Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Wednesday 18 May 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38157A

1. Find the value of

(*a*) $25^{\frac{1}{2}}$,

(1)

(*b*) $25^{-\frac{3}{2}}$.

(2)

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2. Given that $y = 2x^5 + 7 + \dfrac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(*a*) $\dfrac{dy}{dx}$,

(3)

(*b*) $\int y \; dx$.

(4)

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3. The points *P* and *Q* have coordinates (–1, 6) and (9, 0) respectively.

The line *l* is perpendicular to *PQ* and passes through the mid-point of *PQ*.

Find an equation for *l*, giving your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(5)

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4. Solve the simultaneous equations

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

(7)

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5. A sequence a_1, a_2, a_3, …, is defined by

$$a_1 = k,$$
$$a_{n+1} = 5a_n + 3, \quad n \geq 1,$$

where k is a positive integer.

(*a*) Write down an expression for a_2 in terms of k.

(1)

(*b*) Show that $a_3 = 25k + 18$.

(2)

(*c*) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

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6. Given that $\dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(*a*) write down the value of p and the value of q.

(2)

Given that $\dfrac{dy}{dx} = \dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ and that $y = 90$ when $x = 4$,

(*b*) find y in terms of x, simplifying the coefficient of each term.

(5)

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7. $$f(x) = x^2 + (k + 3)x + k,$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

8.

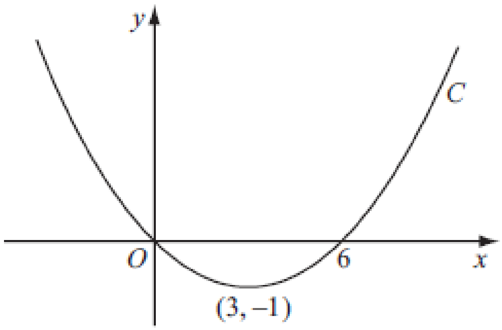

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.
The curve C passes through the origin and through (6, 0).
The curve C has a minimum at the point (3, –1).

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$,

(3)

(b) $y = -f(x)$,

(3)

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$.

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

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9. (*a*) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \text{......} + 100.$$

(3)

(*b*) In the arithmetic series

$$k + 2k + 3k + \text{......} + 100,$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}.$$

(4)

(*c*) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1),\quad (4k + 4),\quad (6k + 7),\ \ldots\ ,$$

giving your answer in its simplest form.

(2)

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10. The curve C has equation

$$y = (x + 1)(x + 3)^2.$$

(*a*) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(*b*) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate –5, lies on C.

(*c*) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(*d*) Find the x-coordinate of B.

(3)

TOTAL FOR PAPER: 75 MARKS

END

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June 2011
Core Mathematics C1 6663
Mark Scheme

Question Number	Scheme	Marks
1. **(a)**	5 (or ± 5)	B1 (1)
(b)	$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}}$ **or** $25^{\frac{3}{2}}=125$ or better	M1
	$\frac{1}{125}$ or 0.008 (or $\pm\frac{1}{125}$)	A1
		(2) **3**
	Notes	
	(a) Give B1 for 5 or ± 5 Anything else is B0 (including just –5)	
	(b) M: Requires reciprocal OR $25^{\frac{3}{2}} = 125$	
	Accept $\frac{1}{5^3}$, $\frac{1}{\sqrt{15625}}$, $\frac{1}{25\times5}$, $\frac{1}{25\sqrt{25}}$, $\frac{1}{\sqrt{25}^3}$ for M1	
	Correct answer with no working (or notation errors in working) scores both marks i.e. M1 A1	
	M1A0 for - $\frac{1}{125}$ without $+\frac{1}{125}$	

Question Number	Scheme	Marks
2. **(a)**	$\frac{dy}{dx} = 10x^4 - 3x^{-4}$ or $10x^4 - \frac{3}{x^4}$	M1 A1 A1 (3)
(b)	$\left(\int =\right) \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} = \frac{x^6}{3} + 7x - \frac{x^{-2}}{2}$ $+ C$	M1 A1 A1 B1 (4) **7**

Notes

(a) M1: Attempt to differentiate $x^n \to x^{n-1}$ (for any of the 3 terms)
i.e. ax^4 or ax^{-4}, where a is any non-zero constant or
the 7 differentiated to give 0 is sufficient evidence for M1

1st A1: One correct (non-zero) term, possibly unsimplified.

2nd A1: Fully correct **simplified** answer.

(b) M1: Attempt to integrate $x^n \to x^{n+1}$

(i.e. ax^6 or ax or ax^{-2}, where a is any non-zero constant).

1st A1: Two correct terms, possibly unsimplified.

2nd A1: All three terms correct and **simplified**.

Allow correct equivalents to printed answer , e.g. $\frac{x^6}{3} + 7x - \frac{1}{2x^2}$ or $\frac{1}{3}x^6 + 7x - \frac{1}{2}x^{-2}$

Allow $\frac{1x^6}{3}$ or $7x^1$

B1: $+ C$ appearing at any stage in part (b) (independent of previous work)

Question Number	Scheme	Marks
3.	Mid-point of PQ is (4, 3)	B1
	PQ: $m = \frac{0-6}{9-(-1)}$, $\left(= -\frac{3}{5}\right)$	B1
	Gradient perpendicular to PQ $= -\frac{1}{m}$ $\left(= \frac{5}{3}\right)$	M1
	$y - 3 = \frac{5}{3}(x-4)$	M1
	$5x - 3y - 11 = 0$ or $3y - 5x + 11 = 0$ or multiples e.g. $10x - 6y - 22 = 0$	A1 (5)
		5
	Notes	
	B1: correct midpoint. B1: correct numerical expression for gradient – need not be simplified 1^{st} M: Negative reciprocal of their numerical value for m 2^{nd} M: Equation of a line through **their** (4, 3) with any gradient except 0 or ∞. If the 4 and 3 are the wrong way round the 2^{nd} M mark can still be given if a correct formula (e.g. $y - y_1 = m(x - x_1)$) is seen, otherwise M0. If (4, 3) is substituted into $y = mx + c$ to find c, the 2^{nd} M mark is for attempting this. A1: Requires integer form with an = zero (see examples above)	

Question Number	Scheme	Marks
4.		
	Either: $y^2 = 4 - 4x + x^2$ — Or: $x^2 = 4 - 4y + y^2$	M1
	Either: $4(4 - 4x + x^2) - x^2 = 11$ or $4(2-x)^2 - x^2 = 11$ — Or: $4y^2 - (4 - 4y + y^2) = 11$ or $4y^2 - (2-y)^2 = 11$	M1
	Either: $3x^2 - 16x + 5 = 0$ — Or: $3y^2 + 4y - 15 = 0$ Correct 3 terms	A1
	Either: $(3x-1)(x-5) = 0, \quad x =$ — Or: $(3y-5)(y+3) = 0, \quad y = ...$	M1
	Either: $x = \frac{1}{3} \quad x = 5$ — Or: $y = \frac{5}{3} \quad y = -3$	A1
	Either: $y = \frac{5}{3} \quad y = -3$ — Or: $x = \frac{1}{3} \quad x = 5$	M1 A1
		(7) **7**

Notes

1st M: Squaring to give 3 or 4 terms (need a middle term)

2nd M: Substitute to give quadratic in one variable (may have just two terms)

3rd M: Attempt to solve a **3 term** quadratic.

4th M: Attempt to find at least one *y* value (or *x* value). (The second variable)

This will be by substitution or by starting again.

If *y* solutions are given as *x* values, or vice-versa, penalise accuracy, so that it is possible to score M1 M1A1 M1 A0 M1 A0.

<u>"Non-algebraic" solutions:</u>

No working, and only one correct solution pair found (e.g. $x = 5, y = -3$):
M0 M0 A0 M1 A0 M1 A0

No working, and both correct solution pairs found, but not demonstrated:
M0 M0 A0 M1 A1 M1 A1

Both correct solution pairs found, and demonstrated: Full marks are possible (send to review)

Question Number	Scheme	Marks
5. **(a)**	$(a_2 =)\ 5k+3$	B1 (1)
(b)	$(a_3 =)\ 5(5k+3)+3$ $= 25k+18$ (*)	M1 A1 cso (2)
(c) **(i)**	$a_4 = 5(25k+18)+3 \quad (=125k+93)$	M1
	$\sum_{r=1}^{4} a_r = k+(5k+3)+(25k+18)+(125k+93)$	M
	$= 156k+114$	A ao
(ii)	$= 6(26k+19)$ (or explain each term is divisible by 6)	A (4) **7**
	Notes (a) $5k+3$ must be seen in (a) to gain the mark (b) 1st M: Substitutes their a_2 into $5a_2+3$ - note the answer is given so working must be seen. (c) 1st M1: Substitutes their a_3 into $5a_3+3$ or uses $125k+93$ 2nd M1: for **their** sum $k+a_2+a_3+a_4$ - must see evidence of **four terms with plus signs and must not be sum of AP** 1st A1: All correct so far 2nd A1ft: Limited ft – previous answer must be divisible by 6 (eg $156k+42$) . This is dependent on second M mark in (c) Allow $\frac{156k+114}{6} = 26k+19$ without explanation. No conclusion is needed.	

Question Number	Scheme	Marks
6.		
(a)	$p=\frac{1}{2},\ q=2$ or $6x^{\frac{1}{2}},\ 3x^2$	B1, B1
		(2)
(b)	$\frac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+\frac{3x^3}{3}$ $\left(=4x^{\frac{3}{2}}+x^3\right)$	M1 A1ft
	$x=4,\ y=90\text{:}\ 32+64+C=90\ \Rightarrow\ C=-6$	M1 A1
	$y=4x^{\frac{3}{2}}+x^3+\text{"}their-6\text{"}$	A1
		(5)
		7
	Notes	
	(a) Accept any equivalent answers, e.g. $p=0.5$, $q=4/2$ (b) 1st M: Attempt to integrate $x^n\to x^{n+1}$ (for either term) 1st A: ft their p and q, but terms need not be simplified (+C not required for this mark) 2nd M: Using x = 4 <u>and</u> y = 90 to form an equation in C. 2nd A: cao 3rd A: answer as shown with simplified correct coefficients and powers – but follow through their value for C If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Numerator and denominator integrated separately</u>: First M mark **cannot** be awarded so only mark available is second M mark. So 1 out of 5 marks.	

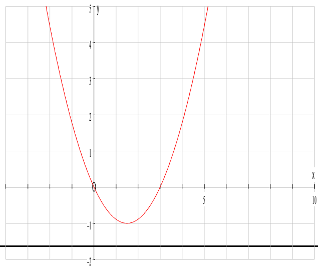
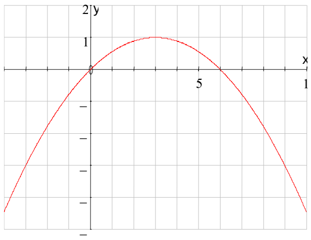
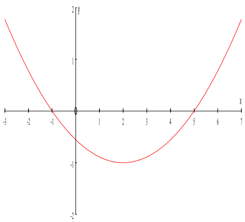
Question Number	Scheme	Marks
7. **(a)**	Discriminant: $b^2 - 4ac = (k+3)^2 - 4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2 - 4k = k^2 + 2k + 9 = (k+1)^2 + 8$	M1 A1 (2)
(c)	For real roots, $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ or $(k+1)^2 + 8 > 0$ $(k+1)^2 \geq 0$ for all k, so $b^2 - 4ac > 0$**,** so roots are real for all k (or equiv.)	M1 A1 cso (2) **6**

Notes

(a) M1: attempt to find discriminant – substitution is required
If formula $b^2 - 4ac$ is seen at least 2 of a, b and c must be correct
If formula $b^2 - 4ac$ is **not** seen all 3 of a, b and c must be correct
Use of $b^2 + 4ac$ is M0
A1: correct unsimplified

(b) M1: Attempt at completion of square (see earlier notes)
A1: both correct (no ft for this mark)

(c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2 + 8$ is greater than 0
A1: The final mark (A1cso) requires $(k+1)^2 \geq 0$ and conclusion. We will allow $(k+1)^2 > 0$ (or word positive) also allow $b^2 - 4ac \geq 0$ and conclusion.

Question Number	Scheme	Marks
8. **(a)**	 Shape ∪ through (0, 0)	B1
	(3, 0)	B1
	(1.5, −1)	B1 (3)
(b)	 Shape ∩	B1
	(0, 0) and (6, 0)	B1
	(3, 1)	B1 (3)
(c)	 Shape ∪ , <u>not</u> through (0, 0)	M1
	Minimum in 4^{th} quadrant	A1
	$(-p, 0)$ and $(6-p, 0)$	B1
	$(3-p, -1)$	B1 (4) **10**

<u>Notes</u>

(a) B1: U shaped parabola through origin
B1: (3,0) stated or 3 labelled on x axis
B1: (1.5, -1) or equivalent e.g. (3/2, -1)

(b) B1: Cap shaped parabola in any position
B1: through origin (may not be labelled) and (6,0) stated or 6 labelled on x - axis
B1: (3,1) shown

(c) M1: U shaped parabola not through origin
A1: Minimum in 4^{th} quadrant (depends on M mark having been given)
B1: Coordinates stated or shown on x axis
B1: Coordinates stated

Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case all minima should be in fourth quadrant)

Question Number	Scheme	Marks
9. **(a)**	Series has 50 terms	B1
	$S=\frac{1}{2}(50)(2+100)=2550$ or $S=\frac{1}{2}(50)(4+49\times 2)=2550$	M1 A1
		(3)
(b) **(i)**	$\frac{100}{k}$	B1
(ii)	Sum: $\frac{1}{2}\left(\frac{100}{k}\right)(k+100)$ or $\frac{1}{2}\left(\frac{100}{k}\right)\left(2k+\left(\frac{100}{k}-1\right)k\right)$	M1 A1
	$=50+\frac{5000}{k}$ (*)	A1 cso
		(4)
(c)	50^{th} term $=a+(n-1)d$	
	$=(2k+1)+49\text{"}(2k+3)\text{"}$ | Or $2k+49(2k)+1+49(3)$	M1
	$=100k+148$ | $=100k+148$	A1
		(2) **9**

Notes

(a) B for seeing attempt to use $n = 50$ or $n = 50$ stated

M for attempt to use $\frac{1}{2}n(a+l)$ or $\frac{1}{2}n(2a+(n-1)d)$ with $a = 2$ and values for other variables (Using $n = 100$ may earn B0 M1A0)

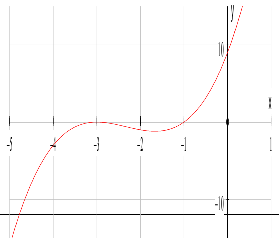
(b) M for use of $a = k$ and $d = k$ or $l = 100$ with their value for n, could be numerical or even letter n in correct formula for sum.

A1: Correct formula with $n = 100/k$

A1: NB Answer is printed – so no slips should have appeared in working

(c) M for use of formula $a+49d$ with $a = 2k + 1$ and with d obtained from difference of terms

A1: Requires this simplified answer

Question Number	Scheme	Marks
10. **(a)**	 Shape (cubic in this orientation) **Touching** x-axis at **–3** **Crossing** at **–1** on x-axis Intersection at **9** on y-axis	B1 B1 B1 B1 (4)
(b)	$y = (x+1)(x^2+6x+9) = x^3+7x^2+15x+9$ or equiv. (possibly unsimplified) Differentiates their polynomial correctly – may be unsimplified $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2+14x+15$ **(*)**	B1 M1 A1 cso (3)
(c)	At $x=-5$: $\frac{\mathrm{d}y}{\mathrm{d}x} = 75-70+15 = 20$ At $x=-5$: $y=-16$ $y-("-16") = "20"(x-(-5))$ or y = "20x" + c with (-5, -"16") used to find c $y = 20x+84$	B1 B1 M1 A1 (4)
(d)	Parallel: $3x^2+14x+15 = "20"$ $(3x-1)(x+5)=0$ $x = \dots$ $x = \frac{1}{3}$	M1 M1 A1 (3) **14**
	Notes (a) Crossing at –3 is B0. Touching at –1 is B0 (b) M: This needs to be correct differentiation here A1: Fully correct simplified answer. (c) M: If the –5 and "-16" are the wrong way round or – omitted the M mark can still be given if a correct formula is seen, (e.g. $y-y_1 = m(x-x_1)$) otherwise M0. m should be numerical and not 0 or infinity and should not have involved negative reciprocal. (d) 1st M: Putting the derivative expression equal to their value for gradient 2nd M: Attempt to solve quadratic (see notes) This may be implied by correct answer.	