Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Monday 10 January 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H35402A

1. (*a*) Find the value of $16^{-\frac{1}{4}}$.

(2)

(*b*) Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$.

(2)

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2. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\ \ \mathrm{d}x\,,$$

giving each term in its simplest form.

(5)

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3. Simplify

$$\frac{5 - 2\sqrt{3}}{\sqrt{3} - 1}\,,$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers.

(4)

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4. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = 2,$$

$$a_{n+1} = 3a_n - c$$

where c is a constant.

(*a*) Find an expression for a_2 in terms of c.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$,

(*b*) find the value of c.

(4)

5.

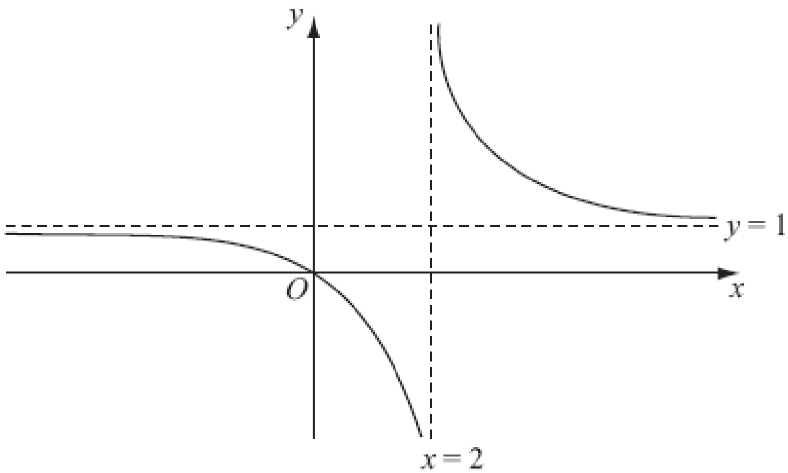

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2.$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x - 1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x - 1)$ crosses the coordinate axes.

(4)

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6. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$.

(2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d,

(1)

(c) find the value of a and the value of d.

(4)

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7. The curve with equation $y = \mathrm{f}(x)$ passes through the point $(-1, 0)$.

Given that

$$\mathrm{f}'(x) = 12x^2 - 8x + 1,$$

find $\mathrm{f}(x)$.

(5)

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8. The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0.$$

(3)

(b) Find the set of possible values of k.

(4)

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9. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point $A(1, 4)$ lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(2)

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10. (*a*) Sketch the graphs of

(i) $y = x(x + 2)(3 - x)$,

(ii) $y = -\dfrac{2}{x}$.

showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(*b*) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x + 2)(3 - x) + \frac{2}{x} = 0.$$

(2)

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11. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0.$$

(*a*) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(*b*) Show that the point $P(4, -8)$ lies on C.

(2)

(*c*) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

TOTAL FOR PAPER: 75 MARKS

END

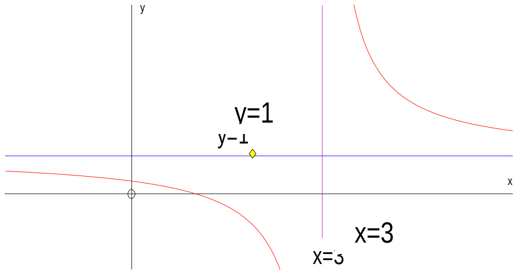
January 2011
Core Mathematics C1 6663
Mark Scheme

Question Number	Scheme	Marks
1.		
(a)	$16^{\frac{1}{4}} = 2$ or $\dfrac{1}{16^{\frac{1}{4}}}$ or better	M1
	$\left(16^{-\frac{1}{4}} =\right) \dfrac{1}{2}$ or 0.5 (ignore $\pm$)	A1
		(2)
(b)	$\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 x^{-\frac{4}{4}}$ or $\dfrac{2^4}{x^{\frac{4}{4}}}$ or equivalent	M1
	$x\left(2x^{-\frac{1}{4}}\right)^4 = 2^4$ or 16	A1 cao
		(2) **4**
	<u>Notes</u>	
(a)	M1 for a correct statement dealing with the $\frac{1}{4}$ or the $-$ power This may be awarded if 2 is seen or for reciprocal of their $16^{\frac{1}{4}}$ s.c ¼ is M1 A0 , also 2^{-1} is M1 A0 $\pm\frac{1}{2}$ is not penalised so M1 A1	
(b)	M1 for **correct** use of the power 4 on both the 2 and the x terms A1 for cancelling the x and simplifying to one of these two forms. Correct answers with no working get full marks	

Question Number	Scheme	Marks
2.	$\left(\int =\right)\frac{12x^6}{6}, -\frac{3x^3}{3}, +\frac{4x^{\frac{4}{3}}}{\frac{4}{3}}, (+c)$ $= \underline{2x^6 - x^3 + 3x^{\frac{4}{3}} + c}$	M1A1,A1,A1 A1 **5**
	<u>Notes</u>	
	M1 for some attempt to integrate: $x^n \to x^{n+1}$ i.e ax^6 or ax^3 or $ax^{\frac{4}{3}}$ or $ax^{1\frac{1}{3}}$, where a is a non zero constant 1st A1 for $\frac{12x^6}{6}$ or better 2nd A1 for $-\frac{3x^3}{3}$ or better 3rd A1 for $\frac{4x^{\frac{4}{3}}}{\frac{4}{3}}$ or better 4th A1 for each term correct and simplified and the $+c$ occurring in the final answer	

Question Number	Scheme	Marks
3.	$\frac{5-2\sqrt{3}}{\sqrt{3}-1}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$	M1
	$=\frac{\ldots}{2}$ denominator of 2	A1
	Numerator = $5\sqrt{3}+5-2\sqrt{3}\sqrt{3}-2\sqrt{3}$	M1
	So $\frac{5-2\sqrt{3}}{\sqrt{3}-1}=-\frac{1}{2}+\frac{3}{2}\sqrt{3}$	A1 **4**
	Alternative: $\left(p+q\sqrt{3}\right)(\sqrt{3}-1)=5-2\sqrt{3}$, and form simultaneous equations in p and q	M1
	$-p + 3q = 5$ and $p - q = -2$	A1
	Solve simultaneous equations to give $p = -\frac{1}{2}$ and $q=\frac{3}{2}$.	M1 A1
	<u>Notes</u>	
	1st M1 for multiplying numerator and denominator by same correct expression 1st A1 for a correct denominator as a single number (NB depends on M mark) 2nd M1 for an attempt to multiply the numerator by $\left(\sqrt{3}\pm1\right)$ and get 4 terms with at least 2 correct. 2nd A1 for the answer as written or $p = -\frac{1}{2}$ and $q=\frac{3}{2}$. Allow -0.5 and 1.5. (Apply isw if correct answer seen, then slip writing p =, q =)	
	Answer only (very unlikely) is full marks if correct – no part marks	

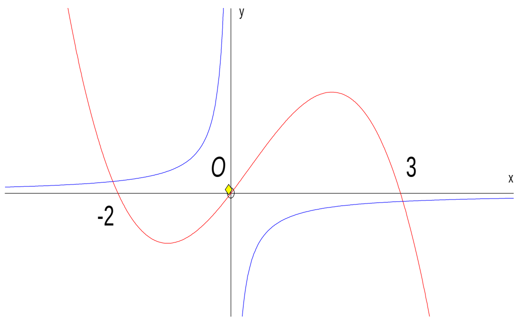
Question Number	Scheme	Marks
4 **(a)**	$(a_2 =)\ 6 - c$	B1 (1)
(b)	$a_3 = 3(\text{their } a_2) - c \qquad (= 18 - 4c)$ $a_1 + a_2 + a_3 = 2 + "(6-c)" + "(18-4c)"$ "$26 - 5c$" $= 0$ So $c = 5.2$	M1 M1 A1ft A1 o.a.e (4) **5**
	Notes	
(b)	1st M1 for attempting a_3. Can follow through their answer to (a) but it must be an expression in c. 2nd M1 for an attempt to find the sum $a_1 + a_2 + a_3$ must see evidence of sum 1st A1ft for their sum put equal to 0. Follow through their values but answer must be in the form $p + qc = 0$ A1 – accept any correct equivalent answer	

Question Number	Scheme	Marks
5. **(a)**	 Correct shape with a single crossing of each axis y = 1 labelled or stated x = 3 labelled or stated	B1 B1 B1 (3)
(b)	Horizontal translation so crosses the x-axis at (1, 0) New equation is $(y=)\dfrac{x\pm1}{(x\pm1)-2}$ When x = 0 y = $=\dfrac{1}{3}$	B1 M1 M1 A1 (4) **7**
	Notes	
(b)	B1 for point (1,0) identified - this may be marked on the sketch as 1 on x axis. Accept x = 1. 1st M1 for attempt at new equation and either numerator or denominator correct 2nd M1 for setting x = 0 in their new equation and solving as far as y =… A1 for $\frac{1}{3}$ or exact equivalent. Must see $y = \frac{1}{3}$ or (0, $\frac{1}{3}$) or point marked on y-axis. **Alternative** $f(-1)=\dfrac{-1}{-1-2}=\dfrac{1}{3}$ scores M1M1A0 unless x =0 is seen or they write the point as (0, $\frac{1}{3}$) or give y = 1/3 Answers only: x = 1, y = 1/3 is full marks as is (1,0) (0, 1/3) Just 1 and 1/3 is B0 M1 M1 A0 Special case : Translates 1 unit to left (a) B0, B1, B0 (b) Mark (b) as before May score B0 M1 M1 A0 so 3/7 or may ignore sketch and start again scoring full marks for this part.	

Question Number	Scheme	Marks
6.		
(a)	$S_{10} = \frac{10}{2}\left[2a+9d\right]$ or	M1
	$S_{10} = a+a+d+a+2d+a+3d+a+4d+a+5da+6d+a+7d+a+8d+a+9d$	
	162 = 10a + 45d *	A1cso
		(2)
(b)	$\left(u_n = a+(n-1)d \quad \Rightarrow \quad\right) 17 = a+5d$	B1
		(1)
	$10\times(b)$ gives $10a+50d = 170$	M1
	(a) is 10a + 45d = 162	
	Subtract 5d = 8 so d = <u>1.6</u> o.e.	A1
	Solving for a a = 17 - 5d	M1
	so a = <u>9</u>	A1
		(4)
		7
	<u>Notes</u>	
(a)	M1 for use of S_n with n = 10	
(b)	1st M1 for an attempt to eliminate a or d from their two linear equations 2nd M1 for using their value of a or d to find the other value.	

Question Number	Scheme	Marks
7.	$\left(\text{f}(x)=\right)\dfrac{12x^3}{3}-\dfrac{8x^2}{2}+x(+c)$	M1 A1 A1
	$\left(\text{f}(-1)=0\Rightarrow\right)\ 0=4\times(-1)-4\times1-1+c$	M1
	$c=\underline{9}$	A1
	$\left[\text{f}(x)=4x^3-4x^2+x+9\right]$	**5**
	Notes	
	1^{st} M1 for an attempt to integrate $x^n\to x^{n+1}$ 1^{st} A1 for at least 2 terms in x correct - needn't be simplified, ignore $+c$ 2^{nd} A1 for all the terms in x correct but they need not be simplified. No need for $+c$ 2^{nd} M1 for using x = -1 and y =0 to form a linear equation in c. No $+c$ gets M0A0 3^{rd} A1 for c = 9. Final form of f(x) is not required.	
8 . **(a)**	$b^2-4ac=(k-3)^2-4(3-2k)$	M1
	$k^2-6k+9-4(3-2k)>0$ or $(k-3)^2-12+8k>0$ or better	M1
	$\underline{k^2+2k-3>0}$ *	A1cso (3)
(b)	$(k+3)(k-1)[=0]$	M1
	Critical values are k = 1 or -3	A1
	(choosing "outside" region)	M1
	$\underline{k>1 \text{ or } k<-3}$	A1 cao (4) **7**
	Notes	
(a)	1^{st} M1 for attempt to find b^2-4ac with one of b or c correct 2^{nd} M1 for a correct inequality symbol and an attempt to expand. A1cso no incorrect working seen	
(b)	1^{st} M1 for an attempt to factorize **or** solve leading to k = (2 values) 2^{nd} M1 for a method that leads them to choose the "outside" region. Can follow through their critical values. 2^{nd} A1 Allow "," instead of "or" $\underline{>}$ loses the final A1 1 < k < -3 scores M1A0 unless a correct version is seen before or after this one.	

Question Number	Scheme	Marks
9. **(a)**	$(8-3-k=0)$ so $\underline{k=5}$	B1 (1)
(b)	$2y=3x+k$ $y=\frac{3}{2}x+\ldots$ and so $m=\frac{3}{2}$ o.e.	M1 A1 (2)
(c)	Perpendicular gradient = $-\frac{2}{3}$ Equation of line is: $y-4=-\frac{2}{3}(x-1)$ $\underline{3y+2x-14=0}$ o.e.	B1ft M1A1ft A1 (4)
(d)	$y=0, \Rightarrow B(7,0)$ or $\underline{x=7}$ $x=7$ or $-\frac{c}{a}$	M1A1ft (2)
(e)	$AB^2=(7-1)^2+(4-0)^2$ $AB=\sqrt{52}$ or $2\sqrt{13}$	M1 A1 (2) **11**
	Notes	
(b)	M1 for an attempt to rearrange to $y = \ldots$ A1 for clear statement that gradient is 1.5, can be m =1.5 o.e.	
(c)	B1ft for using the perpendicular gradient rule correctly on their "1.5" M1 for an attempt at finding the equation of the line through A using their gradient. Allow a sign slip 1st A1ft for a correct equation of the line follow through their changed gradient 2nd A1 as printed or equivalent with integer coefficients – allow $\underline{3y+2x=14}$ or $\underline{3y=14-2x}$	
(d)	M1 for use of y = 0 to find x =…in their equation A1ft for $x=7$ or $-\frac{c}{a}$	
(e)	M1 for an attempt to find AB or AB^2 A1 for any correct surd form- need not be simplified	

Question Number	Scheme	Marks
10. **(a)**	 (i) correct shape (-ve cubic) Crossing at (-2, 0) Through the origin Crossing at (3,0) (ii) 2 branches in correct quadrants not crossing axes One intersection with cubic on each branch	B1 B1 B1 B1 B1 B1 (6)
(b)	"2" solutions Since only "2" intersections	B1ft dB1ft (2) **8**
	Notes	
(b)	B1ft for a value that is compatible with their sketch dB1ft This mark is dependent on the value being compatible with their sketch. For a comment relating the number of solutions to the number of intersections. [Only allow 0, 2 or 4]	
11. **(a)**	$\left(\frac{dy}{dx}=\right)\frac{3}{2}x^2-\frac{27}{2}x^{\frac{1}{2}}-8x^{-2}$	M1A1A1A1 (4)
(b)	$x=4 \quad \Rightarrow \quad y=\frac{1}{2}\times 64-9\times 2^3+\frac{8}{4}+30$ $= 32 - 72 + 2 + 30 \qquad = \underline{-8} \quad *$	M1 A1cso (2)
(c)	$x=4 \quad \Rightarrow \quad y'=\frac{3}{2}\times 4^2-\frac{27}{2}\times 2-\frac{8}{16}$ $= 24-27-\frac{1}{2}= \qquad -\frac{7}{2}$ Gradient of the normal = $-1\div"\frac{7}{2}"$ Equation of normal: $y--8=\frac{2}{7}(x-4)$ $\underline{7y-2x+64=0}$	M1 A1 M1 M1A1ft A1 (6) **12**

Question Number	Scheme	Marks
	<u>Notes</u>	
(a)	1^{st} M1 for an attempt to differentiate $x^n \to x^{n-1}$ 1^{st} A1 for one correct term in x 2^{nd} A1 for 2 terms in x correct 3^{rd} A1 for all correct x terms. No 30 term and no $+c$.	
(b)	M1 for substituting $x = 4$ into $y =$ and attempting $4^{\frac{3}{2}}$ A1 note this is a printed answer	
(c)	1^{st} M1 Substitute x = 4 into y' (allow slips) A1 Obtains –3.5 or equivalent 2^{nd} M1 for correct use of the perpendicular gradient rule using their gradient. (May be slip doing the division) Their gradient must have come from y' 3^{rd} M1 for an attempt at equation of tangent or normal at P 2^{nd} A1ft for correct use of their changed gradient to find **normal** at P. Depends on 1^{st}, 2^{nd} and 3^{rd} Ms 3^{rd} A1 for any equivalent form with integer coefficients	