

(2)

(5)

8.

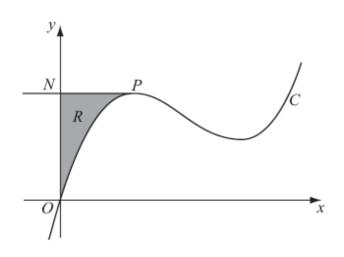




Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where *k* is a constant.

The point *P* on *C* is the maximum turning point.

Given that the *x*-coordinate of *P* is 2,

(a) show that k = 28.

(3)

The line through *P* parallel to the *x*-axis cuts the *y*-axis at the point *N*. The region *R* is bounded by *C*, the *y*-axis and *PN*, as shown shaded in Figure 2.

(*b*) Use calculus to find the exact area of *R*.

(6)

9. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

| (<i>a</i>) | Show that the predicted adult population at the end of Year 2 is 25 750. | |
|--------------|--------------------------------------------------------------------------|-----|
| | | (1) |
| <i>(b)</i> | Write down the common ratio of the geometric sequence. | |

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(*c*) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(*d*) Find the value of *N*.

(2)

(3)

(1)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

| 10. | The circle <i>C</i> has centre $A(2,1)$ and passes through the point $B(10, 7)$. | |
|-----|-----------------------------------------------------------------------------------|-----|
| | (a) Find an equation for C. | (4) |
| | The line l_1 is the tangent to C at the point B. | |
| | (b) Find an equation for l_1 . | (4) |
| | The line l_2 is parallel to l_1 and passes through the mid-point of AB. | |
| | Given that l_2 intersects C at the points P and Q, | |
| | (c) find the length of PQ, giving your answer in its simplest surd form. | (3) |
| | TOTAL FOR PAPER: 75 MAI | RKS |

END

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June 2010 Core Mathematics C2 6664 Mark Scheme

| Question Number | Scheme | Marks | |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|----|
| 1. | (a) 2.35, 3.13, 4.01 (One or two correct B1 B0, all correct B1 B1) <u>Important</u> : If part (a) is blank, or if answers have been crossed out and no replacement answers are visible, please send to Review as 'out of clip'. | B1 B1 (| 2) |
| | (b) $\frac{1}{2} \times 0.2$ (or equivalent numerical value) | B1 | |
| | $k \{(1+5)+2(1.65+p+q+r)\}, k \text{ constant}, k \neq 0 \text{(See notes below)} \\ = 2.828 (awrt 2.83, allowed even after minor slips in values)$ | M1 A1 A1 | |
| | The fractional answer $\frac{707}{250}$ (or other fraction wrt 2.83) is also acceptable. | (| 4) |
| | Answers with no working score no marks. | | 6 |
| | (a) Answers must be given to 2 decimal places. <u>No marks</u> for answers given to only 1 decimal place. | | |
| | (b) The <i>p</i>, <i>q</i> and <i>r</i> below are positive numbers, none of which is equal to any of: 1, 5, 1.65, 0.2, 0.4, 0.6 or 0.8 | | |
| | M1 A1: $k\{(1+5)+2(1.65+p+q+r)\}$ M1 A0: $k\{(1+5)+2(1.65+p+q)\}$ or $k\{(1+5)+2(p+q+r)\}$ M0 A0: $k\{(1+5)+2(1.65+p+q+r+other value(s))\}$ | | |
| | Note that if the only mistake is to <u>omit</u> a value from the second bracket, this is considered as a slip and the M mark is allowed. | | |
| | Bracketing mistake: i.e. $\frac{1}{2} \times 0.2(1+5) + 2(1.65+2.35+3.13+4.01)$ | | |
| | instead of $\frac{1}{2} \times 0.2\{(1+5)+2(1.65+2.35+3.13+4.01)\}$, so that only | | |
| | the $(1 + 5)$ is multiplied by 0.1 scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). | | |
| | <u>Alternative</u> : Separate trapezia may be used, and this can be marked equivalently. | | |
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| Question Number | Scheme | Marks |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|
| 2 | (a) Attempting to find $f(3)$ or $f(-3)$ | M1 |
| | $f(3) = 3(3)^3 - 5(3)^2 - (58 \times 3) + 40 = 81 - 45 - 174 + 40 = -98$ | A1 (2) |
| | (b) $\{3x^3 - 5x^2 - 58x + 40 = (x - 5)\}$ $(3x^2 + 10x - 8)$ | M1 A1 |
| | Attempt to <u>factorise</u> 3-term quadratic, or to use the quadratic formula (see general principles at beginning of scheme). This mark may be implied by the correct solutions to the quadratic. | M1 |
| | $(3x-2)(x+4) = 0$ $x =$ or $x = \frac{-10 \pm \sqrt{100+96}}{6}$ $\frac{2}{3}$ (or exact equiv.), -4, 5 (Allow 'implicit' solns, e.g. f(5) = 0, etc.) | A1 ft |
| | $\frac{2}{3}$ (or exact equiv.), -4, 5 (Allow 'implicit' solns, e.g. $f(5) = 0$, etc.) | A1 (5) |
| | Completely correct solutions without working: full marks. | 7 |
| < / | tive (long division): <u>'Grid' met</u> | hod |
| | by $(x-3)$ to get $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1] 3 3 -5 | |
| $(3x^2 +$ | (4x - 46), and -98 seen. [A1] 0 9 | 12 -138 -46 -98 |
| (If con | tinues to say 'remainder = 98', isw) 3 4 - | -46 –98 |
| (b) 1st M | requires use of $(x-5)$ to obtain $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$. <u>'Grid' met</u> | |
| (V | Vorking need not be seen this could be done 'by inspection'.) $3 \begin{array}{c} 3 \\ -5 \\ 0 \\ 15 \end{array}$ | |
| | $(3x^2 + 10x - 8) \blacktriangleleft \qquad $ | -8 0 |
| 2^{nd} M f | For the attempt to factorise their 3-term quadratic, or to solve it using the quadratic | ratic formula. |
| | Factorisation: $(3x^2 + ax + b) = (3x + c)(x + d)$, where $ cd =$ | = b . |
| A1ft: C | orrect factors for their 3-term quadratic <u>followed by a solution</u> (at least one va might be incorrect), <u>or</u> numerically correct expression from the quadratic for 3-term quadratic. | |
| W | herefore that if the quadratic is correctly factorised but no solutions are given, vill be lost. | the last 2 marks |
| | ative (first 2 marks): | |
| (x-5) | (3x2 + ax + b) = 3x3 + (a - 15)x2 + (b - 5a)x - 5b = 0, | |
| | then compare coefficients to find <u>values</u> of a and b a = 10, b = -8 | o. [M1] [A1] |
| | <u>ative 1</u> : (factor theorem) nding that $f(-4) = 0$ | |
| A1: Sta | ating that $(x + 4)$ is a factor. | |
| M1: Fi | nding third factor $(x-5)(x+4)(3x\pm 2)$. | |
| | lly correct factors (no ft available here) <u>followed by a solution</u> , (which might l solutions correct. | be incorrect). |
| | <u>ative 2</u> : (direct factorisation) actors $(x-5)(3x+p)(x+q)$ A1: $pq = -8$ | |
| | $(x-5)(3x\pm 2)(x\pm 4)$ | |
| | A marks as in Alternative 1. | |
| Throug | hout this scheme, allow $\left(x \pm \frac{2}{3}\right)$ as an alternative to $(3x \pm 2)$. | |

| Question Number | Scheme | Marks |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------|----------|
| 3 | (a) $\left(\frac{dy}{dx}\right) = 2x - \frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. + <i>C</i> , is A0) | M1 A1 |
| | dy = dy | (2) |
| | (b) Substituting $x = 4$ into their $\frac{dy}{dx}$ and 'compare with zero' (The mark is allowed for : <, >, =, \leq , \geq) | M1 |
| | $8 - \frac{k}{4} < 0$ $k > 32$ (or $32 < k$) <u>Correct inequality needed</u> | A1 |
| | | (2) 4 |
| | (a) M: $x^2 \to cx$ or $k\sqrt{x} \to cx^{-\frac{1}{2}}$ (<i>c</i> constant, $c \neq 0$) | |
| | (b) Substitution of $x = 4$ into y scores M0. However, $\frac{dy}{dx}$ is sometimes | |
| | <u>called</u> y, and in this case the M mark can be given. dy | |
| | $\frac{dy}{dx} = 0$ may be 'implied' for M1, when, for example, a value of k or an inequality solution for k is found. | |
| | <u>Working</u> must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0. | |
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| Question Number | Scheme | Marks |
|--------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| 4 | (a) $(1 + ax)^7 = 1 + 7ax$ or $1 + 7(ax)$ (<u>Not</u> unsimplified versions) | B1 |
| | $+\frac{7\times 6}{2}(ax)^2 + \frac{7\times 6\times 5}{6}(ax)^3$ Evidence from <u>one</u> of these terms is enough | M1 |
| | $+21a^2x^2$ or $+21(ax)^2$ or $+21(a^2x^2)$ | A1 |
| | $+35a^{3}x^{3}$ or $+35(ax)^{3}$ or $+35(a^{3}x^{3})$ | A1 |
| | (b) $21a^2 = 525$ | (4) M1 |
| | | |
| | $a = \pm 5$ (Both values are required) (The answer $a = 5$ with no working scores M1 A0) | A1 (2) 6 |
| | (a) The terms can be 'listed' rather than added. | |
| | M1: Requires correct structure: a correct binomial coefficient in any form (perhaps from Pascal's triangle) with the correct power of x. Allow missing a's and wrong powers of a, e.g. $\frac{7 \times 6}{2} ax^2, \qquad \frac{7 \times 6 \times 5}{3 \times 2} x^3$ However, $21 + a^2x^2 + 35 + a^3x^3$ or similar is M0. $1 + 7ax + 21 + a^2x^2 + 35 + a^3x^3 = 57 + \dots$ scores the B1 (isw). $\binom{7}{2}$ and $\binom{7}{3}$ or equivalent such as 7C_2 and 7C_3 are acceptable, | |
| | but $\underline{\text{not}}\left(\frac{7}{2}\right)$ or $\left(\frac{7}{3}\right)$ (unless subsequently corrected). 1 st A1: Correct x^2 term. 2 nd A1: Correct x^3 term (The binomial coefficients <u>must</u> be simplified). | |
| | Special case: If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost | |
| | A1 A0 can be given if $21ax^2$ and $35ax^3$ are <u>both</u> achieved. | |
| | <u><i>a</i>'s omitted throughout</u> : Note that only the M mark is available in this case. | |
| | (b) M: Equating their coefficient of x^2 to 525. An equation in a or a^2 alone is required for this M mark, but allow 'recovery' that shows the required coefficient, e.g. $21a^2x^2 = 525 \implies 21a^2 = 525$ is acceptable, but $21a^2x^2 = 525 \implies a^2 = 25$ is not acceptable. After $21ax^2$ in the answer for (a), allow 'recovery' of a^2 in (b) so that | |
| | full marks are available for (b) (but not retrospectively for (a)). | |

| Question Number | Scheme | Marks |
|-----------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|
| 5 | (a) $\tan \theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found) | B1 (1) |
| | Requires the correct value with no incorrect working seen.(b) awrt 21.8 (α) | B1 |
| | _ | Ы |
| | (Also allow awrt 68.2, ft from $\tan \theta = \frac{5}{2}$ in (a), but no other ft) | |
| | (This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9) | |
| | 180 + α (= 201.8), or 90 + ($\alpha/2$) (if division by 2 has already occurred) (α found from tan 2x = or tan x = or sin 2x = ± or cos 2x = ±) | M1 |
| | 360 + α (= 381.8), or 180 + ($\alpha/2$) (α found from tan 2x = or sin 2x = or cos 2x =) | M1 |
| | OR $540 + \alpha$ (= 561.8), or $270 + (\alpha/2)$ (α found from tan $2x =$) | |
| | Dividing at least one of the angles by 2 (α found from $\tan 2x = \dots$ or $\sin 2x = \dots$ or $\cos 2x = \dots$) | M1 |
| | x = 10.9, 100.9, 190.9, 280.9 (Allow awrt) | A1 (5) |
| <u>Comm</u> 10.9 ar 10.9 ar <u>Alterna</u> | | rded) |
| (1) $2\cos 2$ | | 81 //1 |
| | | лт Л1 |
| | | M1 |
| (ii) 25sin | $x^{2} 2x = 4\cos^{2} 2x = 4(1 - \sin^{2} 2x)$ | |
| 29 | $\sin^2 2x = 4$ $2x = 21.8$ B1 | |
| <u>Using</u> B1: Ca | marks are scored as in the main scheme, but extra solutions will be likely, los radians: in be given for awrt 0.38 (β) | sing the A mark. |
| | or $\pi + \beta$ or $180 + \beta$ | |
| | or $2\pi + \beta$ or $3\pi + \beta$ (Must now be consistently radians) or dividing at least one of the angles by 2 | |
| A1: Fo | r this mark, the answers must be in degrees. | |
| B and | ct) answers only (or by graphical methods): M marks can be awarded by implication, e.g. | |
| | cores B1 M0 M0 M1 A0 00.9 scores B1 M1 M0 M1 A0 | |
| 10.9, 1 | 00.9, 190.9, 280.9 scores full marks. | |
| Using | 11, etc. instead of 10.9 can still score the M marks by implication. | |

| Question Number | Scheme | Marks |
|--------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| 6 | (a) $r\theta = 9 \times 0.7 = 6.3$ (Also allow 6.30, or awrt 6.30) | M1 A1 (2 |
| | (b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 81 \times 0.7 = 28.35$ (Also allow 28.3 or 28.4, or awrt 28.3 or 28.4) (Condone 28.35 ² written instead of 28.35 cm ²) | M1 A1 (2 |
| | (c) $\tan 0.7 = \frac{AC}{9}$ AC = 7.58 (Allow awrt) <u>NOT</u> 7.59 (see below) | M1 A1 (2 |
| | (d) Area of triangle $AOC = \frac{1}{2}(9 \times \text{their } AC)$ (or other complete method) | M1 |
| | Area of $R = "34.11" - "28.35"$ (triangle – sector) or (sector – triangle) (needs a <u>value</u> for each) = 5.76 (Allow awrt) | M1 A1 (3 |
| | (a) M: Use of $r\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). | |
| | (b) M: Use of $\frac{1}{2}r^2\theta$ (with θ in radians), or equivalent (could be working in degrees with a correct degrees formula). | |
| | (c) M: Other methods must be fully correct, e.g. $\frac{AC}{\sin 0.7} = \frac{9}{\sin(\frac{\pi}{2} - 0.7)}$ $(\pi - 0.7)$ instead of $(\frac{\pi}{2} - 0.7)$ here is <u>not</u> a fully correct method. | |
| | $\frac{\text{Premature approximation (e.g. taking angle C as 0.87 radians)}{This will often result in loss of A marks, e.g. AC = 7.59 in (c) is A0.$ | |
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| Question Number | Scheme | Marks | |
|---------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|---------------|--|
| 7 | (a) $2\log_3(x-5) = \log_3(x-5)^2$ | B1 | |
| | $\log_3(x-5)^2 - \log_3(2x-13) = \log_3\frac{(x-5)^2}{2x-13}$ | M1 | |
| | $\log_3 3 = 1$ seen or used correctly | B1 | |
| | $\log_3\left(\frac{P}{Q}\right) = 1 \implies P = 3Q \qquad \left\{\frac{(x-5)^2}{2x-13} = 3 \implies (x-5)^2 = 3(2x-13)\right\}$ | M1 | |
| | $x^2 - 16x + 64 = 0 \tag{(*)}$ | A1 cso (5) | |
| | (b) $(x-8)(x-8) = 0 \implies x = 8$ Must be seen in part (b). | M1 A1 | |
| | Or: Substitute $x = 8$ into original equation and verify. | (2) | |
| | Having additional solution(s) such as $x = -8$ loses the A mark. | (2) | |
| | x = 8 with no working scores both marks. | 7 | |
| (a) Marks | may be awarded if equivalent work is seen in part (b). | | |
| 1st M: $\log_3(x-5)^2 - \log_3(2x-13) = \frac{\log_3(x-5)^2}{\log_3(2x-13)}$ is M0 | | | |
| $2\log_3(x-5) - \log_3(2x-13) = 2\log\frac{x-5}{2x-13}$ is M0 | | | |
| 2 nd M: After the first mistake above, this mark is available only if there is 'recovery' to the required | | | |
| $\log_3\left(\frac{P}{Q}\right) = 1 \implies P = 3Q$. Even then the final mark (cso) is lost. | | | |
| <u>'Cancelling logs'</u> , e.g. $\frac{\log_3(x-5)^2}{\log_3(2x-13)} = \frac{(x-5)^2}{2x-13}$ will also lose the 2 nd M. | | | |
| A typical wrong solution: | | | |
| $\log_3 \frac{(x-5)^2}{2x-13} = 1 \implies \log_3 \frac{(x-5)^2}{2x-13} = 3 \implies \frac{(x-5)^2}{2x-13} = 3 \implies (x-5)^2 = 3(2x-13)$ | | | |
| (Wrong step here) This, with no evidence elsewhere of $\log_3 3 = 1$, scores B1 M1 B0 M0 A0 | | | |
| However, $\log_3 \frac{(x-5)^2}{2x-13} = 1 \implies \frac{(x-5)^2}{2x-13} = 3$ is correct and could lead to full marks. | | | |
| (Here $\log_3 3 = 1$ is implied). | | | |
| No log methods shown: | | | |
| It is <u>not</u> acceptable to jump immediately to $\frac{(x-5)^2}{2x-13} = 3$. The only mark this scores is the 1 st B1 (by | | | |
| generou | s implication). | | |
| (b) M1: A | ttempt to solve the given quadratic equation (usual rules), so the factors $(x-8)$ | (x-8) with no | |

(b) M1: Attempt to solve the given quadratic equation (usual rules), so the factors (x-8)(x-8) with no solution is M0.

| Question Number | Scheme | Marks |
|--------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| 8 | (a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required) | M1 A1 |
| | At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*) | A1 cso |
| | N.B. The '= 0' must be seen at some stage to score the final mark. | |
| | <u>Alternatively</u> : (using $k = 28$) | |
| | $\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1) | |
| | 'Assuming' $k = 28$ only scores the final cso mark if there is justification | (3) |
| | that $\frac{dy}{dx} = 0$ at $x = 2$ represents the <u>maximum</u> turning point. | |
| | (b) $\int (x^3 - 10x^2 + 28x) dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$ | M1 A1 |
| | $\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = \dots \qquad \left(=4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$ | M1 |
| | (With limits 0 to 2, substitute the limit 2 into a 'changed function') | |
| | y-coordinate of $P = 8 - 40 + 56 = 24$ (The B1 for 24 may be scored by implication from later working) Area of rectangle = $2 \times$ (their y - coordinate of P) | B1 |
| | Area of $R = (\text{their } 48) - \left(\text{their } \frac{100}{3}\right) = \frac{44}{3} \left(14\frac{2}{3} \text{ or } 14.6\right)$ | M1 A1 |
| | If the subtraction is the 'wrong way round', the final A mark is lost. | (6) 9 |
| | (a) M: xⁿ → cxⁿ⁻¹ (c constant, c ≠ 0) for one term, seen <u>in part (a)</u>. (b) 1st M: xⁿ → cxⁿ⁺¹ (c constant, c ≠ 0) for one term. Integrating the <u>gradient function</u> loses this M mark. | |
| | 2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated). | |
| | Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle. | |
| | A1: Must be <u>exact</u> , not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$. | |
| | Alternative: (effectively finding area of rectangle by integration) | |
| | $\int \left\{ 24 - (x^3 - 10x^2 + 28x) \right\} dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2} \right), \text{ etc.}$ | |
| | This can be marked equivalently, with the 1^{st} A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2^{nd} M. If the subtraction is the 'wrong way round', the final A mark is lost. | |

| Question Number | Scheme | Marks | | |
|------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|-----------|--|
| | (a) $25\ 000 \times 1.03 = 25750$ | | | |
| 9 | $\left\{25000 + 750 = 25750, \text{ or } 25000 \frac{(1 - 0.03^2)}{1 - 0.03} = 25750\right\} $ (*) | B1 | (1) | |
| | (b) $r = 1.03$ Allow $\frac{103}{100}$ or $1\frac{3}{100}$ but no other alternatives | B1 | (1) | |
| | (c) $25000r^{N-1} > 40000$ (Either letter <i>r</i> or their <i>r</i> value) Allow '= ' or '<' | M1 | | |
| | $r^{M} > 1.6 \Rightarrow \log r^{M} > \log 1.6$ Allow '= ' or '<' (See below) | | | |
| | OR (by change of base), $\log_{1.03} 1.6 < M \implies \frac{\log 1.6}{\log 1.03} < M$ | M1 | | |
| | $(N-1)\log 1.03 > \log 1.6$ (Correct bracketing required) (*) | A1 cso | | |
| | Accept work for part (c) seen in part (d) | | (3) | |
| | (d) Attempt to evaluate $\frac{\log 1.6}{\log 1.03} + 1$ {or $25000(1.03)^{15}$ and $25000(1.03)^{16}$ } | M1 | | |
| | $N = 17 (\underline{\text{not}} \ 16.9 \text{ and not e.g. } N \ge 17) \qquad \text{Allow `} 17^{\text{th}} \text{ year'}$ Accept work for part (d) seen in part (c) | A1 | (2) | |
| | (e) Using formula $\frac{a(1-r^n)}{1-r}$ with values of <i>a</i> and <i>r</i> , and <i>n</i> = 9, 10 or 11 | M1 | | |
| l | $25000(1-1.03^{10})$ | A1 | | |
| | 1-1.03 287 000 (must be rounded to the nearest 1 000) Allow 287000.00 | A1 | (3) 10 | |
| (c) 2^{nd} M: | Requires $\frac{40000}{25000}$ to be dealt with, and 'two' logs introduced. | I | | |
| | say, N instead of $N-1$, this mark is still available. | | | |
| - | ng straight from $1.03^{N-1} > 1.6$ to $(N-1)\log 1.03 > \log 1.6$ can only M1 M0 A0. | | | |
| (The intermediate step $\log 1.03^{N-1} > \log 1.6$ must be seen). | | | | |
| Longe | <u>r methods</u> require correct log work throughout for 2 nd M, e.g.: | | | |
| $\log(25000r^{N-1}) > \log 40000 \implies \log 25000 + \log r^{N-1} > \log 40000 \implies$ | | | | |
| $\log r^{N-1} > \log 40000 - \log 25000 \Rightarrow \qquad \log r^{N-1} > \log 1.6$ | | | | |
| · · · | t answer with no working scores both marks. (16) | | | |
| Evalua | ting $\log\left(\frac{1.6}{1.03}\right) + 1$ does <u>not</u> score the M mark. | | | |
| (Allo 1 st A1 To the | n also be scored by a "year by year" method, <u>with terms added</u> . w the M mark if there is evidence of adding 9, 10 or 11 terms). is scored if the 10 correct terms have been added (allow terms to be to the nea nearest 100, these terms are: 0, 25800, 26500, 27300, 28100, 29000, 29900, 30700, 31700, 32600 | rest 100). | | |
| <u>No</u> working shown: Special case: 287 000 scores 1 mark, scored on ePEN as 1, 0, 0. (Other answers with no working score no marks). | | | | |

| Question Number | Scheme | Marks |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| 10 | (a) $(10-2)^2 + (7-1)^2$ or $\sqrt{(10-2)^2 + (7-1)^2}$ | M1 A1 |
| | $(x \pm 2)^2 + (y \pm 1)^2 = k$ (k a positive <u>value</u>) | M1 |
| | $(x-2)^{2} + (y-1)^{2} = 100$ (Accept 10 ² for 100) | A1 |
| | (Answer only scores full marks) | (4) |
| | (b) (Gradient of radius =) $\frac{7-1}{10-2} = \frac{6}{8}$ (or equiv.) Must be seen in part (b) | B1 |
| | Gradient of tangent = $\frac{-4}{3}$ (Using perpendicular gradient method) | M1 |
| | $y-7 = m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞) | M1 |
| | $y-7 = \frac{-4}{3}(x-10)$ or equiv (ft gradient of <u>radius</u> , dep. on <u>both</u> M marks) | A1ft |
| | ${3y = -4x + 61}$ (N.B. The A1 is only available as <u>ft</u> after B0) The unsimplified version scores the A mark (isw if necessary subsequent mistakes in simplification are not penalised here. The equation must at some stage be <u>exact</u> , not, e.g. $y = -1.3x + 20.3$ | |
| | The equation must at some stage be \underline{oxact} , not, e.g. $y = 1.3x + 20.5$ | (4) |
| | (c) $\sqrt{r^2 - \left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag. | M1 |
| | $=\sqrt{10^2-5^2}$ or numerically exact equivalent | A1 |
| | $PQ(=2\sqrt{75})=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark | A1 |
| | | (3) |
| | (b) 2 nd M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞). | 11 |
| | <u>Alternative</u> : 2^{nd} M: Using (10, 7) and an <i>m</i> value in $y = mx + c$ to find a value of <i>c</i> . | |
| | (b) <u>Alternative</u> for first 2 marks (differentiation): | |
| | $2(x-2) + 2(y-1)\frac{dy}{dx} = 0$ or equiv. B1 | |
| | Substitute $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$ M1 | |
| | (This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit'). | |
| | (c) <u>Alternatives</u> : | |
| | To score M1, must be a <u>fully</u> correct method to obtain $\frac{1}{2}PQ$ or PQ. | |
| | 1^{st} A1: For alternative methods that find PQ directly, this mark is for an exact numerically correct version of PQ. | |