Write your name here


Statistics S1
Advanced/Advanced Subsidiary

| Wednesday 15 June 2016 - Morning | Paper Reference |
| :--- | :--- |
| Time: $\mathbf{1}$ hour $\mathbf{3 0}$ minutes | $\mathbf{6 6 8 3 / 0 1}$ |

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You must have:
Mathematical Formulae and Statistical Tables (Pink)
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Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them. Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy. Information
- The total mark for this paper is 75 .
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question. Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. A biologist is studying the behaviour of bees in a hive. Once a bee has located a source of food, it returns to the hive and performs a dance to indicate to the other bees how far away the source of the food is. The dance consists of a series of wiggles. The biologist records the distance, $d$ metres, of the food source from the hive and the average number of wiggles, $w$, in the dance.

| Distance, $\boldsymbol{d} \mathbf{~ m}$ | 30 | 50 | 80 | 100 | 150 | 400 | 500 | 650 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average number <br> of wiggles, $\boldsymbol{w}$ | 0.725 | 1.210 | 1.775 | 2.250 | 3.518 | 6.382 | 8.185 | 9.555 |

[You may use $\sum w=33.6 \sum d w=13833 \mathrm{~S}_{d d}=394600 \mathrm{~S}_{w w}=80.481$ (to 3 decimal places)]
(a) Show that $\mathrm{S}_{d w}=5601$.
(b) State, giving a reason, which is the response variable.
(c) Calculate the product moment correlation coefficient for these data.
(d) Calculate the equation of the regression line of $w$ on $d$, giving your answer in the form $w=a+b d$.

A new source of food is located 350 m from the hive.
(e) (i) Use your regression equation to estimate the average number of wiggles in the corresponding dance.
(ii) Comment, giving a reason, on the reliability of your estimate.
(Total 11 marks)
2. The discrete random variable $X$ has the following probability distribution, where $p$ and $q$ are constants.

| $x$ | -2 | -1 | $\frac{1}{2}$ | $\frac{3}{2}$ | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | $p$ | $q$ | 0.2 | 0.3 | $p$ |

(a) Write down an equation in $p$ and $q$.

Given that $\mathrm{E}(X)=0.4$,
(b) find the value of $q$.
(c) Hence find the value of $p$.

Given also that $\mathrm{E}\left(X^{2}\right)=2.275$,
(d) find $\operatorname{Var}(X)$.

Sarah and Rebecca play a game.
A computer selects a single value of $X$ using the probability distribution above.
Sarah's score is given by the random variable $S=X$ and Rebecca's score is given by the random variable $R=\frac{1}{X}$.
(e) Find $\mathrm{E}(R)$.

Sarah and Rebecca work out their scores and the person with the higher score is the winner. If the scores are the same, the game is a draw.
(f) Find the probability that
(i) Sarah is the winner,
(ii) Rebecca is the winner.
3. Before going on holiday to Seapron, Tania records the weekly rainfall ( $x \mathrm{~mm}$ ) at Seapron for 8 weeks during the summer. Her results are summarised as

$$
\sum x=86.8 \quad \sum x^{2}=985.88
$$

(a) Find the standard deviation, $\sigma_{x}$, for these data.

Tania also records the number of hours of sunshine ( $y$ hours) per week at Seapron for these 8 weeks and obtains the following

$$
\bar{y}=58 \quad \sigma_{y}=9.461 \text { (correct to } 4 \text { significant figures) } \quad \sum x y=4900.5
$$

(b) Show that $\mathrm{S}_{y y}=716$ (correct to 3 significant figures).
(c) Find $\mathrm{S}_{x y}$.
(d) Calculate the product moment correlation coefficient, $r$, for these data.

During Tania's week-long holiday at Seapron there are 14 mm of rain and 70 hours of sunshine.
(e) State, giving a reason, what the effect of adding this information to the above data would be on the value of the product moment correlation coefficient.
(Total 10 marks)
4. The Venn diagram shows the probabilities of customer bookings at Harry's hotel.
$R$ is the event that a customer books a room
$B$ is the event that a customer books breakfast
$D$ is the event that a customer books dinner $u$ and $t$ are probabilities.

(a) Write down the probability that a customer books breakfast but does not book a room.

Given that the events $B$ and $D$ are independent,
(b) find the value of $t$.
(c) Hence find the value of $u$.
(d) Find
(i) $\mathrm{P}(D \mid R \cap B)$,
(ii) $\mathrm{P}\left(D \mid R \cap B^{\prime}\right)$.

A coach load of 77 customers arrive at Harry's hotel.
Of these 77 customers
40 have booked a room and breakfast
37 have booked a room without breakfast
(e) Estimate how many of these 77 customers will book dinner.
5. A midwife records the weights, in kg , of a sample of 50 babies born at a hospital. Her results are given in the table below.

| Weight ( $\boldsymbol{w} \mathbf{~ k g )}$ | Frequency ( $\boldsymbol{f}$ ) | Weight midpoint $(\boldsymbol{x}$ ) |
| :---: | :---: | :---: |
| $0 \leq w<2$ | 1 | 1 |
| $2 \leq w<3$ | 8 | 2.5 |
| $3 \leq w<3.5$ | 17 | 3.25 |
| $3.5 \leq w<4$ | 17 | 3.75 |
| $4 \leq w<5$ | 7 | 4.5 |

[You may use $\sum \mathrm{f} \mathrm{x}^{2}=611.375$ ]
A histogram has been drawn to represent these data.
The bar representing the weight $2 \leq w<3$ has a width of 1 cm and a height of 4 cm .
(a) Calculate the width and height of the bar representing a weight of $3 \leq w<3.5$.
(b) Use linear interpolation to estimate the median weight of these babies.
(c) (i) Show that an estimate of the mean weight of these babies is 3.43 kg .
(ii) Find an estimate of the standard deviation of the weights of these babies.

Shyam decides to model the weights of babies born at the hospital, by the random variable $W$, where $W \sim \mathrm{~N}\left(3.43,0.65^{2}\right)$.
(d) Find $\mathrm{P}(W<3)$.
(e) With reference to your answers to (b), (c)(i) and (d) comment on Shyam's decision.

A newborn baby weighing 3.43 kg is born at the hospital.
(f) Without carrying out any further calculations, state, giving a reason, what effect the addition of this newborn baby to the sample would have on your estimate of the
(i) mean,
(ii) standard deviation.
6. The time, in minutes, taken by men to run a marathon is modelled by a normal distribution with mean 240 minutes and standard deviation 40 minutes.
(a) Find the proportion of men that take longer than 300 minutes to run a marathon.

Nathaniel is preparing to run a marathon. He aims to finish in the first $20 \%$ of male runners.
(b) Using the above model estimate the longest time that Nathaniel can take to run the marathon and achieve his aim.

The time, $W$ minutes, taken by women to run a marathon is modelled by a normal distribution with mean $\mu$ minutes.

Given that $\mathrm{P}(W<\mu+30)=0.82$,
(c) find $\mathrm{P}(W<\mu-30 \mid W<\mu)$.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1.(a) | $\begin{gathered} \mathrm{S}_{d v}=13833-\frac{" 1960 " \times 33.6}{8} \text { or } 13833-\frac{65856}{8} \text { (But } 13833-8232 \text { is M0) } \\ =\underline{\mathbf{5 6 0 1}} \quad(*) \end{gathered}$ | M1 A1 cso |
| (b) | $w$, since the number of wiggles depends on the distance or $w$ depends on $d$ | B1 (1) |
| (c) | $r=\frac{5601}{\sqrt{394600 \times 80.481}},=0.99389 \ldots \quad \text { awrt } \underline{\mathbf{0 . 9 9 4}}$ | M1,A1 |
| (d) | $b=\frac{5601}{394600}, \quad=0.014194 \ldots . \quad(\text { awrt } 0.014)$ | M1, A1 |
| (e) | $a=\frac{33.6}{8}-" 0.01419 \ldots . . \times \frac{" 1960 "}{8}=4.2-" 0.01419 \ldots . . \times 245[=0.72244 . .]$ | M1 |
|  | $\underline{w}=0.722+0.0142 d$ | A1 (4) |
|  | (i) $[0.722+0.0142 \times 350=] \quad$ awrt: $\underline{\mathbf{5 . 7}}$ or $\underline{\mathbf{5 . 6}}$ <br> (ii) Reliable since 350 m is in the range of the data | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ |
|  |  | (2) <br> [11 marks] |
|  | Notes |  |
| (a) | $\left.\begin{array}{ll}\text { M1 } & \text { for clear attempt to find } \Sigma d \text { and use in a correct formula. Accept } 1300<\Sigma d<2500 \\ & \text { For the M1 we can condone a single slip e.g. using } 1383 \text { instead of } 13833 \text { etc }\end{array}\right]$ |  |
| (b) | B1 Must select $w$ (or wiggles) and reason based on the idea that $w$ is dependent on $d$ Allow $w$ "changes according to"/ "is determined/affected by" Must mention $w$ and $d$ <br> B0 for " $w$ is measured" or " $d$ is explanatory/indep't" or " $w$ can't be controlled" or " $w$ responds to $d$ " |  |
| (c) | M1 for a correct expression (Allow ft of their incorrect $\mathrm{S}_{d w}$ ) <br> A1 for awrt 0.994 (Answer only 2/2) [Answer only of 0.99 scores M1 | 0] |
| (d) | $1^{\text {st }}$ M1 for a correct expression for $b$. (Allow ft of their incorrect $\mathrm{S}_{d w}$ ) <br> $1^{\text {st }} \mathrm{A} 1$ for awrt 0.014 No fractions. [Answer only $2 / 2$ ] Can be given at final equation. <br> [Must come from correct formula not gradient of line from e.g. (650, 9.555) to $(30,0.725)$ ] <br> $2^{\text {nd }}$ M1 for a correct method for $a$. Follow through their value of $b$ and their $\Sigma d$ <br> $2^{\text {nd }} \mathrm{A} 1$ for a correct equation for $w$ and $d$ with $a=$ awrt 0.722 and $b=$ awrt 0.0142 No fractions Equation in $x$ and $y$ is A0 Answer only 4/4 |  |
| (e) | $1^{\text {st }}$ B1 for awrt 5.7 or awrt 5.6 <br> $2^{\text {nd }} \mathrm{B} 1$ for a reason citing $350(\mathrm{~m})$ or mentioning $d$ is in the range of the data and stating reliable. Allow "Interpolation (or not extrapolation) therefore reliable". <br> Saying " 5.7 (or $w$ or just "it") is in the range" is B0 "accurate" instead of "reliable" is B0 "strong correlation" (without mention of interpolation o.e.) is B0 Apply ISW if a correct comment is seen. |  |






| Question Number | Scheme Marks |
| :---: | :---: |
| 6.(a) |  |
|  | Notes |
| (a) <br> (b) | $1^{\text {st }}$ M1 for standardising with 300, 240 and 40. May be implied by use of 1.5 Allow $\pm$ <br> $2^{\text {nd }} \mathrm{M} 1$ for $1-\mathrm{P}(Z<" 1.5$ " $)$ i.e. a correct method for finding $\mathrm{P}(Z>$ " 1.5 " $)$ <br> e.g. $1-p$ where $0.5<p<0.99$ <br> A1 for awrt 0.0668 (Answer only 3/3) <br> M1 for an attempt to standardise with 240, 40 and $n$ and set $= \pm z(0.8<\|z\|<0.9)$ <br> B1 for $z= \pm 0.8416$ (or better) used as a $z$ value. Do not allow for $1-0.8416$ <br> Calc gives $0.8416212 \ldots$ [May be implied by awrt 206.34, give B1 as well as A1 if seen] <br> A1 for awrt 206 (can be scored for using a $z$ value of 0.84 or even 0.85 ) <br> Must follow from correct working but a range of possible $z$ values are OK <br> If answer is awrt 206 score M1B0A1 (unless of course $z=0.8416$ seen) but awrt 206.34 scores $3 / 3$ <br> M1 for the correct ratio expression (Not $\mathrm{P}([W<30-\mu] \cap[W<\mu])$ on numerator) <br> Condone use of $Z$ instead of $W$ only if they later get a correct numerical ratio otherwise M0 However they may write $\mathrm{P}\left(Z<\frac{-30}{\sigma}\right)$ etc which is of course fine <br> $1^{\text {st }} \mathrm{A} 1$ for a correct numerical ratio <br> May see use of $z=0.92$ or better (calc: $0.9153650 \ldots$ ) or $\sigma=32.6 \sim 32.8$ allow: <br> $1^{\text {st }}$ M1 for $\frac{\mathrm{P}(Z<-0.92)}{\mathrm{P}(Z<0)}$ and $1^{\text {st }} \mathrm{A} 1$ for $\frac{1-0.8212}{0.5}$ or $\frac{0.1788}{0.5}$ <br> $2^{\text {nd }} \mathrm{A} 1$ for 0.36 or an exact equivalent e.g. $\frac{9}{25}$ (Answer only M1A1A0) <br> The final answer of 0.36 must come from exact values; 0.36 rounded from 0.3576 etc is A0 |

