Paper Reference(s)

6677/01

Edexcel GCE

Mechanics M1

Advanced/Advanced Subsidiary

Friday 6 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

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Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M1), the paper reference (6677), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper.

The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

1.

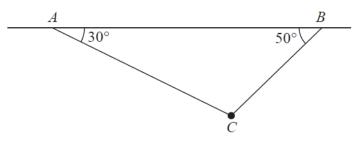


Figure 1

A particle of weight W newtons is attached at C to two light inextensible strings AC and BC. The other ends of the strings are attached to fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at 30° and 50° respectively, as shown in Figure 1.

Given that the tension in BC is 6 N, find

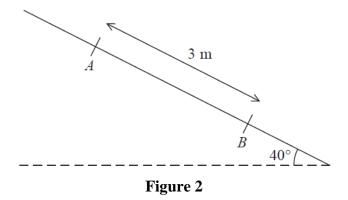
(a) the tension in AC,

(3)

(b) the value of W.

(3)

2.



A rough plane is inclined at 40° to the horizontal. Two points A and B are 3 metres apart and lie on a line of greatest slope of the inclined plane, with A above B, as shown in Figure 2. A particle P of mass m kg is held at rest on the plane at A. The coefficient of friction between P and the plane is $\frac{1}{2}$. The particle is released.

(a) Find the acceleration of P down the plane.

(5)

(b) Find the speed of P at B.

(2)

- 3. A ball of mass 0.3 kg is released from rest at a point which is 2 m above horizontal ground. The ball moves freely under gravity. After striking the ground, the ball rebounds vertically and rises to a maximum height of 1.5 m above the ground, before falling to the ground again. The ball is modelled as a particle.
 - (a) Find the speed of the ball at the instant before it strikes the ground for the first time.

(2)

(b) Find the speed of the ball at the instant after it rebounds from the ground for the first time.

(2)

(c) Find the magnitude of the impulse on the ball in the first impact with the ground.

(2)

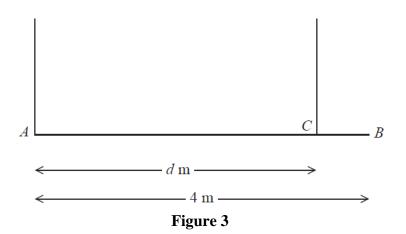
(d) Sketch, in the space provided, a velocity-time graph for the motion of the ball from the instant when it is released until the instant when it strikes the ground for the second time.

(3)

(e) Find the time between the instant when the ball is released and the instant when it strikes the ground for the second time.

(4)

4.



A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where AC = d metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(a) Find the value of d.

A small load of weight kW newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

- (b) Find the value of k. (6)
- **5.** A particle *P* of mass 0.5 kg is moving under the action of a single force $(3\mathbf{i} 2\mathbf{j})$ N.
 - (a) Show that the magnitude of the acceleration of P is $2\sqrt{13}$ m s⁻². (4)

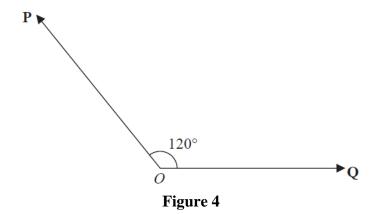
At time t = 0, the velocity of P is $(\mathbf{i} + 3\mathbf{j})$ m s⁻¹.

(b) Find the velocity of P at time t = 2 seconds. (3)

Another particle Q moves with constant velocity $\mathbf{v} = (2\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$.

- (c) Find the distance moved by Q in 2 seconds. (2)
- (d) Show that at time t = 3.5 seconds both particles are moving in the same direction. (3)

6.



Two forces \mathbf{P} and \mathbf{Q} act on a particle at O. The angle between the lines of action of \mathbf{P} and \mathbf{Q} is 120° as shown in Figure 4. The force \mathbf{P} has magnitude 20 N and the force \mathbf{Q} has magnitude X newtons. The resultant of \mathbf{P} and \mathbf{Q} is the force \mathbf{R} .

Given that the magnitude of \mathbf{R} is 3X newtons, find, giving your answers to 3 significant figures,

(a) the value of X, (5)

(b) the magnitude of $(\mathbf{P} - \mathbf{Q})$. (4)

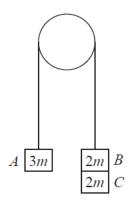


Figure 5

Three particles A, B and C have masses 3m, 2m and 2m respectively. Particle C is attached to particle B. Particles A and B are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 5. The system is released from rest and A moves upwards.

- (a) (i) Show that the acceleration of A is $\frac{g}{7}$.
 - (ii) Find the tension in the string as A ascends.

(7)

At the instant when A is 0.7 m above its original position, C separates from B and falls away. In the subsequent motion, A does not reach the pulley.

(b) Find the speed of A at the instant when it is 0.7 m above its original position.

(2)

(c) Find the acceleration of A at the instant after C separates from B.

(4)

(d) Find the greatest height reached by A above its original position.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1a	Resolving horizontally: $T \cos 30^{\circ} = 6 \cos 50^{\circ}$	M1A1
	T = 4.45 (N), 4.5 (N), or better	A1
		(3)
b	Resolving vertically: $W = 6\cos 40^{\circ} + T\cos 60^{\circ}$	M1A1
	= 6.82 (N), 6.8 (N), or better	A1
		(3)
		[6]

Question 1(a)

First M1 for resolving horizontally with correct no. of terms and both T_{AC} and '6' terms resolved. First A1 for a correct equation in T_{AC} only.

Second A1 for 4.5 (N), 4.45 (N) or better. (4.453363194)

N.B. The M1 is for a <u>complete method</u> to find the tension so where two resolution equations, neither horizontal, are used, the usual criteria for an M mark must be applied to *both* equations and the first A1 is for a correct equation in T_{AC} only (i.e. W eliminated correctly)

Alternatives:

Triangle of Forces:
$$\frac{T_{AC}}{\sin 40^{\circ}} = \frac{6}{\sin 60^{\circ}}$$
 (same equation as \rightarrow resolution) M1A1

Or

Lami's Theorem:
$$\frac{T_{AC}}{\sin 140^{\circ}} = \frac{6}{\sin 120^{\circ}}$$
 (same equation as \rightarrow resolution) M1A1

Question 1(b)

First M1 for resolving vertically with correct no. of terms and both T_{AC} (does not need to be substituted) and '6' terms resolved.

First A1 for a correct equation in T_{AC} and W.

Second A1 for 6.8 (N), 6.82 (N) or better. (6.822948256)

Alternatives:

Triangle of Forces:
$$\frac{6}{\sin 60^{\circ}} = \frac{W}{\sin 80^{\circ}}$$
 M1A1

Or Lami's Theorem:
$$\frac{6}{\sin 120^{\circ}} = \frac{W}{\sin 100^{\circ}}$$
 M1A1

Or Resolution in another direction e.g. along one of the strings M1 (usual criteria) A1 for a correct equation.

Question Number	Scheme	Marks
2(a)	$R = mg \cos 40$	B1
	Use of $F = \mu R$	B1
	$mg\sin 40 - F = \pm ma$	M1A1
	$acc = 2.55 \text{ (m s}^{-2}) \text{ or } 2.5 \text{ (m s}^{-2})$	A1 (5)
(b)	$v^2 = u^2 + 2as = 2 \times a \times 3$ Speed at B is 3.9 (m s ⁻¹) or 3.91(m s ⁻¹)	M1A1 (2)
		[7]

(Deduct only 1 mark in **whole question** for not giving an answer to either 2 sf or 3 sf, following use of g = 9.8)

Question 2(a)

 $\overline{\text{First B1 for } R} = mg\cos 40^{\circ}$

Second B1 for $F = \mu R$ seen or implied(can be on diagram)

M1 for resolving parallel to plane, correct no. of terms, mg resolved (F does not need to be substituted)

First A1 for a correct equation Second A1 for 2.5 (ms⁻²) or 2.55 (ms⁻²) Must be **positive.**

S.C. If *m* is given a specific numerical value, can score max B1B1M1A0A0

Question 2(b)

M1 is for a complete method for finding speed (usually $v^2=u^2+2as$) A1 for 3.9 (ms⁻¹) or 3.91(ms⁻¹)

Question Number	Scheme	Marks
3a	Using $v^2 = u^2 + 2as$: $v^2 = 4g$, $v = \sqrt{4g}$ or 6.3 or 6.26 (m s ⁻¹)	M1,A1 (2)
b	Rebounds to 1.5 m, $0 = u^2 - 3g$, $u = \sqrt{3g}$, 5.4 or 5.42 (m s ⁻¹)	M1A1 (2)
c	Impulse = $0.3(6.3+5.4) = 3.5$ (Ns)	M1A1 (2)
d	If speed downwards is taken to be positive: First line Second line -u,u,	B1 B1 B1 (3)
e.	Use of suvat to find t_1 or t_2 ,	
	$\sqrt{4g} = gt_1$ $t_1 = \sqrt{\frac{4}{g}} = 0.64 \text{ s}$ $\sqrt{3g} = gt_2$ $t_2 = \sqrt{\frac{3}{g}} = 0.55 \text{ s}$ Total time = $t_1 + 2t_2 = 1.7 \text{ s}$ or 1.75 s	$M1A1$ $(t_1 \text{ or } t_2)$ $DM1A1$ (4) $[13]$

N.B. Deduct only 1 mark in **whole question** for not giving an answer to either 2 sf or 3 sf, following use of g = 9.8 or use of g = 9.81

Question 3(a)

M1 is for a complete method for finding speed (usually $v^2=u^2+2as$) A1 for $v = 6.3 \text{ (ms}^{-1})$ or $6.26 \text{ (ms}^{-1})$ or $\sqrt{4g} \text{ (ms}^{-1})$ (must be positive)

Allow $0 = u^2 - 4g$ or $v^2 = 4g$ but not $0 = u^2 + 4g$ or $v^2 = -4g$

Question 3(b)

M1 is for a complete method for finding speed

Allow $0 = u^2 - 3g$ or $v^2 = 3g$ but not $0 = u^2 + 3g$ or $v^2 = -3g$ A1 for 5.4 (ms⁻¹) or 5.42 (ms⁻¹) or $\sqrt{3}g$ (ms⁻¹) (must be positive)

Question 3(c)

M1 is for ± 0.3 (their (b) \pm their (a)) (unless they are definitely adding the momenta

i.e. using I = m (v + u) which is M0). N.B. Extra g is M0

A1 for 3.5 (Ns) or 3.50 (Ns) (must be positive)

Question 3(d)

First B1 for a straight line from origin to their v which must be marked on the axis.

Second B1 for a parallel straight line correctly positioned (if continuous vertical lines are clearly *included as part of the graph then B0)*

Third B1 for their –u and u correctly marked, provided their second line is correctly positioned **N.B.** A reflection of the graph in the t-axis (upwards +ve) is also acceptable

Question 3(e)

First M1 for use of *suvat* or area under their v-t graph to find either t_1 or t_2 or $2t_2$

First A1 for correct value for either t_1 or t_2 (can be in terms of g at this stage or surds or unsimplified e.g.6.3/9.8)

Second M1 **dependent on the first M1** for their $t_1 + 2t_2$

Second A1 for 1.7 (s) or 1.75 (s).

Question Number	Scheme	Marks
4a	Resolving vertically: $T + 2T (= 3T) = W$	M1A1
	Moments about A: $2W = 2T \times d$	M1A1
	Substitute and solve: $2W = 2\frac{W}{3}d$	DM1
	d=3	A1
		(6)
b	Resolving vertically: $T + 4T = W + kW$ $(5T = W(1+k))$	M1A1 ft
	Moments about A: $2W + 4kW = 3 \times 4T$	M1A1 ft
	Substitute and solve: $2W + 4kW = \frac{12}{5}W(1+k)$	DM1
	$2+4k = \frac{12}{5} + \frac{12}{5}k$	
	$\frac{8}{5}k = \frac{2}{5},$ $k = \frac{1}{4}$	A1 (6)
		[12]

N.B. In moments equations, for the M mark, all terms must be force x distance but take care in the cases when the distance is 1.

Question 4(a)

N.B. If Wg is used, mark as a misread. If T and 2T are reversed, mark as per scheme NOT as a misread.

First M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the mid-pt), with usual rules.

First A1 for a correct equation.

Second M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the mid-pt), with usual rules.

Second A1 for a correct equation.

Third M1, dependent on first and second M marks, for solving for d

Third A1 for d = 3 cso

N.B. If a single equation is used (see below) by taking moments about the mid-point of the rod, 2T = 2T(d-2), this scores M2A2 (-1 each error)

Third M1, dependent on first and second M marks, for solving for d

Third A1 for d = 3 cso

Question 4(b)

N.B. If Wg and kWg are used, mark as a misread.

If they use any results from (a), can score max M1A1 in (b) for one equation.

If T and 4T are reversed, mark as per scheme NOT as a misread.

First M1 for an equation in W and a tension T_1 and possibly their d or their d and k (either resolve vertically or moments about any point), with usual rules.

First A1 **ft** on their d, for a correct equation.

Second M1 for an equation in W and **the same tension** T_1 and possibly their d or their d and k (either resolve vertically or moments about any point), with usual rules.

Second A1 **ft** on their *d*, for a correct equation.

Third M1, dependent on first and second M marks, for solving to give a numerical value of k Third A1 for k = 1/4 oe cso

Question Number	Scheme	Marks
5a	$\mathbf{F} = m\mathbf{a} : 3\mathbf{i} - 2\mathbf{j} = 0.5\mathbf{a}$	M1
	$a = 6\mathbf{i} - 4\mathbf{j}$	A1
	$ a = \sqrt{6^2 + (-4)^2} = 2\sqrt{13} (\mathbf{m} \mathbf{s}^{-2}) **$	M1A1 (4)
b	v = u + at: $v = (i + 3j) + 2(6i - 4j)= 13i - 5j m s-1$	M1A1 ft
	$= 13\mathbf{i} - 5\mathbf{j} \text{ m s}^{-1}$	A1
		(3)
С	Distance = $2 \mathbf{v} = 2\sqrt{4+1} = 2\sqrt{5} = 4.47$ (m)	M1A1
		(2)
d	When $t = 3.5$, velocity of <i>P</i> is $(i + 3j) + 3.5(6i - 4j) = 22i - 11j$	M1A1 ft
	Given conclusion reached correctly. E.g. $22\mathbf{i} - 11\mathbf{j} = 11(2\mathbf{i} - \mathbf{j})$	A1 (3)
		[12]

Question 5(a)

Either:

First M1 for use of $\mathbf{F} = m \mathbf{a}$

First A1 for $\mathbf{a} = 6\mathbf{i} - 4\mathbf{j}$ Second M1 for $a = \sqrt{(6^2 + (-4)^2)}$ (Allow $\sqrt{(6^2 + 4^2)}$) Second A1 for $a = 2\sqrt{13}$ (ms⁻²) **Given answer**

First M1 for $F = \sqrt{(3^2 + (-2)^2)}$ (Allow $\sqrt{(3^2 + 2^2)}$)

First A1 $F = \sqrt{13}$

Second M1 for $\sqrt{13} = 0.5 a$

Second A1 for $a = 2\sqrt{13}$ (ms⁻²) **Given answer**

Question 5(b)

M1 for (i + 3j) + (2 x their a)

First A1 ft for a correct expression

Second A1 for $13\mathbf{i} - 5\mathbf{j}$; isw if they go on to find the speed

Question 5(c)

M1 for $2\sqrt{(2^2 + (-1)^2)}$ or $\sqrt{(4^2 + (-2)^2)}$

A1 for $2\sqrt{5}$ or $\sqrt{20}$ or 4.5 or 4.47 or better

Question 5(d)

M1 for $(\mathbf{i} + 3\mathbf{j}) + (3.5 \text{ x their } \mathbf{a})$, or possibly, their $(\mathbf{b}) + (1.5 \text{ x their } \mathbf{a})$

First A1 ft for a correct expression of form $a\mathbf{i} + b\mathbf{j}$

Second A1 for given conclusion reached correctly e.g. 22i-11j=11(2i-j) oe Given answer

Question Number	Scheme	Marks
6a	3X 20 60° X	
	Resolve and use Pythagoras $(X - 20\cos 60)^2 + (20\cos 30)^2 = (3X)^2$	M1 A1
	$8X^2 + 20X - 400 = 0$	A1
	$X = \frac{-5 \pm \sqrt{25 + 800}}{4} = 5.93 $ (3 SF)	M1A1 (5)
6a alt	Cosine rule $(3X)^2 = 20^2 + X^2 - 2.20X \cos 60$	M1A1
	$8X^2 + 20X - 400 = 0$	A1
	$X = \frac{-5 \pm \sqrt{25 + 800}}{4} = 5.93 \text{ (3SF)}$	M1A1 (5)
b	$\left \mathbf{P} - \mathbf{Q} \right ^2 = 20^2 + X^2 - 2X \times 20 \times \cos 120$	M1A1
	$ \mathbf{P} - \mathbf{Q} = 23.5 \text{ (N) } (\mathbf{3SF})$	DM1 A1 (4)
6b alt	$ \mathbf{P} - \mathbf{Q} ^2 = (X + 20\cos 60)^2 + (20\cos 30)^2$	M1A1
	$ \mathbf{P} - \mathbf{Q} = 23.5 \text{ (N) } (\mathbf{3SF})$	DM1 A1
		(4) [9]
	I.	[/]

In this question a misquoted Cosine Rule is M0.

The question asks for both answers to 3 SF but only penalise under or over accuracy once in this question.

Question 6(a)

First M1 for a complete method to give an **equation in** X **only** i.e. producing two components *and* usually squaring and adding and equating to $(3X)^2$ (condone sign errors and consistent incorrect trig. in the components for this M mark **BUT** the x-component must be a difference)

First A1 for a correct unsimplified equation in *X only*

e.g., allow $(\pm (X - 20\cos 60^{\circ}))^{2} + (\pm (20\cos 30^{\circ}))^{2} = (3X)^{2}$

Second A1 for any correct fully numerical 3 term quadratic = 0

Second M1(**independent**) for solving *a 3 term* quadratic

Third A1 for 5.93

Alternative using cosine rule:

First M1 for use of cosine rule with cos60° (M0 if they use 120°)

First A1 for a correct equation unsimplified e.g., allow $\cos 60^{\circ}$ and $(3X)^{2}$

Second A1 for any correct fully numerical 3 term quadratic = 0

Second M1(**independent**) for solving *a 3 term* quadratic

Third A1 for 5.93

Alternative using 2 applications of the sine rule:

First M1 for using $3X / \sin 60 = X / \sin a$ **AND**

Either: $X / \sin a = 20 / \sin (120^{\circ} - a)$

Or: $3X / \sin 60^{\circ} = 20 / \sin (120^{\circ} - a)$

(These could be in terms of b where $b = (120^{\circ} - a)$)

First A1 for two correct equations

Second A1 for a = 16.778... (or b = 103.221...)

Second M1 for solving:

 $X / \sin a = 20 / \sin (120^{\circ} - a)$ or $3X / \sin 60^{\circ} = 20 / \sin (120^{\circ} - a)$

with their a or b, to find X

Third A1 for 5.93

Question 6(b)

First M1 for use of cosine rule unsimplified with cos120° (M0 if they use 60°)

First A1 for a correct expression for $|\mathbf{P} - \mathbf{Q}|$ in terms of X (does not need to be substituted)

Second M1, **dependent on first M1**, for substituting for their X and solving for $|\mathbf{P} - \mathbf{Q}|$

Second A1 for 23.5

Alternative using components:

First M1 for a complete method i.e. producing two components *and* squaring and adding (no square root needed) (condone sign errors and consistent incorrect trig. in the components for this M mark

BUT the x-component must be a sum)

First A1 for a correct expression for $|\mathbf{P} - \mathbf{Q}|$

 $(e.g. allow (\pm (X + 20\cos 60^{\circ}))^{2} + (\pm (20\cos 30^{\circ}))^{2}$

Second M1, **dependent on first M1**, for substituting for their X and solving for $|\mathbf{P} - \mathbf{Q}|$

Second A1 for 23.5

Question Number	Scheme	Marks
7(a)	4mg - T = 4ma	M1A1
	T-3mg=3ma	M1A1
	Condone the use of $4mg - 3mg = 4ma + 3ma$ in place of one of these equations.	M1A1
	Reach given answer $a = \frac{g}{7}$ correctly ***	A1
	Form an equation in T : $T = 3mg + 3\left(mg - \frac{T}{4}\right), T = 3mg + 3m\frac{g}{7}, \text{ or } T = 4mg - 4m\frac{g}{7}$	M1
	$T = \frac{24}{7}mg \text{ or equivalent, } 33.6m, 34m$	A1 (7)
(b)	$v^2 = u^2 + 2as = 2 \times \frac{g}{7} \times 0.7 = 1.96$, $v = 1.4$ ms ⁻¹	M1A1 (2)
(c)	$3mg - T = 3ma$ $T - 2mg = 2ma$ $a = \frac{g}{5}$	M1A1 A1 A1 (4)
(d)	$0 = 1.96 - 2 \times \frac{g}{5} \times s$	M1
	$0 = 1.96 - 2 \times \frac{g}{5} \times s$ $s = \frac{5 \times 1.96}{2g} = 0.5 \text{ (m)}$	A1
	Total height = $0.7 + 0.5 = 1.2$ (m)	A1 ft (3)
Alt d	Using energy: $3mgs - 2mgs = \frac{1}{2}3m \times 1.4^2 + \frac{1}{2}2m \times 1.4^2$	M1
	$s = \frac{2.5 \times 1.96^2}{g} = 0.5 \text{ (m)}$	A1
	Total height = $0.7 + 0.5 = 1.2 \text{ (m)}$	A1 ft (3)
		[16]

Question 7(a)(i) and (ii)

First M1 for resolving vertically (up or down) for B+C, with correct no. of terms.

First A1 for a correct equation.

Second M1 for resolving vertically (up or down) for A, with correct no. of terms.

Second A1 for a correct equation.

Third A1 for g/7, obtained correctly. Given answer (1.4 A0)

Third M1 for an equation in *T* only

Fourth A1 for 24mg/7 oe or 33.6m or 34m

N.B. If they omit m throughout (which gives a = g/7), can score max M1A0M1A0A0M1A0 for part (a) BUT CAN SCORE ALL OF THE MARKS in parts (b), (c) and (d).

Question 7(b)

M1 for an equation in v only (usually $v^2=u^2+2as$)

A1 for 1.4 (ms⁻¹) allow $\sqrt{(g/5)}$ oe.

Question 7(c)

First M1 for resolving vertically (up or down) for A or B, with correct no. of terms. (**N.B.** M0 if they use the tension from part (a))

First A1 for a correct equation for *A*.

Second A1 for a correct equation for *B*.

N.B. 'Whole system' equation: 3mg - 2mg = 5ma earns first 3 marks but any error loses all 3 Third A1 for g/5 oe or 1.96 or 2.0 (ms⁻²) (allow a negative answer)

Question 7(d)

M1 for an equation in s only using their v from (b) and a from (c).

either $0 = 1.4^2 - 2(g/5)s$ or $1.4^2 = 0 + 2(g/5)s$

First A1 for s = 0.5 (m) correctly obtained

Second A1 ft for their 0.5 + 0.7 = 1.2 (m)

Alternative using conservation of energy

M1 for an equation in s only, with correct number of terms, using their v from (b):-

 $(3mgs - 2mgs) = \frac{1}{2} 3m (1.4)^{2} + \frac{1}{2} 2m (1.4)^{2}$

First A1 for s = 0.5 (m) correctly obtained

Second A1 **ft** for their 0.5 + 0.7 = 1.2 (m)