## Core Mathematics C1

## Advanced Subsidiary

## Monday 14 January 2013 - Morning

## Time: 1 hour 30 minutes

Materials required for examination<br>Mathematical Formulae (Pink)<br>Items included with question papers Nil

Calculators may NOT be used in this examination.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (2).
There are 11 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. Factorise completely $x-4 x^{3}$.

## (3)

2. Express $8^{2 x+3}$ in the form $2^{y}$, stating $y$ in terms of $x$.

## (2)

3. (i) Express

$$
(5-\sqrt{ } 8)(1+\sqrt{ } 2)
$$

in the form $a+b \sqrt{ } 2$, where $a$ and $b$ are integers.
(ii) Express

$$
\sqrt{80}+\frac{30}{\sqrt{5}}
$$

in the form $c \sqrt{ } 5$, where $c$ is an integer.
4. A sequence $u_{1}, u_{2}, u_{3}, \ldots$, satisfies

$$
u_{n+1}=2 u_{n}-1, \quad n \geq 1 .
$$

Given that $u_{2}=9$,
(a) find the value of $u_{3}$ and the value of $u_{4}$,
(b) evaluate $\sum_{r=1}^{4} u_{r}$.
5. The line $l_{1}$ has equation $y=-2 x+3$.

The line $l_{2}$ is perpendicular to $l_{1}$ and passes through the point $(5,6)$.
(a) Find an equation for $l_{2}$ in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
(3)

The line $l_{2}$ crosses the $x$-axis at the point $A$ and the $y$-axis at the point $B$.
(b) Find the $x$-coordinate of $A$ and the $y$-coordinate of $B$.

Given that $O$ is the origin,
(c) find the area of the triangle $O A B$.
6.


Figure 1
Figure 1 shows a sketch of the curve with equation $y=\frac{2}{x}, x \neq 0$.
The curve $C$ has equation $y=\frac{2}{x}-5, x \neq 0$, and the line $l$ has equation $y=4 x+2$.
(a) Sketch and clearly label the graphs of $C$ and $l$ on a single diagram.

On your diagram, show clearly the coordinates of the points where $C$ and $l$ cross the coordinate axes.
(b) Write down the equations of the asymptotes of the curve $C$.
(c) Find the coordinates of the points of intersection of $y=\frac{2}{x}-5$ and $y=4 x+2$.
7. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.
He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.
(a) Find the number of points that Lewis scored for capturing his 20th spaceship.
(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured $n$ dragons and the total number of points that she scored for capturing all $n$ dragons was 8500 .

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her $n$th dragon,
(c) find the value of $n$.
8.

$$
\frac{\mathrm{d} y}{\mathrm{~d} x}=-x^{3}+\frac{4 x-5}{2 x^{3}}, \quad x \neq 0 .
$$

Given that $y=7$ at $x=1$, find $y$ in terms of $x$, giving each term in its simplest form.
9. The equation

$$
(k+3) x^{2}+6 x+k=5, \text { where } k \text { is a constant, }
$$

has two distinct real solutions for $x$.
(a) Show that $k$ satisfies

$$
k^{2}-2 k-24<0 .
$$

(b) Hence find the set of possible values of $k$.
10.

$$
4 x^{2}+8 x+3 \equiv a(x+b)^{2}+c .
$$

(a) Find the values of the constants $a, b$ and $c$.

## (3)

(b) Sketch the curve with equation $y=4 x^{2}+8 x+3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.
11. The curve $C$ has equation

$$
y=2 x-8 \sqrt{ } x+5, \quad x \geq 0 .
$$

(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$, giving each term in its simplest form.

The point $P$ on $C$ has $x$-coordinate equal to $\frac{1}{4}$.
(b) Find the equation of the tangent to $C$ at the point $P$, giving your answer in the form $y=a x+b$, where $a$ and $b$ are constants.

The tangent to $C$ at the point $Q$ is parallel to the line with equation $2 x-3 y+18=0$.
(c) Find the coordinates of $Q$.

## END

## J anuary 2013 <br> 6663 Core Mathematics C1 <br> Mark Scheme

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1. | $x\left(1-4 x^{2}\right)$ <br> Accept $x\left(-4 x^{2}+1\right)$ or $-x\left(4 x^{2}-1\right)$ or $-x\left(-1+4 x^{2}\right)$ or even $4 x\left(\frac{1}{4}-x^{2}\right)$ or equivalent Factorises quadratic (or initial cubic) into two brackets $x(1-2 x)(1+2 x) \text { or }-x(2 x-1)(2 x+1) \text { or } x(2 x-1)(-2 x-1)$ | B1 <br> M1 <br> A1 <br> [3] |
|  |  | 3 marks |
|  | Notes |  |
|  | B1: Takes out a factor of $x$ or $-x$ or even $4 x$. This line may be implied by correct final answer, but if this stage is shown it must be correct. So $\mathbf{B 0}$ for $x\left(1+4 x^{2}\right)$ <br> M1: Factorises the quadratic resulting from their first factorisation using usual rules (see note 1 in General Principles). e.g. $x(1-4 x)(x-1)$. Also allow attempts to factorise cubic such as $\left(x-2 x^{2}\right)(1+2 x)$ etc N.B. Should not be completing the square here. <br> A1: Accept either $x(1-2 x)(1+2 x)$ or $-x(2 x-1)(2 x+1)$ or $x(2 x-1)(-2 x-1)$. (No fractions for this final answer) |  |
|  | Specific situations |  |
|  | Note: $x\left(1-4 x^{2}\right)$ followed by $x(1-2 x)^{2}$ scores B1M1A0 as factors follow quadratic factorisation criteria And $x\left(1-4 x^{2}\right)$ followed by $x(1-4 x)(1+4 x)$ B1M0A0. |  |
|  | Answers with no working: Correct answer gets all three marks B1M1A1 |  |
|  | : $x(2 x-1)(2 x+1)$ gets B0M1A0 if no working as $x\left(4 x^{2}-1\right)$ would earn B0 |  |
|  | Poor bracketing: e.g. $\left(-1+4 x^{2}\right)-x$ gets B0 unless subsequent work implies bracket round the $-x$ in which case candidate may recover the mark by the following correct work. |  |
|  | N.B. If correct factors are followed by $x=0, x=\frac{1}{2}, x=-\frac{1}{2}$ then ignore this as subsequent work. |  |
|  | But these answers- $x=0, x=\frac{1}{2}, x=-\frac{1}{2}$ - with no working, or no factors, gets B0M0A0. |  |
|  | Ignore " $=0$ " written at the end of lines and mark LHS as in the scheme above. Candidate who changes the question to $4 x^{3}-x=x\left(4 x^{2}-1\right)=x(2 x-1)(2 x+1)$ would earn B0 M1 A0 $1 / 3$ |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 2. | $\left(8^{2 x+3}=\left(2^{3}\right)^{2 x+3}\right)=2^{3(2 x+3)}$ or $2^{a x+b}$ with $a=6$ or $b=9$ $=2^{6 x+9}$ or $=2^{3(2 x+3)}$ as final answer with no errors or $(y=) 6 x+9$ or $3(2 x+3)$ | M1 <br> A1 <br> [2] |
|  |  | 2 marks |
|  | Notes |  |
|  | M1: Uses $8=2^{3}$, and multiplies powers $3(2 x+3)$. Does not add powers. ( Just $8=2^{3}$ or $8^{\frac{1}{3}}=2$ is M0 ) A1: Either $2^{6 x+9}$ or $=2^{3(2 x+3)}$ or $\quad(y=) 6 x+9$ or $3(2 x+3)$ |  |
|  | Note: Examples: $2^{6 x+3}$ scores M1A0 $: 8^{2 x+3}=\left(2^{3}\right)^{2 x+3}=2^{3+2 x+3} \text { gets M0A0 }$ <br> Special case: : $\quad=2^{6 x} 2^{9}$ without seeing as single power M1A0 <br> Alternative method using logs: $8^{2 x+3}=2^{y} \Rightarrow(2 x+3) \log 8=y \log 2 \Rightarrow y=\frac{(2 x+3) \log 8}{\log 2}$ <br> So $(y=) 6 x+9$ or $3(2 x+3)$ | M1 <br> A1 [2] |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 3. (i) | $\begin{aligned} & (5-\sqrt{8})(1+\sqrt{2}) \\ = & 5+5 \sqrt{2}-\sqrt{8}-4 \\ = & 5+5 \sqrt{2}-2 \sqrt{2}-4 \\ = & 1+3 \sqrt{2} \end{aligned} \quad \sqrt{8}=2 \sqrt{2} \text {, seen or implied at any point. }$ | $\begin{array}{\|lr} \text { M1 } & \\ \text { B1 } & \\ \text { A1 } & \text { [3] } \end{array}$ |
| (ii) | $\begin{array}{lll} \hline \text { Method 1 } & \text { Method 2 } \\ \text { Either } & \sqrt{80}+\frac{30}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right) & \text { Or }\left(\frac{\sqrt{400}+3}{\sqrt{5}}\right. \\ =4 \sqrt{5}+\ldots & =\left(\frac{20+. .}{. .}\right) . . \\ =4 \sqrt{5}+6 \sqrt{5} & =\left(\frac{50 \sqrt{5}}{5}\right) \\ & =10 \sqrt{5} \end{array}$ | $\begin{array}{ll}\text { M1 } \\ \text { B1 } & \\ \\ \\ \text { A1 } & \\ & \\ & \\ \end{array}$ |
| Alternative for (i) for (i) | $\begin{array}{rr} \hline(5-2 \sqrt{2})(1+\sqrt{2}) & \text { This earns the B1 mark and is entered on epen as B1 } \\ =5+5 \sqrt{2}-2 \sqrt{2}-2 \sqrt{2} \sqrt{2} & \text { Multiplies out correctly with } 2 \sqrt{2} \text {. This may be seen } \\ \text { or implied and may be simplified } \\ \text { e.g. }=5+3 \sqrt{2}-2 \sqrt{4} \text { o.e. } \\ =1+3 \sqrt{2} & \text { For earlier use of } 2 \sqrt{2} \\ 1+3 \sqrt{2} \text { or } a=1 \text { and } b=3 . \end{array}$ | $\begin{aligned} & \text { M1 } \\ & \\ & \text { B1 } \\ & \text { A1 [3] } \\ & 6 \text { marks } \\ & \hline \end{aligned}$ |
|  | Notes |  |
| (i) (ii) | M1: Multiplies out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) - can appear as table <br> B1: $\sqrt{8}=2 \sqrt{2}$, seen or implied at any point <br> A1: Fully and correctly simplified to $1+3 \sqrt{2}$ or $a=1$ and $b=3$. <br> M1: Rationalises denominator i.e. Multiplies $\left(\frac{k}{\sqrt{5}}\right)$ by $\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ or $\left(\frac{-\sqrt{5}}{-\sqrt{5}}\right)$, seen or implied or uses <br> Method 3 or similar e.g. $\left(\frac{30}{\sqrt{5}}\right)=\frac{6 \times 5}{\sqrt{5}}=6 \sqrt{5}$ <br> B1: (Independent mark) States $\sqrt{80}=4 \sqrt{5}$ Or either $\sqrt{400}=20$ or $\sqrt{80} \sqrt{5}=20$ at any point if they use Method 2. <br> A1: $10 \sqrt{5}$ or $c=10$. |  |
|  | N.B There are other methods e.g. $\sqrt{80}=\frac{20}{\sqrt{5}}$ (B1) then add $\frac{20}{\sqrt{5}}+\frac{30}{\sqrt{5}}=\frac{50}{\sqrt{5}}$ then M1 A1 as before Those who multiply initial expression by $\sqrt{5}$ to obtain $\sqrt{400}+30=20+30=50$ earn M0 B1 A0 |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 4. (a) | $\begin{aligned} & u_{2}=9, u_{n+1}=2 u_{n}-1, \quad n \ldots 1 \\ & u_{3}=2 u_{2}-1=2(9)-1 \quad(=17) \\ & u_{4}=2 u_{3}-1=2(17)-1=33 \end{aligned}$ $u_{3}=2(9)-1$ <br> Can be implied by $u_{3}=17$ <br> Both $u_{3}=17$ and $u_{4}=33$ | M1 A1 <br> [2] |
| (b) | $\begin{aligned} & \sum_{r=1}^{4} u_{r}=u_{1}+u_{2}+u_{3}+u_{4} \\ & \left(u_{1}\right)=5 \\ & \sum_{r=1}^{4} u_{r}=" 5 "+9+" 17 "+" 33 "=64 \end{aligned}$ $\left(u_{1}\right)=5$ <br> Adds their first four terms obtained legitimately (see notes below) | B1 <br> (M1 on epen) <br> M1 <br> A1 <br> [3] <br> 5 marks |
|  | Notes |  |
|  | M1: Substitutes 9 into RHS of iteration formula <br> A1: Needs both 17 and 33 (but allow if either or both seen in part (b) ) <br> B1: (Appears as M1 on epen) for $u_{1}=5$ (however obtained - may appear in (a)) May be called $a=5$ <br> M1: Uses their $u_{1}$ found from $u_{2}=2 u_{1}-1$ stated explicitly, or uses $u_{1}=4$ or $5 \frac{1}{2}$, and adds it to $u_{2}$, their $u_{3}$ and their $u_{4}$ only. (See special cases below). <br> There should be no fifth term included. <br> Use of sum of AP is irrelevant and scores M0 <br> A1: 64 |  |
|  | Note: Special cases: A candidate who adds $u_{2}, u_{3}, u_{4}$ and $u_{5}$ scores B0M0A0. (M0M0A0 on epen) Such candidates will usually give a final answer of $9+17+33+65=124$. <br> Candidates who invent an arbitrary (wrong) value for $u_{1}$ will also score B0 M0 A0. (M0M0A0 on epen) Uses $u_{1}=4$ to obtain sum (usually 63) get B0 M1 A0 (M0 M1 A0 on epen) <br> Uses $u_{1}=5 \frac{1}{2}$ to obtain sum (usually $64 \frac{1}{2}$ ) also get B0 M1 A0 (M0 M1 A0 on epen) |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| (a) | Gradient of $l_{2}$ is $\quad \frac{1}{2}$ or 0.5 or $\frac{-1}{-2}$ | B1 |
|  | Either $y-6=" \frac{1}{2} "(x-5) \quad$ or $y=" \frac{1}{2} " x+c$ and $6=" \frac{1}{2} "(5)+c \Rightarrow c=\left(" \frac{7}{2} "\right)$ $x-2 y+7=0$ or $-x+2 y-7=0$ <br> or $k(x-2 y+7)=0$ with $\boldsymbol{k}$ an integer | M1 <br> A1 <br> [3] |
|  | Puts $x=0$, or $y=0$ in their equation and solves to find appropriate co-ordinate | M1 |
| (b) | $x$-coordinate of $A$ is -7 and $y$-coordinate of $B$ is $\frac{7}{2}$. | A1 сао |
|  | Applies $\pm \frac{1}{2}$ (base)(height) | M1 |
| (c) | Area $O A B=\frac{1}{2}(7)\left(\frac{7}{2}\right)=\frac{49}{4}(\text { units })^{2} \quad \frac{49}{4}$ | A1cso |
|  |  | [2] |
|  |  | 7 marks |
|  | Notes |  |
| (a) (b) (c) | B1: Must have $1 / 2$ or 0.5 or $\frac{-1}{-2}$ o.e. stated and stops, or used in their line equation <br> M1: Full method to obtain an equation of the line through $(5,6)$ with their " $m$ ". So $y-6=m(x-5)$ with their gradient or uses $y=m x+c$ with $(5,6)$ and their gradient to find $c$. Allow any numerical gradient here including -2 or -1 but not zero . (Allow (6,5) as a slip if $y-y_{1}=m\left(x-x_{1}\right)$ is quoted first ) <br> A1: Accept any multiple of the correct equation, provided that the coefficients are integers and equation $=0$ e.g. $-x+2 y-7=0$ or $k(x-2 y+7)=0$ or even $2 y-x-7=0$ <br> M1: Either one of the $x$ or $y$ coordinates using their equation <br> A1: Needs both correct values. Accept any correct equivalent.. Need not be written as co-ordinates. Even just -7 and 3.5 with no indication which is which may be awarded the A1. <br> M1: Any correct method for area of triangle $A O B$, with their values for co-ordinates of $A$ and $B$ (may include negatives) Method usually half base times height but determinants could be used. <br> A1: Any exact equivalent to 49/4, e.g. 12.25. (negative final answer is A0 but replacing by positive is A1) Do not need units. <br> c.s.o. implies if A0 is scored in (b) then A0 is scored in (c) as well. However if candidate has correct line equation in (a) of wrong form may score A0 in (a) and A1 in (b) and (c) |  |
|  | Note: Special cases: $\frac{1}{2}(-7)\left(+\frac{7}{2}\right)=-\frac{49}{4}$ (units) ${ }^{2}$ is M1 A0 but changing sign to area $=+\frac{49}{4}$ gets M1A1 (recovery) <br> N.B. Candidates making sign errors in (b) and obtaining +7 and $-\frac{7}{2}$. may also get $\frac{49}{4}$ as their answer following previous errors. They should be awarded A0 as this answer is not ft and is for correct solution only <br> Special Case: In (a) and (b): Produces parallel line instead of perpendicular line: So uses $m=-2$ This is not treated as a misread as it simplifies the question. The marks will usually be B0 M1 A0, M1 A0, M1 A0 i.e. maximum of 3/7 |  |


| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 6. (a) |  | $y=\frac{2}{x}$ is translated up or down. | M1 |
|  |  | $y=\frac{2}{x}-5$ is in the correct position. | A1 |
|  | $\longrightarrow$ - | Intersection with $x$-axis at $\left(\frac{2}{5},\{0\}\right)$ only Independent mark. | B1 |
|  |  | $y=4 x+2$ : attempt at straight line, with positive gradient with positive $y$ intercept. | B1 |
|  | Check graph in question for possible answers and space below graph for answers to part (b) | Intersection with $x$-axis at $\left(-\frac{1}{2},\{0\}\right)$ and $y$-axis at $(\{0\}, 2)$. | B1 [5] |
| (b) | Asymptotes : $x=0$ (or $y$-axis) and $y=-5$. (Lose second B mark for extra asymptotes) | An asymptote stated correctly. Independent of (a) These two lines only. Not ft their graph. | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ |
| (c) | (Lose second B mark for extra asymptotes) <br> Method 1: $\frac{2}{x}-5=4 x+2$ | Method 2: $\quad \frac{y-2}{4}=\frac{2}{y+5}$ | M1 |
|  | $\begin{aligned} & 4 x^{2}+7 x-2=0 \Rightarrow x= \\ & x=-2, \frac{1}{4} \end{aligned}$ <br> When $x=-2, y=-6$, When $x=\frac{1}{4}, y=3$ | $\begin{aligned} & y^{2}+3 y-18=0 \rightarrow y= \\ & y=-6,3 \end{aligned}$ | dM1 <br> A1 |
|  |  | When $y=-6, x=-2$ When $y=3, x=\frac{1}{4}$. | M1A1 <br> [5] |
|  |  |  | 12 marks |
|  | Notes |  |  |

(a) M1: Curve implies $y$ axis as asymptote and does not change shape significantly. Changed curve needs horizontal asymptote (roughly) Asymptote(s) need not be shown but shape of curve should be implying asymptote(s) parallel to $x$ axis. Curve should not remain where it was in the given figure. Both sections move in the same direction. There should be no reflection
A1: Crosses positive $x$ axis. Hyperbola has moved down. Both sections move by almost same amount. See sheet on page 19 for guidance.
B1: Check diagram and text of answer. Accept $2 / 5$ or 0.4 shown on $x$-axis or $x=2 / 5$, or $(2 / 5,0)$ stated clearly in text or on graph. This is independent of the graph. Accept $(0,2 / 5)$ if clearly on $x$ axis. Ignore any intersection points with $y$ axis. Do not credit work in table of values for this mark.
B1: Must be attempt at astraight line, with positive gradient \& with positive $y$ intercept (need not cross $x$ axis)
B1: Accept $x=-1 / 2$, or -0.5 shown on $x$-axis or $(-1 / 2,0)$ or $(-0.5,0)$ in text or on graph and similarly accept 2 on $y$ axis or $y=2$ or ( 0,2 ) in text or on graph. Need not cross curve and allow on separate axes.
(b) B1: For either correct asymptote equation. Second B1: For both correct (lose this if extras e.g. $x= \pm 1$ are given also). These asymptotes may follow correctly from equation after wrong graph in (a)
Just $y=-5$ is B1 B0 This may be awarded if given on the graph. However for other B mark it must be clear that $x=0$ (or the $y$-axis) is an asymptote. NB $x \neq 0, y \neq-5$ is B1B0
(c) M1: Either of these equations is enough for the method mark (May appear labelled as part (b))
dM1: Attempt to solve a 3 term quadratic by factorising, formula, completion of square or implied by correct answers.
(see note 1) This mark depends on previous mark.
A1: Need both correct $x$ answers (Accept equivalents e.g. 0.25) or both correct $y$ values (Method 2)
M1: At least one attempt to find second variable (usually $y$ ) using their first variable (usually $x$ ) related to line meeting curve. Should not be substituting $x$ or $y$ values from part (a) or (b). This mark is independent of previous marks.
Candidate may substitute in equation of line or equation of curve.
A1: Need both correct second variable answers Need not be written as co-ordinates (allow as in the scheme)
Note: Special case: Answer only with no working in part (c) can have 5 marks if completely correct, with both points found. If coordinates of just one of the points is correct - with no working - this earns M0 M0 A0 M1 A0 (i.e. 1/5)


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 8. | $\left(\frac{\mathrm{d} y}{\mathrm{~d} x}=\right) \quad-x^{3}+22^{\prime \prime} x^{-2}-2\left(\frac{5}{2}\right)$ "x ${ }^{-3}$ | M1 |
|  | $(y=) \quad-\frac{1}{4} x^{4}+\frac{" 2 " x^{-1}}{(-1)}-"\left(\frac{5}{2}\right) " \frac{x^{-2}}{(-2)}(+c) \quad \begin{array}{r\|r} \text { Raises power correctly on any one term. } \\ \text { Any two follow through terms correct. } \end{array}$ | M1 A1ft |
|  | $(y=) \quad-\frac{1}{4} x^{4}+\frac{2 x^{-1}}{(-1)}-\frac{5}{2} \frac{x^{-2}}{(-2)}(+c) \quad \text { This is not follow through }- \text { must be correct }$ | A1 |
|  | Given that $y=7$, at $x=1$, then $7=-\frac{1}{4}-2+\frac{5}{4}+c \Rightarrow c=$ | M1 |
|  | So, $(y=) \quad-\frac{1}{4} x^{4}-2 x^{-1}+\frac{5}{4} x^{-2}+c, \quad c=8 \quad$ or $(y=)-\frac{1}{4} x^{4}-2 x^{-1}+\frac{5}{4} x^{-2}+8$ | A1 |
|  |  | [6] |
|  |  | 6 marks |
|  | Notes |  |
|  | M1: Expresses as three term polynomial with powers 3, -2 and -3 . Allow slips in coefficients. This may be implied by later integration having all three powers $4,-1$ and -2 . <br> M1: An attempt to integrate at least one term so $x^{n} \rightarrow x^{n+1}$ (not a term in the numerator or denominator) <br> A1ft: Any two integrations are correct - coefficients may be unsimplified (follow through errors in coefficients only here) so should have two of the powers $4,-1$ and -2 after integration - depends on $2^{\text {nd }}$ method mark only. There should be a maximum of three terms here. <br> A1: Correct three terms - coefficients may be unsimplified- do not need constant for this mark Depends on both Method marks <br> M1: Need constant for this mark. Uses $y=7$ and $x=1$ in their changed expression in order to find $c$, and attempt to find $c$. This mark is available even after there is suggestion of differentiation. <br> A1: Need all four correct terms to be simplified and need $c=8$ here. |  |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 9. (a) | $\begin{array}{ll} \hline \text { Method 1: } & \text { Attempts } b^{2}-4 a c \text { for } a=(k+3), b=6 \text { and their } c . \quad c \neq k \\ b^{2}-4 a c=6^{2}-4(k+3)(k-5) \\ \left(b^{2}-4 a c=\right) \quad-4 k^{2}+8 k+96 \quad \text { or }-\left(b^{2}-4 a c=\right) \quad 4 k^{2}-8 k-96 \text { (with no prior algebraic } \\ \text { errors) } \\ \text { As } b^{2}-4 a c>0, \text { then }-4 k^{2}+8 k+96>0 \quad \text { and so, } k^{2}-2 k-24<0 \end{array}$ | M1 <br> A1 <br> B1 <br> (M1 on epen) A1 * |
|  | Method 2: $\quad$ Considers $b^{2}>4 a c$ for $a=(k+3), b=6$ and their $c . \quad c \neq k$ $6^{2}>4(k+3)(k-5)$ $4 k^{2}-8 k-96<0 \text { or }-4 k^{2}+8 k+96>0 \quad \text { or } 9>(k+3)(k-5)$ <br> (with no prior algebraic errors) and so, $k^{2}-2 k-24<0$ following correct work | M1 <br> A1 <br> B1 <br> (M1 on epen) <br> A1 * |
| (b) | Attempts to solve $k^{2}-2 k-24=0$ to give $k=$ ( $\Rightarrow$ Critical values, $k=6,-4$.) $k^{2}-2 k-24<0$ gives $-4<k<6$ | M1 <br> M1 A1 |
|  | Notes |  |
| (a) | Method 1: M1: Attempts $b^{2}-4 a c$ for $a=(k+3), b=6$ and their $c . c \neq k$ or uses quadratic formula and has this expression under square root. (ignore $>0,<0$ or $=0$ for first 3 marks) <br> A1: Correct expression for $b^{2}-4 a c-$ need not be simplified (may be under root sign) <br> B1: Uses algebra to manipulate result without error into one of these three term quadratics. Again may be under root sign in quadratic formula. (This mark is given as second $M$ on epen). If inequality is used early in "proof" may see <br> $4 k^{2}-8 k-96<0$ and B1 would be given for $4 k^{2}-8 k-96$ correctly stated. <br> A1: Applies $b^{2}-4 a c>0$ correctly ( or writes $b^{2}-4 a c>0$ ) to achieve the result given in the question. No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by -1 , or taking every term to other side of inequality. If doubtful send to review. Need conclusion i.e. printed answer. <br> Method 2: M1: Allow $b^{2}>4 a c, b^{2}<4 a c$ or $b^{2}=4 a c$ for $a=(k+3), b=6$ and their $c . c \neq k$ <br> A1: Correct expressions on either side (ignore >, < or =). <br> B1: Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 again without error <br> A1: Produces result with no errors seen from initial consideration of $b^{2}>4 a c$. |  |
| (b) | M1: Uses factorisation, formula, completion of square method to find two values for $k$, or finds two correct answers with no obvious method <br> M1: Their Lower Limit $<k<$ Their Upper Limit . Allow the M mark mark for $\leq$. (Allow $k<$ upper and $k>$ lower) <br> A1: $-4<k<6$ Lose this mark for $\leq$ Allow ( $-4,6$ ) [not square brackets] or $k>-4$ and $k<6$ (must be and not or) Can also use intersection symbol $\cap$ NOT $k>-4, k<6$ (M1A0) |  |
|  | Special case : In part (a) uses $c=k$ instead of $k-5$ - scores 0 . Allow $k+5$ for method marks |  |
|  | Special Case: In part (b) Obtaining $-6<k<4$ This is a common wrong answer. Give M1 M1 A0 special case. |  |
|  | Special Case: In part (b) Use of $x$ instead of $k-$ M1M1A0 |  |
|  | Special Case: $-4<k<6$ and $k<-4, k>6$ both given is M0A0 for last two marks. Do not treat as isw. |  |



(c) $\quad$ Special case: Erroneous method Tangent at $Q$ is perpendicular to $2 x-3 y+18=0$

Uses - 3/2
So, " $2-\frac{4}{\sqrt{x}} "="-\frac{3}{2} " \quad$ Sets their gradient function $=$ their numerical gradient.

Substitutes their found $x$ into $y$.

## See next page for notes on graphs in qu 6:

Q6 Examples for Gudance.






MIAO (stops shoutd

