Paper Reference(s)
6663/01

## Edexcel GCE

## Core Mathematics C1

## Advanced Subsidiary

## Monday 13 May 2013 - Afternoon

## Time: 1 hour 30 minutes

Materials required for examination Mathematical Formulae (Pink)<br>Items included with question papers Nil

Calculators may NOT be used in this examination.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2)
There are 11 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Simplify

$$
\frac{7+\sqrt{ } 5}{\sqrt{5-1}}
$$

giving your answer in the form $a+b \sqrt{5}$, where $a$ and $b$ are integers.
2. Find

$$
\int\left(10 x^{4}-4 x-\frac{3}{\sqrt{ } x}\right) \mathrm{d} x,
$$

giving each term in its simplest form.
3. (a) Find the value of $8^{\frac{5}{3}}$.
(b) Simplify fully $\frac{\left(2 x^{\frac{1}{2}}\right)^{3}}{4 x^{2}}$.
4. A sequence $a_{1}, a_{2}, a_{3}, \ldots$ is defined by

$$
\begin{aligned}
& a_{1}=4, \\
& a_{n+1}=k\left(a_{n}+2\right), \quad \text { for } n \geq 1
\end{aligned}
$$

where $k$ is a constant.
(a) Find an expression for $a_{2}$ in terms of $k$.

Given that $\sum_{i=1}^{3} a_{i}=2$,
(b) find the two possible values of $k$.
5. Find the set of values of $x$ for which
(a) $2(3 x+4)>1-x$,
(b) $3 x^{2}+8 x-3<0$.
6. The straight line $L_{1}$ passes through the points $(-1,3)$ and $(11,12)$.
(a) Find an equation for $L_{1}$ in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.

The line $L_{2}$ has equation $3 y+4 x-30=0$.
(b) Find the coordinates of the point of intersection of $L_{1}$ and $L_{2}$.
7. A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week $N$.
(a) Find the value of $N$.

The company then plans to continue to make 600 mobile phones each week.
(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.
8.


Figure 1
Figure 1 shows a sketch of the curve with equation $y=f(x)$ where

$$
\mathrm{f}(x)=(x+3)^{2}(x-1), \quad x \in \mathbb{R}
$$

The curve crosses the $x$-axis at $(1,0)$, touches it at $(-3,0)$ and crosses the $y$-axis at $(0,-9)$.
(a) Sketch the curve $C$ with equation $y=\mathrm{f}(x+2)$ and state the coordinates of the points where the curve $C$ meets the $x$-axis.
(b) Write down an equation of the curve $C$.
(c) Use your answer to part (b) to find the coordinates of the point where the curve $C$ meets the $y$-axis.
9.

$$
\mathrm{f}^{\prime}(x)=\frac{\left(3-x^{2}\right)^{2}}{x^{2}}, \quad x \neq 0
$$

(a) Show that $\mathrm{f}^{\prime}(x)=9 x^{-2}+A+B x^{2}$, where $A$ and $B$ are constants to be found.
(b) Find $\mathrm{f}^{\prime \prime}(x)$.

Given that the point $(-3,10)$ lies on the curve with equation $y=\mathrm{f}(x)$,
(c) find $f(x)$.
10. Given the simultaneous equations

$$
\begin{array}{r}
2 x+y=1 \\
x^{2}-4 k y+5 k=0
\end{array}
$$

where $k$ is a non zero constant,
(a) show that $x^{2}+8 k x+k=0$.

Given that $x^{2}+8 k x+k=0$ has equal roots,
(b) find the value of $k$.
(c) For this value of $k$, find the solution of the simultaneous equations.
11.


Figure 2
Figure 2 shows a sketch of the curve $H$ with equation $y=\frac{3}{x}+4, x \neq 0$.
(a) Give the coordinates of the point where $H$ crosses the $x$-axis.
(b) Give the equations of the asymptotes to $H$.
(c) Find an equation for the normal to $H$ at the point $P(-3,3)$.

This normal crosses the $x$-axis at $A$ and the $y$-axis at $B$.
(d) Find the length of the line segment $A B$. Give your answer as a surd.

| Question <br> Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 1 | $\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$ | Multiplies top and bottom by a correct expression. This statement is sufficient. | M1 |
|  | (Allow to multiply top and bottom by $k(\sqrt{5}+1)$ ) |  |  |
|  | $=\frac{\cdots}{4}$ | Obtains a denominator of 4 or sight of $(\sqrt{5}-1)(\sqrt{5}+1)=4$ | A1cso |
|  | Note that M0A1 is not possible. The 4 must come from a correct method. |  |  |
|  | $(7+\sqrt{5})(\sqrt{5}+1)=7 \sqrt{5}+5+7+\sqrt{5}$ | An attempt to multiply the numerator by ( $\pm \sqrt{5} \pm 1$ ) and get 4 terms with at least 2 correct for their $( \pm \sqrt{5} \pm 1)$. (May be implied) | M1 |
|  | $3+2 \sqrt{5}$ | Answer as written or $a=3$ and $b=2$. (Allow $2 \sqrt{5}+3$ ) | A1cso |
|  | Correct answer with no working scores full marks |  |  |
|  |  |  | [4] |
| Way 2 | $\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{(-\sqrt{5}-1)}{(-\sqrt{5}-1)}$ | Multiplies top and bottom by a correct expression. This statement is sufficient. | M1 |
|  | (Allow to multiply top and bottom by $k(-\sqrt{5}-1)$ ) |  |  |
|  | $=\frac{\cdots}{-4}$ | Obtains a denominator of -4 | A1cso |
|  | $(7+\sqrt{5})(-\sqrt{5}-1)=-7 \sqrt{5}-5-7-\sqrt{5}$ | An attempt to multiply the numerator by ( $\pm \sqrt{5} \pm 1$ ) and get 4 terms with at least 2 correct for their $( \pm \sqrt{5} \pm 1)$. (May be implied) | M1 |
|  | $3+2 \sqrt{5}$ | Answer as written or $a=3$ and $b=2$ | A1cso |
|  | Correct answer with no working scores full marks |  |  |
|  |  |  | [4] |
|  | Alternative using Simultaneous Equations: $\frac{(7+\sqrt{5})}{\sqrt{5}-1}=a+b \sqrt{5} \Rightarrow 7+\sqrt{5}=(a-b) \sqrt{5}+5 b-a \text { M1 }$ <br> Multiplies and collects rational and irrational parts $a-b=1, \quad 5 b-a=7 \mathrm{~A} 1$ <br> Correct equations $a=3, b=2$ <br> M1 for attempt to solve simultaneous equations A1 both answers correct |  |  |



| Question <br> Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 3(a) | $8^{\frac{1}{3}}=2$ or $8^{5}=32768$ | A correct attempt to deal with the $\frac{1}{3}$ or the 5 . $8^{\frac{1}{3}}=\sqrt[3]{8} \text { or } 8^{5}=8 \times 8 \times 8 \times 8 \times 8$ | M1 |
|  | $\left(8^{\frac{5}{3}}=\right) 32$ | Cao | A1 |
|  | A correct answer with no working scores full marks |  |  |
|  | Alternative$\begin{aligned} 8^{\frac{5}{3}}=8 \times 8^{\frac{2}{3}}=8 \times 2^{2} & =\text { M1 (Deals with the } 1 / 3) \\ & =32 \text { A1 } \end{aligned}$ |  |  |
|  |  |  | (2) |
| (b) | $\left(2 x^{\frac{1}{2}}\right)^{3}=2^{3} x^{\frac{3}{2}}$ | One correct power either $2^{3}$ or $x^{\frac{3}{2}}$. $\left(2 x^{\frac{1}{2}}\right) \times\left(2 x^{\frac{1}{2}}\right) \times\left(2 x^{\frac{1}{2}}\right)$ on its own is not sufficient for this mark. | M1 |
|  | $\frac{8 x^{\frac{3}{2}}}{4 x^{2}}=2 x^{-\frac{1}{2}} \text { or } \frac{2}{\sqrt{x}}$ | M1: Divides coefficients of $x$ and subtracts their powers of $x$. <br> Dependent on the previous M1 | dM1A1 |
|  |  | A1: Correct answer |  |
|  | Note that unless the power of $x$ implies that they have subtracted their powers you would need to see evidence of subtraction. E.g. $\frac{8 x^{\frac{3}{2}}}{4 x^{2}}=2 x^{\frac{1}{2}}$ would score dM0 unless you see some evidence that $3 / 2-2$ was intended for the power of $x$. |  |  |
|  | Note that there is a misconception that $\frac{\left(2 x^{\frac{1}{2}}\right)^{3}}{4 x^{2}}=\left(\frac{2 x^{\frac{1}{2}}}{4 x^{2}}\right)^{3}$ - this scores $0 / 3$ |  |  |
|  |  |  | (3) |
|  |  |  | [5] |


| $\begin{array}{c}\text { Question } \\ \text { Number }\end{array}$ | Scheme |  | Marks |
| :---: | :---: | :---: | :--- |
|  | For this question, mark (a) and (b) together and ignore labelling. |  |  |$]$| (a) |
| :--- |
| 4(a) |


| Question <br> Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 5 (a) | $6 x+x>1-8$ | Attempts to expand the bracket and collect $x$ terms on one side and constant terms on the other. Condone sign errors and allow one error in expanding the bracket. Allow $<, \leq, \geq,=$ instead of $>$. | M1 |
|  | $x>-1$ | Cao | A1 |
|  | Do not isw here, mark their final answer. |  |  |
|  |  |  | (2) |
| (b) | $\begin{aligned} & (x+3)(3 x-1)[=0] \\ & \Rightarrow x=-3 \text { and } \frac{1}{3} \end{aligned}$ | M1: Attempt to solve the quadratic to obtain two critical values | M1A1 |
|  |  | A1: $x=-3$ and $\frac{1}{3}$ (may be implied by their inequality). Allow all equivalent fractions for -3 and $1 / 3$. (Allow 0.333 for $1 / 3$ ) |  |
|  | $-3<x<\frac{1}{3}$ | M1: Chooses "inside" region (The letter $x$ does not need to be used here) | M1A1ft |
|  |  | A1ft: Allow $x<\frac{1}{3}$ and $x>-3$ or $\left(-3, \frac{1}{3}\right)$ or $x<\frac{1}{3} \cap x>-3$. Follow through their critical values. (must be in terms of $x$ here) Allow all equivalent fractions for -3 and $1 / 3$. <br> Both ( $x<\frac{1}{3}$ or $x>-3$ ) and ( $x<\frac{1}{3}, x>-3$ ) as a final answer score A0. |  |
|  |  |  | (4) |
|  |  |  | [6] |
|  | Note that use of $\leq$ or $\geq$ appearing in an otherwise correct answer in (a) or (b) should only be penalised once, the first time it occurs. |  |  |


| Question <br> Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 6 | $(-1,3) \quad, \quad(11,12)$ |  |  |
| (a) | $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{12-3}{11-(-1)},=\frac{3}{4}$ | M1:Correct method for the gradient <br> A1: Any correct fraction or decimal | M1,A1 |
|  | $\begin{gathered} y-3=3 / 4(x+1) \text { or } y-12=3 / 4(x-11) \\ \text { or } y=3 / 4 x+c \text { with attempt at } \\ \text { substitution to find } c \end{gathered}$ | Correct straight line method using either of the given points and a numerical gradient. | M1 |
|  | $4 y-3 x-15=0$ | Or equivalent with integer coefficients (= 0 is required) | A1 |
|  | This A1 should only be awarded in (a) |  |  |
|  |  |  | (4) |
| (a) Way 2 | $\frac{y-y_{1}}{y_{2}-y_{1}}=\frac{x-x_{1}}{x_{2}-x_{1}} \Rightarrow \frac{y-3}{12-3}=\frac{x+1}{11+1}$ | M1: Use of a correct formula for the straight line | M1A1 |
|  |  | A1: Correct equation |  |
|  | $12(y-3)=9(x+1)$ | Eliminates fractions | M1 |
|  | $4 y-3 x-15=0$ | Or equivalent with integer coefficients (= 0 is required) | A1 |
|  |  |  | (4) |
| (b) | Solves their equation from part (a) and $L_{2}$ simultaneously to eliminate one variable | Must reach as far as an equation in $x$ only or in $y$ only. (Allow slips in the algebra) | M1 |
|  | $x=3$ or $y=6$ | One of $x=3$ or $y=6$ | A1 |
|  | Both $x=3$ and $y=6$ | Values can be un-simplified fractions. | A1 |
|  | Fully correct answers with no working can score 3/3 in (b) |  |  |
|  |  |  | (3) |
| (b)$\text { Way } 2$ | $\begin{aligned} & (-1,3) \rightarrow-a+3 b+c=0 \\ & (11,12) \rightarrow 11 a+12 b+c=0 \end{aligned}$ | Substitutes the coordinates to obtain two equations | M1 |
|  | $\therefore a=-\frac{3}{4} b, b=-\frac{4}{15} c$ | Obtains sufficient equations to establish values for $a, b$ and $c$ | A1 |
|  | e.g. $c=1 \Rightarrow b=-\frac{4}{15}, \quad a=\frac{3}{15}$ | Obtains values for $a, b$ and $c$ | M1 |
|  | $\frac{3}{15} x-\frac{4}{15} y+1=0 \Rightarrow 4 y-3 x-15=0$ | Correct equation | A1 |
|  |  |  | (4) |
|  |  |  | [7] |


| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 7(a) | $600=200+(N-1) 20 \Rightarrow N=.$. | Use of 600 with a correct formula in an attempt to find $N$. A correct formula could be implied by a correct answer. | M1 |
|  | $N=21$ | cso | A1 |
|  | Accept correct answer only. |  |  |
|  | $\begin{gathered} 600=200+20 \mathrm{~N} \Rightarrow N=20 \text { is M0A0 (wrong formula) } \\ \frac{600-200}{20}=20 \therefore N=21 \text { is M1A1 (correct formula implied) } \end{gathered}$ |  |  |
|  | Listing: All terms must be listed up to 600 and 21 correctly identified. A solution that scores 2 if fully correct and 0 otherwise. |  |  |
|  |  |  | (2) |
| (b) | Look for an AP first: |  |  |
|  | $S=\frac{21}{2}(2 \times 200+20 \times 20) \text { or } \frac{21}{2}(200+600)$ <br> or $\begin{gathered} S=\frac{20}{2}(2 \times 200+19 \times 20) \text { or } \frac{20}{2}(200+580) \\ (=8400 \text { or } 7800) \end{gathered}$ | M1: Use of correct sum formula with their integer $n=N$ or $N-1$ from part (a) where $3<N<52$ and $a=200$ and $d=20$. <br> A1: Any correct un-simplified numerical expression with $n=20$ or $n=21$ (No follow through here) | M1A1 |
|  | Then for the constant terms: |  |  |
|  | $600 \times(52-" N ")(=18600)$ | M1: $600 \times k$ where $k$ is an integer and $3<k<52$ |  |
|  |  | A1: A correct un-simplified follow through expression with their $k$ consistent with $n$ so that $n+k=52$ | M1A1ft |
|  | So total is 27000 | Cao | A1 |
|  | Note that for the constant terms, they may correctly use an AP sum with $d=0$. |  |  |
|  | There are no marks in (b) for just finding $\mathbf{S}_{52}$ |  |  |
|  |  |  | (5) |
|  |  |  | [7] |
|  | If they obtain $N=20$ in (a) ( $0 / 2$ ) and then in (b) proceed with, $S=\frac{20}{2}(2 \times 200+19 \times 20)+32 \times 600=7800+19200=27000$ <br> allow them to 'recover' and score full marks in (b) Similarly <br> If they obtain $N=22$ in (a) ( $0 / 2$ ) and then in (b) proceed with, $S=\frac{21}{2}(2 \times 200+20 \times 20)+31 \times 600=8400+18600=27000$ <br> allow them to 'recover' and score full marks in (b) |  |  |



| Question <br> Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 9 (a) | $\left(3-x^{2}\right)^{2}=9-6 x^{2}+x^{4}$ | An attempt to expand the numerator obtaining an expression of the form $9 \pm p x^{2} \pm q x^{4}, \quad p, q \neq 0$ | M1 |
|  | $9 x^{-2}+x^{2}$ | Must come from $\frac{9+x^{4}}{x^{2}}$ | A1 |
|  | -6 | Must come from $\frac{-6 x^{2}}{x^{2}}$ | A1 |
|  | Alternative 1: Writes $\frac{\left(3-x^{2}\right)^{2}}{x^{2}}$ as $\left(3 x^{-1}-x\right)^{2}$ and attempts to expand $=$ M1 then A1A1 as in the scheme. |  |  |
|  | Alternative 2: Sets $\left(3-x^{2}\right)^{2}=9+A x^{2}+B x^{4}$, expands $\left(3-x^{2}\right)^{2}$ and compares coefficients $=$ M1 then A1A1 as in the scheme. |  |  |
|  |  |  | (3) |
|  | $\left(\mathrm{f}^{\prime}(x)=9 x^{-2}-6+x^{2}\right)$ |  |  |
| (b) | $-18 x^{-3}+2 x$ | M1: $x^{n} \rightarrow x^{n-1}$ on separate terms at least once. Do not award for $A \rightarrow 0$ (Integrating is M0) | M1 A1ft |
|  |  | A1ft: $-18 x^{-3}+2$ " $B^{\prime \prime} x$ with a numerical $B$ and no extra terms. (A may have been incorrect or even zero) |  |
|  |  |  | (2) |
| (c) | $\mathrm{f}(x)=-9 x^{-1}-6 x+\frac{x^{3}}{3}(+c)$ | M1: $x^{n} \rightarrow x^{n+1}$ on separate terms at least once. (Differentiating is M0) | M1A1ft |
|  |  | A1ft: $-9 x^{-1}+A x+\frac{B x^{3}}{3}(+c)$ with numerical $A$ and $B, A, B \neq 0$ |  |
|  | $\begin{aligned} & 10=\frac{-9}{-3}-6(-3)+\frac{(-3)^{3}}{3}+c \text { so } c \\ & =\ldots \end{aligned}$ | Uses $x=-3$ and $y=10$ in what they think is $\mathrm{f}(x)$ (They may have differentiated here) but it must be a changed function i.e. not the original $\mathrm{f}^{\prime}(x)$, to form a linear equation in $c$ and attempts to find $c$. No $+c$ gets M0 and A0 unless their method implies that they are correctly finding a constant. | M1 |
|  | $c=-2$ | CSO | A1 |
|  | $(f(x)=)-9 x^{-1}-6 x+\frac{x^{3}}{3}+$ their <br> c | Follow through their $c$ in an otherwise (possibly un-simplified) correct expression. $\text { Allow }-\frac{9}{x} \text { for }-9 x^{-1} \text { or even } \frac{9 x^{-1}}{-1} .$ | A1ft |
|  | Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available. |  |  |
|  |  |  | (5) |
|  |  |  | [10] |


| Question <br> Number | Scheme | Marks |  |
| :---: | :---: | :---: | :---: |
| 10(a) | $x^{2}-4 k(1-2 x)+5 k(=0)$ | Makes $y$ the subject from the first equation and substitutes into the second equation ( $=0$ not needed here) or eliminates $y$ by a correct method. | M1 |
|  | So $x^{2}+8 k x+k=0$ * | Correct completion to printed answer. There must be no incorrect statements. | A1cso |
|  |  |  | (2) |
| (b) | $(8 k)^{2}-4 k$ | M1: Use of $b^{2}-4 a c$ (Could be in the quadratic formula or an inequality, $=0$ not needed yet). There must be some correct substitution but there must be no $x$ 's. No formula quoted followed by e.g. $8 k^{2}-4 k=0 \text { is } \mathrm{M} 0$ <br> A1: Correct expression. Do not condone missing brackets unless they are implied by later work but can be implied by $(8 k)^{2}>4 k$ etc. | M1 A1 |
|  | $k=\frac{1}{16}(\mathrm{oe})$ | Cso (Ignore any reference to $k=0$ ) but there must be no contradictory earlier statements. A fully correct solution with no errors. | A1 |
|  |  |  | (3) |
| (b) <br> Way 2 <br> Equal <br> roots | $\begin{gathered} \Rightarrow x^{2}+8 k x+k=(x+\sqrt{k})^{2} \\ \Rightarrow 8 k=2 \sqrt{k} \end{gathered}$ | M1: Correct strategy for equal roots | M1A1 |
|  |  | A1: Correct equation |  |
|  | $k=\frac{1}{16}$ (oe) | Cso (Ignore any reference to $k=0$ ) | A1 |
| (b) <br> Way 3 | Completes the Square$\begin{aligned} & x^{2}+8 k x+k=(x+4 k)^{2}-16 k^{2}+k \\ & \Rightarrow 16 k^{2}-k=0 \end{aligned}$ | M1: $(x \pm 4 k)^{2} \pm p \pm k, p \neq 0$ | M1A1 |
|  |  | A1: Correct equation |  |
|  | $k=\frac{1}{16}$ (oe) | Cso (Ignore any reference to $k=0$ ) | A1 |
| (c) | $\begin{aligned} & x^{2}+\frac{1}{2} x+\frac{1}{16}=0 \text { so } \\ & \left(x+\frac{1}{4}\right)^{2}=0 \Rightarrow x= \end{aligned}$ |  | (3) |
|  |  | Substitutes their value of $k$ into the given quadratic and attempt to solve their 2 or 3 term quadratic as far as $x=$ (may be implied by substitution into the quadratic formula) or starts again and substitutes their value of $k$ into the second equation and solves simultaneously to obtain a value for $x$. | M1 |
|  | $x=-\frac{1}{4}, y=1 \frac{1}{2}$ | First A1 one answer correct, second A1 both answers correct. | A1A1 |
|  | Special Case: $x^{2}+\frac{1}{2} x+\frac{1}{16}=0 \Rightarrow x=-\frac{1}{4}, \frac{1}{4} \Rightarrow y=1 \frac{1}{2}, \frac{1}{2}$ allow M1A1A0 |  |  |
|  |  |  | (3) |
|  |  |  | [8] |



