

Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Friday 13 January 2012 – Morning

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for the parts of questions are shown in round brackets, e.g. (2).

There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

1. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(a) $\frac{dy}{dx}$, (3)

(b) $\int y \, dx$. (3)

2. (a) Simplify

$$\sqrt{32} + \sqrt{18},$$

giving your answer in the form $a\sqrt{2}$, where a is an integer. (2)

(b) Simplify

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}},$$

giving your answer in the form $b\sqrt{2} + c$, where b and c are integers. (4)

3. Find the set of values of x for which

(a) $4x - 5 > 15 - x$, (2)

(b) $x(x - 4) > 12$. (4)

4. A sequence x_1, x_2, x_3, \dots is defined by

$$x_1 = 1,$$

$$x_{n+1} = a x_n + 5, \quad n \geq 1,$$

where a is a constant.

- (a) Write down an expression for x_2 in terms of a .

(1)

- (b) Show that $x_3 = a^2 + 5a + 5$.

(2)

Given that $x_3 = 41$

- (c) find the possible values of a .

(3)

5. The curve C has equation $y = x(5 - x)$ and the line L has equation $2y = 5x + 4$.

- (a) Use algebra to show that C and L do not intersect.

(4)

- (b) Sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes.

(4)

6.

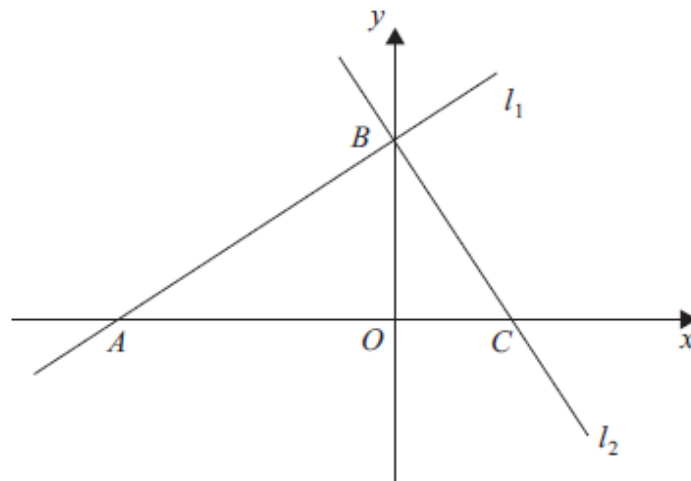


Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$.

(a) Find the gradient of l_1 . (1)

The line l_1 crosses the x -axis at the point A and the y -axis at the point B , as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B .

(b) Find an equation of l_2 . (3)

The line l_2 crosses the x -axis at the point C .

(c) Find the area of triangle ABC . (4)

7. A curve with equation $y = f(x)$ passes through the point $(2, 10)$. Given that

$$f'(x) = 3x^2 - 3x + 5,$$

find the value of $f(1)$. (5)

8. The curve C_1 has equation

$$y = x^2(x + 2).$$

(a) Find $\frac{dy}{dx}$.

(2)

(b) Sketch C_1 , showing the coordinates of the points where C_1 meets the x -axis.

(3)

(c) Find the gradient of C_1 at each point where C_1 meets the x -axis.

(2)

The curve C_2 has equation

$$y = (x - k)^2(x - k + 2),$$

where k is a constant and $k > 2$.

(d) Sketch C_2 , showing the coordinates of the points where C_2 meets the x and y axes.

(3)

9. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is $\pounds P$.

Salary increases by $\pounds(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is $\pounds(P + 1800)$.

Salary increases by $\pounds T$ each year, forming an arithmetic sequence.

(a) Show that the total earned under Salary Scheme 1 for the 10-year period is

$$\pounds(10P + 90T).$$

(2)

For the 10-year period, the total earned is the same for both salary schemes.

(b) Find the value of T .

(4)

For this value of T , the salary in Year 10 under Salary Scheme 2 is $\pounds 29\,850$.

(c) Find the value of P .

(3)

10.

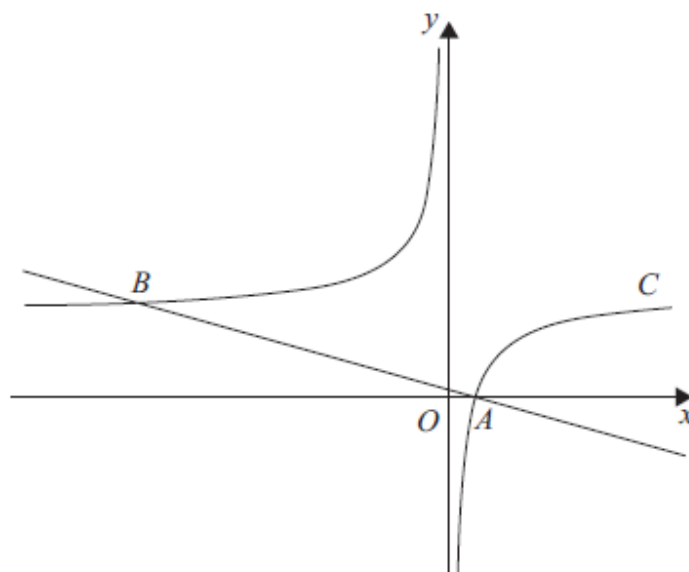


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \quad x \neq 0.$$

The curve crosses the x -axis at the point A .

(a) Find the coordinates of A . (1)

(b) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0. \quad (6)$$

The normal to C at A meets C again at the point B , as shown in Figure 2.

(c) Find the coordinates of B . (4)

TOTAL FOR PAPER: 75 MARKS

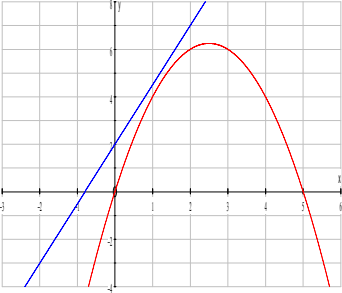
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January 2012
C1 6663
Mark Scheme

Question	Scheme	Marks
1.		
(a)	$4x^3 + 3x^{-\frac{1}{2}}$	M1A1A1 (3)
(b)	$\frac{x^5}{5} + 4x^{\frac{3}{2}} + C$	M1A1A1 (3)
Notes		6 marks
(a)	<p>M1 for $x^n \rightarrow x^{n-1}$ i.e. x^3 or $x^{-\frac{1}{2}}$ seen</p> <p>1st A1 for $4x^3$ <u>or</u> $6 \times \frac{1}{2} \times x^{-\frac{1}{2}}$ (o.e.) (ignore any + c for this mark)</p> <p>2nd A1 for simplified terms i.e. <u>both</u> $4x^3$ <u>and</u> $3x^{-\frac{1}{2}}$ or $\frac{3}{\sqrt{x}}$ and no +c $\left[\frac{3}{1} x^{-\frac{1}{2}} \text{ is A0} \right]$</p> <p>Apply ISW here and award marks when first seen</p>	
(b)	<p>M1 for $x^n \rightarrow x^{n+1}$ applied to y only so x^5 or $x^{\frac{3}{2}}$ seen. Do not award for integrating their answer to part (a)</p> <p>1st A1 for $\frac{x^5}{5}$ or $\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$ (or better). Allow $1/5x^5$ here but not for 2nd A1</p> <p>2nd A1 for fully correct and simplified answer with +C. Allow $(1/5)x^5$ If + C appears earlier but not on a line where 2nd A1 could be scored then A0</p>	

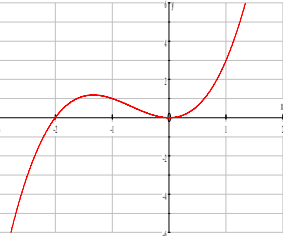

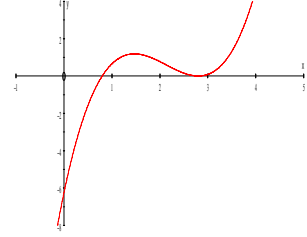
Question	Scheme	Marks
<p>3. (a)</p> <p>(b)</p>	<p>$5x > 20$</p> <p style="text-align: center;"><u>$x > 4$</u></p> <p>$x^2 - 4x - 12 = 0$ $(x+2)(x-6) [= 0]$</p> <p style="text-align: center;">$x = 6, -2$ $x < -2, x > 6$</p>	<p>M1 A1 (2)</p> <p>M1 A1 M1, A1ft (4)</p> <p>6 marks</p>
Notes		
	<p>(a) M1 for reducing to the form $px > q$ with one of p or q correct Using $px = q$ is M0 unless $>$ appears later on A1 $x > 4$ only</p> <p>(b) 1st M1 for multiplying out and attempting to solve a 3TQ with at least $\pm 4x$ or ± 12 See General Principles for definitions of “attempt to solve” 1st A1 for 6 and -2 seen. Allow $x > 6, x > -2$ etc to score this mark. Values may be on a sketch. 2nd M1 for choosing the “outside region” for their critical values. Do not award simply for a diagram or table – they must have chosen their “outside” regions 2nd A1ft follow through their 2 distinct critical values. Allow “,” “or” or a “blank” between answers. Use of “and” is M1A0 i.e. loses the final A1 $-2 > x > 6$ scores M1A0 i.e. loses the final A1 but apply ISW if $x > 6, x < -2$ has been seen Accept $(-\infty, -2) \cup (6, \infty)$ (o.e) Use of \leq instead of $<$ (or \geq instead of $>$) loses the final A mark in (b) unless A mark was lost in (a) for $x \geq 4$ in which case allow it here.</p>	

Question	Scheme	Marks
<p>4. (a)</p> <p>(b)</p> <p>(c)</p>	<p>$(x_2 =) a + 5$</p> <p>$(x_3) = a(a+5) + 5$ $= a^2 + 5a + 5 \quad (*)$</p> <p>$41 = a^2 + 5a + 5 \Rightarrow a^2 + 5a - 36 (= 0)$ or $36 = a^2 + 5a$ $(a + 9)(a - 4) = 0$ $a = 4$ or -9</p>	<p>B1 (1)</p> <p>M1 A1cso (2)</p> <p>M1 M1 A1 (3)</p> <p>6 marks</p>
Notes		
<p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>B1 accept $a + 5$ or $1 \times a + 5$ (etc)</p> <p>M1 must see a (their x_2) + 5 A1cso must have seen $a(a + 5) + 5$ (etc or better) Must have both brackets (...) and no incorrect working seen</p> <p>1st M1 for forming a suitable equation using x_3 and 41 and an attempt to collect like terms and reduce to 3TQ (o.e). Allow one error in sign. Accept for example $a^2 + 5a + 46 (= 0)$ If completing the square should get to $(a \pm \frac{5}{2})^2 = 36 + \frac{25}{4}$</p> <p>2nd M1 Attempting to solve their relevant 3TQ (see General Principles) A1 for both 4 and -9 seen. If $a = 4$ and -9 is followed by $-9 < a < 4$ apply ISW. No working or trial and improvement leading to <u>both</u> answers scores 3/3 but no marks for only one answer. Allow use of other letters instead of a</p>	

Question	Scheme	Marks
<p>5. (a)</p> <p>(b)</p>	$x(5-x) = \frac{1}{2}(5x+4) \quad (\text{o.e.})$ $2x^2 - 5x + 4 (= 0) \quad (\text{o.e.}) \text{ e.g. } x^2 - 2.5x + 2 (= 0)$ $b^2 - 4ac = (-5)^2 - 4 \times 2 \times 4$ $= 25 - 32 < 0, \text{ so no roots } \underline{\text{or}} \text{ no intersections } \underline{\text{or}} \text{ no solutions}$  <p>Curve: \cap shape and passing through (0, 0) \cap shape and passing through (5, 0)</p> <p>Line : +ve gradient and no intersections with C. If no C drawn score B0</p> <p>Line passing through (0, 2) and (-0.8, 0) marked on axes</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (4)</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1 (4)</p> <p>8 marks</p>
Notes		
<p>(a)</p> <p>ALT</p> <p>(b)</p> <p>SC</p>	<p>1st M1 for forming a suitable equation in one variable</p> <p>1st A1 for a correct 3TQ equation. Allow missing “= 0” Accept $2x^2 + 4 = 5x$ etc</p> <p>2nd M1 for an attempt to evaluate discriminant for their 3TQ. Allow for $b^2 > 4ac$ or $b^2 < 4ac$</p> <p>Allow if it is part of a solution using the formula e.g. $(x =) \frac{5 \pm \sqrt{25 - 32}}{4}$</p> <p>Correct formula quoted and some correct substitution or a correct expression</p> <p>False factorising is M0</p> <p>2nd A1 for correct evaluation of discriminant for a correct 3TQ e.g. $25 - 32$ (or better) <u>and</u> a comment indicating no roots or equivalent. For <u>contradictory</u> statements score A0</p> <p>2nd M1 for attempt at completing the square $a \left[\left(x \pm \frac{b}{2a} \right)^2 - q \right] + c$</p> <p>2nd A1 for $\left(x - \frac{5}{4} \right)^2 = -\frac{7}{16}$ and a suitable comment</p> <p>Coordinates must be seen <u>on</u> the diagram. Do not award if only in the body of the script.</p> <p>“Passing through” means <u>not</u> stopping at and <u>not</u> touching.</p> <p>Allow (0, x) and (y, 0) if marked on the correct places on the correct axis.</p> <p>1st B1 for correct shape and passing through origin. Can be assumed if it passes through the intersection of axes</p> <p>2nd B1 for correct shape and 5 marked on x-axis</p> <p>for \cap shape stopping at <u>both</u> (5, 0) <u>and</u> (0, 0) award B0B1</p> <p>3rd B1 for a line of positive gradient that (if extended) has no intersection with their C (possibly extended). Must have both graphs on same axes for this mark. If no C given score B0</p> <p>4th B1 for straight line passing through -0.8 on x-axis and 2 on y-axis</p> <p>Accept exact fraction equivalents to -0.8 or 2 (e.g. $\frac{4}{5}$)</p>	

Question	Scheme	Marks
6. (a)	$(m =) \frac{2}{3}$ (or exact equivalent)	B1 (1)
(b)	B: (0, 4) [award when first seen – may be in (c)] Gradient: $\frac{-1}{m} = -\frac{3}{2}$	B1 M1
(c)	A: (-6, 0) [award when first seen – may be in (b)] C: $\frac{3x}{2} = 4 \Rightarrow x = \frac{8}{3}$ [award when first seen – may be in (b)] Area: Using $\frac{1}{2}(x_C - x_A)y_B$	B1 B1ft
ALT	$= \frac{1}{2}\left(\frac{8}{3} + 6\right)4 = \frac{52}{3} \left(= 17\frac{1}{3}\right)$	M1 A1 cso (4)
ALT	$BC = \frac{4}{6}\sqrt{52}$ (from similar triangles) (or possibly using C) Area: Using $\frac{1}{2}(AB \times BC)$ N.B. $AB = \sqrt{6^2 + 4^2} = \sqrt{52}$ $= \frac{1}{2} \times \sqrt{52} \times \left(\frac{2}{3}\sqrt{52}\right) = \frac{52}{3} \left(= 17\frac{1}{3}\right)$	2 nd B1ft M1 A1
	Notes	8 marks
Use of Det	<p>(a) B1 for $\frac{2}{3}$ seen. Do not award for $\frac{2}{3}x$ and must be in part (a)</p> <p>(b) B1 for coordinates of B. Accept 4 marked on y-axis (clearly labelled) M1 for use of perpendicular gradient rule. Follow through their value for m A1 for a correct equation (any form, need not be simplified). Answer only 3/3</p> <p>(c) 1st B1 for the coordinates of A (clearly labelled). Accept - 6 marked on x-axis 2nd B1ft for the coordinates of C (clearly labelled) or $AC = \frac{26}{3}$. Accept $x = \frac{8}{3}$ marked on x-axis. Follow through from l_2 if >0 M1 for an expression for the area of the triangle (all lengths > 0). Ft their 4, - 6 and $\frac{8}{3}$ A1 cso for $\frac{52}{3}$ or exact equivalent seen but must be a single fraction or $17\frac{1}{3}$ or $17\frac{2}{6}$ etc $17\frac{1}{3}$ on its own can only score full marks if A, B and C are all correct.</p> <p>ALT 2nd B1ft If they use this approach award this mark for C (if seen) or BC</p> <p>2nd M1 must get as far as: $\frac{1}{2} x_A \times y_B - x_C \times y_B$</p>	

Question	Scheme	Marks
7.	$[f(x) = \frac{3x^3}{3} - \frac{3x^2}{2} + 5x + c] \quad \text{or} \quad \left\{ x^3 - \frac{3}{2}x^2 + 5x + c \right\}$ $10 = 8 - 6 + 10 + c$ $c = -2$ $f(1) = 1 - \frac{3}{2} + 5 \quad "-2" = \underline{\underline{\frac{5}{2}}} \quad (\text{o.e.})$	M1A1 M1 A1 A1ft (5) 5 marks
Notes		
1 st M1 for attempt to integrate $x^n \rightarrow x^{n+1}$ 1 st A1 all correct, possibly unsimplified. Ignore +c here. 2 nd M1 for using $x = 2$ <u>and</u> $f(2) = 10$ to form a linear equation in c . Allow sign errors. They should be substituting into a <u>changed</u> expression 2 nd A1 for $c = -2$ 3 rd A1ft for $\frac{9}{2} + c$ Follow through their <u>numerical</u> $c (\neq 0)$ This mark is dependent on 1 st M1 and 1 st A1 only.		

Question	Scheme	Marks
<p>8. (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>[$y = x^3 + 2x^2$] so $\frac{dy}{dx} = 3x^2 + 4x$</p>  <p>Shape </p> <p>Touching x-axis at origin Through and not touching or stopping at -2 on x-axis. Ignore extra intersections.</p> <p>At $x = -2$: $\frac{dy}{dx} = 3(-2)^2 + 4(-2) = 4$</p> <p>At $x = 0$: $\frac{dy}{dx} = 0$ (Both values correct)</p>  <p>Horizontal translation (touches x-axis still) $k - 2$ and k marked on positive x-axis $k^2(2 - k)$ (o.e.) marked on negative y-axis</p>	<p>M1A1 (2)</p> <p>B1 B1 B1 (3)</p> <p>M1 A1 (2)</p> <p>M1 B1 B1 (3)</p> <p>10 marks</p>
Notes		
<p>Prod Rule</p>	<p>(a) M1 for attempt to multiply out and then some attempt to differentiate $x^n \rightarrow x^{n-1}$ Do not award for $2x(x + 2)$ or $2x(1 + 2)$ etc Award M1 for a correct attempt: 2 products with a + and at least one product correct A1 for both terms correct. (If +c or extra term is included score A0)</p> <p>(b) 1st B1 for correct shape (anywhere). Must have 2 clear turning points. 2nd B1 for graph touching at origin (not crossing or ending) 3rd B1 for graph passing through (not stopping or touching at) -2 on x axis and -2 marked on axis</p> <p>SC B0B0B1 for $y = x^3$ <u>or</u> cubic with straight line between $(-2, 0)$ and $(0, 0)$</p> <p>(c) M1 for attempt at $y'(0)$ or $y'(-2)$. Follow through their 0 or -2 and their $y'(x)$ <u>or</u> for a <u>correct</u> statement of zero gradient for an identified point on their curve that touches x-axis A1 for both correct answers</p> <p>(d) For the M1 in part (d) ignore any coordinates marked – just mark the shape. M1 for a horizontal translation of their (b). Should still touch x – axis if it did in (b) <u>Or</u> for a graph of correct shape with min. and intersection in correct order on +ve x-axis 1st B1 for k and $k - 2$ on the positive x-axis. Curve must pass through $k - 2$ and touch at k 2nd B1 for a correct intercept on negative y-axis in terms of k. Allow $(0, 2k^2 - k^3)$ (o.e.) seen in script if curve passes through $-ve$ y-axis</p>	

Question	Scheme	Marks
<p>9. (a)</p> <p>(b)</p> <p>(c)</p>	<p>$S_{10} = \frac{10}{2}[2P + 9 \times 2T]$ <u>or</u> $\frac{10}{2}(P + [P + 18T])$ e.g. $5[2P + 18T]$ <u>=</u> (£) $(10P + 90T)$ <u>or</u> (£) $10P + 90T$ (*)</p> <p>Scheme 2: $S_{10} = \frac{10}{2}[2(P + 1800) + 9T] = \{10P + 18000 + 45T\}$ $10P + 90T = 10P + 18000 + 45T$ $90T = 18000 + 45T$ $T = 400$ (only)</p> <p>Scheme 2, Year 10 salary: $[a + (n - 1)d] = (P + 1800) + 9T$ $P + 1800 + "3600" = 29850$ $P = (\text{£}) \underline{24450}$</p>	<p>M1 A1cso (2)</p> <p>M1A1 M1 A1 (4)</p> <p>B1ft M1 A1 (3)</p> <p>9 marks</p>
Notes		
<p>(a)</p> <p>List</p> <p>(b)</p> <p>List</p> <p>(c)</p> <p>MR</p>	<p>M1 for identifying $a = P$ or $d = 2T$ and attempt at S_{10}. Using $n = 10$ and one of a or d correct. Must see evidence for M mark, at least one line before the answer. A1cso for simplifying to given answer. No incorrect working seen. Do not penalise missing end bracket in working eg $5(2P + 18T)$ M1A1 for a full list seen (with + signs or written in columns) and no incorrect working seen. Any missing terms is M0A0</p> <p>1st M1 for attempting S_{10} for scheme 2 (allow missing (...) brackets e.g. $2P + 1800 + 9T$) Using $n = 10$ and at least one of a or d correct. 1st A1 for a correct expression for S_{10} using scheme 2 (needn't be multiplied out) Allow M1A1 if they reach $10P + 18000 + 45T$ with no incorrect working seen $10P + 18000 + 45T$ with no working is M1A1 2nd M1 for forming an equation using the two sums that would enable P to be eliminated. Follow through their expressions provided P would disappear. 2nd A1 for $T = 400$ Answer only (4/4)</p> <p>B1 for using u_{10} for scheme 2 . Can be $9T$ or follow through their <u>value</u> of T M1 for forming an equation based on u_{10} for scheme 2 and using 29850 and their <u>value</u> of T A1 for 24450 seen Answer only (3/3)</p> <p>If they misread scheme 2 as scheme 1 in part (c) apply MR rule and award B0M1A0 max for an equation based on u_{10} for scheme 1 and using 29850 and their <u>value</u> of T</p>	

Question	Scheme	Marks
<p>10. (a)</p> <p>(b)</p> <p>(c)</p>	$\left(\frac{1}{2}, 0\right)$ $\frac{dy}{dx} = x^{-2}$ <p>At $x = \frac{1}{2}$, $\frac{dy}{dx} = \left(\frac{1}{2}\right)^{-2} = 4$ ($= m$)</p> <p>Gradient of normal $= -\frac{1}{m} \left(= -\frac{1}{4} \right)$</p> <p>Equation of normal: $y - 0 = -\frac{1}{4}\left(x - \frac{1}{2}\right)$</p> $2x + 8y - 1 = 0 \quad (*)$ $2 - \frac{1}{x} = -\frac{1}{4}x + \frac{1}{8}$ $[= 2x^2 + 15x - 8 = 0] \quad \text{or} \quad [8y^2 - 17y = 0]$ <p>$(2x - 1)(x + 8) = 0$ leading to $x = \dots$</p> $x = \left[\frac{1}{2}\right] \text{ or } -8$ $y = \frac{17}{8} \text{ (or exact equivalent)}$	<p>B1 (1)</p> <p>M1A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1cso (6)</p> <p>M1</p> <p>A1</p> <p>A1ft (4)</p> <p>11 marks</p>

Notes

- (a)** B1 accept $x = \frac{1}{2}$ if evidence that $y = 0$ has been used. Can be written on graph. Use ISW
- (b)** 1st M1 for kx^{-2} even if the '2' is not differentiated to zero. If no evidence of $\frac{dy}{dx}$ seen then 0/6
- 1st A1 for x^{-2} (o.e.) only
- 2nd A1 for using $x = 0.5$ to get $m = 4$ (correctly) (or $m = 1/0.25$)
- To score final A1cso must see at least one intermediate equation for the line after $m = 4$
- 2nd M1 for using the perpendicular gradient rule on their m coming from their $\frac{dy}{dx}$
- Their m must be a value not a letter.
- 3rd M1 for using a changed gradient (based on y') and their A to find equation of line
- 3rd A1cso for reaching printed answer with no incorrect working seen.
- Accept $2x + 8y = 1$ or equivalent equations with $\pm 2x$ and $\pm 8y$
- Trial and improvement requires sight of first equation.
- (c)** 1st M1 for attempt to form a suitable equation in one variable. Do not penalise poor use of brackets etc.
- 2nd M1 for simplifying their equation to a 3TQ and attempting to solve. May be \Rightarrow by $x = -8$
- 1st A1 for $x = -8$ (ignore a second value). If found y first allow ft for x if $x < 0$
- 2nd A1ft for $y = \frac{17}{8}$ Follow through their x value in line or curve provided answer is > 0
- This second A1 is dependent on both M marks