MEI

Mathematics in Education and Industry

Curriculum 2004

Methods for Advanced Mathematics (C3) Coursework

Coursework for Methods for Advanced Mathematics (C3)

Introduction

The 2004 specification contains an assignment to investigate the solution of equations by numerical methods. This task was attached to the module P2 in previous specifications; in this specification it is attached to C3 and it will contribute 20% to the assessment.

This revised edition of the Coursework Bank reflects the changes in the coursework requirements for this module.

Teachers may offer to their students a list of at least 10 equations from which they should be able to freely choose in order to demonstrate the methods working and failing. If teachers do this then they should submit the list to the external moderator at the time that the assignments are submitted, and they should change the list each session.

Teachers are reminded that **the part of this bank marked FOR TEACHERS ONLY should not be made available to students**.

MEI would be pleased to update this bank in accordance with the needs of teachers. Any comments or suggestions should be sent to:

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Methods for Advanced Mathematics (C3) Coursework

Solving equations by numerical methods

Introduction

The coursework in C3 is designed to provide a focus for students' learning of the numerical methods for solving equations. The aim is that on completing the coursework they should have mastered a set of useful techniques which they can apply confidently as the need arises. The coursework also forms the assessment of this syllabus topic.

In presenting these pages of guidance we would like to express the hope that their students will enjoy doing this coursework.

Numerical Methods

The coursework in C3 involves the solution of equations by numerical methods. Before looking at the requirements in detail, some general points about numerical methods should be borne in mind.

- Numerical methods should not be regarded as somehow inferior to analytical ones but as an important and complementary part of the reality of mathematics. It should always be remembered that most real life problems cannot be solved using only analytic methods.

- A numerical method should not be used when an analytical one is available. It would be wrong, for example, to solve a quadratic equation numerically since it can be solved analytically using the quadratic formula, by completing the square or in some cases by factorisation. There may, however, be times when an analytical method is not known to a student and in such cases it is entirely reasonable to use numerical methods.

- There are circumstances in which particular numerical methods break down and it is important that students learn about these; this is emphasised within the coursework requirements. Both for teaching and coursework, it may be most satisfactory when demonstrating the failure of a method to use an example where the answer (which the numerical method is failing to obtain) is known. In these special circumstances it is reasonable to attempt to use numerical methods on problems for which analytical methods are available. It may be helpful to think of this as finding a counter example. In such cases, however, the analytical solution should not be trivial (C3.5),), nor should a quadratic equation be used to demonstrate failure.

- There are two parts to a numerical method: estimation of the answer to the problem in hand, and establishing error bounds for the given answer. An answer derived using a numerical method and stated without any reference to its level of accuracy is valueless. Since a numerical method should only be applied to a situation where an analytical method is not available, the accuracy (or error) must be established within the numerical method. It is not acceptable to determine error by referring to a known, correct answer.

Terminology

It is important that students understand and use correctly the mathematical language associated with this work. Consequently we have taken advantage of the extra marks in this new specification to allocate one mark specifically for correct notation and terminology.

Expression

An expression is a number of terms that are added or subtracted.
E.g. $x^3 + x - 7$ is an expression.

Function

A function is a way of describing an expression. A function of x is usually denoted $f(x)$.
The function may be defined, e.g. $f(x) = x^3 + x - 7$.
The letter y may also be used to describe a function of x, e.g. $y = x^3 + x - 7$.
A graph of a function may therefore be drawn, on which the value of the function for different values of x are given.

Equation

An equation is an expression set equal to 0, or some other number, or one expression set equal to another.

The solution of an equation

The terms *root* and *solution* are often confused. The equation $x^3 - x = 0$ has three roots, namely $x = -1$, $x = 0$ and $x = +1$. The solution of the equation is $x = -1, 0$ or $+1$. Consequently to solve an equation, you must find all its roots. A method which misses one or more roots has failed to solve the equation.

In these notes, a general equation is represented by $f(x) = 0$. The roots of this are the x-values of the points where the curve $y = f(x)$ cuts the x-axis. A common mistake among students is to call the equation $y = f(x)$, or even just $f(x)$, rather than $f(x) = 0$.

Coursework Requirements

The requirements, as stated in the specification, are as follows.

TASK: Candidates will investigate the solution of equations using the following three methods.

(i) *Systematic search for change of sign using one of the methods: bisection, decimal search, linear interpolation.*

(ii) *Fixed point iteration using the Newton-Raphson method.*

(iii) *Fixed point iteration after rearranging the equation $f(x) = 0$ into the form $x = g(x)$.*

In doing so candidates are expected to meet the following requirements.

1. Each method must be shown working. In the case of Newton-Raphson all the roots (i.e. at least 2) of the equation must be found; for Rearrangement and Change of Sign it is sufficient to find one root. A different equation must be used for each method.

2. Each method must be shown failing. In this context failure is taken to mean:
   not finding all the roots of the equation;
   or finding a root other than that expected;
   or finding a false root.

- In all situations candidates must show the process graphically. Candidates should do this clearly, using their chosen equations. Diagrams should be easy to follow. There is no need to show more than a few steps.

- Error or solution bounds should be established for at least one root in the case of the Change of Sign and Newton-Raphson methods. They should be established by looking for a change of sign, not just stated, and should be given numerically as either error bounds ($x = 2.614 \pm 0.0005$) or solution bounds ($2.6135<x<2.6145$). Roots should be found to at least 3 decimal place accuracy when applying the Change of Sign method and to at least 5 significant figures when applying the Newton Raphson and Rearrangement methods.

- Students should compare the three methods, discussing their ease of use and speed of convergence. In order to do this they must find one root of one of their equations by all three methods and this work should form the basis of their discussion. They should state what technology they are using since this may affect their view of a particular method.

- The coursework is expected to take about 6–8 hours and the work involved should be consistent with that duration, both in quantity and level of sophistication.

Coursework Advice

Technology
There are no particular requirements on technology. The coursework may be done on a calculator or on a computer. Where software has taken much of the humdrum out of the work, students must demonstrate that they understand what the software has done and how they could have performed the calculations themselves; they should appreciate that the use of such software allows them more time to spend on investigational work (for example when making their choice of equations).

Geometrical Understanding
To understand the various numerical methods for solving equations, students must have an appreciation of what is happening graphically. The first step in solving any equation should be to draw a sketch graph of the function involved and this should be included in the write-up. Each sketch should be annotated to show how the method works.

Extra work
The requirements set out above are the minimum that a student must do in order to obtain full marks on the coursework. However, the primary intention of the coursework is that students should learn from doing it and it would be entirely within this spirit if they were to do more than the bare minimum, for example demonstrating different ways in which a particular method may break down.

Investigatory work
It is permissible for teachers to offer a list of at least 10 equations (which must be submitted with the coursework sample called for external moderation and changed regularly). In this case, students should include some explanation of their choices within their write-up.
However, students usually learn more if they select their own equations. They will benefit from spending a fair amount of time investigating the graphs of various functions, using graphical calculators or graph-drawing packages, and should thereby come to a better understanding of the behaviour of functions. In cases of failure, their explanation may take the form of sketch graphs in which the important features are highlighted.

Discontinuous functions
The required methods are described in the next section. All the equations in this section are those of continuous functions but discontinuous functions are a rich source of difficulties for numerical methods. Candidates who wish to use these to demonstrate failure are at liberty to do so, providing they do not select trivial cases.

Disallowed functions
A number of equations are used in some detail in these notes, or in the MEI A2 Core Mathematics textbooks.Teachers are free to use these particular equations as classroom examples but they should be regarded as off limits for the coursework and care should be taken to ensure that students are using their own equations whose roots they do not know in advance; they should not use equations from the examples or exercises in a text book as the solutions are given in the answers.

Trivial Equations

Students should avoid trivial equations both when solving them, and when demonstrating failure. For an equation to be non-trivial it must pass two tests.

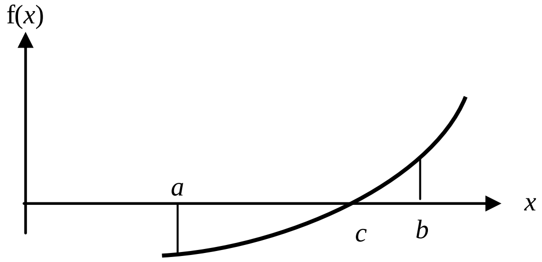
(i) It should be an equation they would expect to work on rather than just write down the solution (if it exists); for instance $\frac{1}{(x-a)}$ = 0 is definitely not acceptable; nor is any polynomial expressed as a product of linear factors.

(ii) Constructing a table of values for integer values of x should not, in effect, solve the equation. Thus $x^3 - 6x^2 + 11x - 6 = 0$ (roots at $x = 1$, 2 and 3) is not acceptable. A typical equation that is used incorrectly in this context is one which has a repeated root at an integer point. The argument is that in constructing the table of values there is no change of sign and therefore the root cannot be found. But if a value of $f(x) = 0$ appears in the table then the root has been found and so the method has not failed.

Notes on the required methods

1. Interval estimation: systematic search for a change of sign

This method involves finding an *interval* in which f(x) changes sign. If f(x) is a continuous function, it follows that it has a root within that interval.



f(a) < 0 and f(b) > 0 $\Rightarrow$ f(c) = 0 for some c between a and b.

Example
Solve f(x) = 0 where f(x) = $x^3 - 3x^2 - 4x + 11$.

Here is the table of values

x	–3	–2	–1	0	1	2	3	4
f(x)	–31	–1	11	11	5	–1	–1	11

This shows that there are three intervals containing roots:
[–2,–1], [1,2] and [3,4].

In order to fulfil the requirements of the coursework, students need to draw and annotate the graph of the function, to show that the change of sign shows that there is a root of the equation in the stated interval.

There are three main ways of homing in on the root; Interval Bisection, Decimal Search and Linear Interpolation.

Bisection
In this method the interval is successively halved by looking at the value of f(x) at its mid-point. For the root in the interval [3,4], you next try an x-value of 3.5.

$$f(3.5) = 3.125$$

Since this is positive you conclude that the root lies in the interval [3,3.5]. The next value you try is the midpoint of the new interval, 3.25; and so on.

Decimal Search
To return to the example of finding the root in the interval [3,4] of the equation

$$f(x) = x^3 - 3x^2 - 4x + 11 = 0.$$

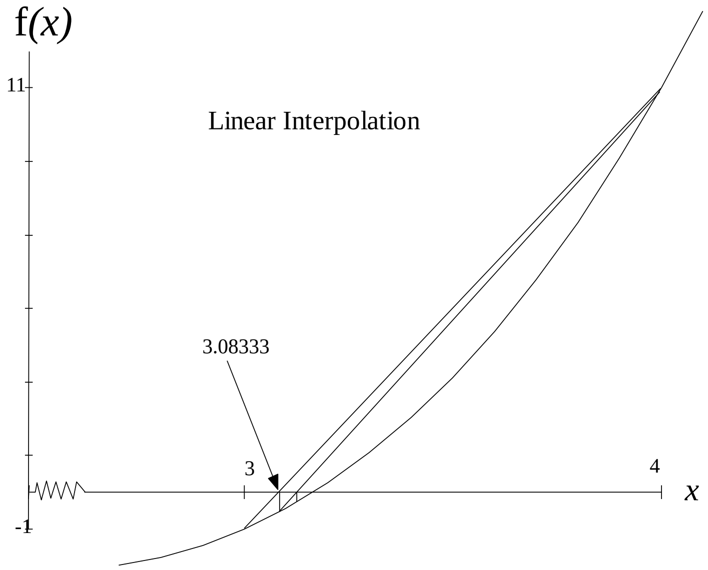
We found earlier that f(3) = –1 and f(4) = 11.

In Decimal Search, instead of trying 3.5 next, try 3.1 (still negative), then 3.2. Since f(3.2) is positive, one would conclude the root lies within the interval [3.1,3.2] and start trying to fix the next decimal place by looking at the signs of f(3.11), f(3.12) and so on until f(3.17) to find the sign change; by then the interval would have been narrowed to [3.16,3.17] and the next step would be to start searching for the third decimal place.

Note. When finding the root in [1,2] using Decimal Search, f(1) = 5, f(2) = –1. Since f(2) is closer to the x-axis, the method is speeded up by considering f(1.9) (negative), f(1.8) (still negative) and f(1.7) (positive). This means that the root in [1.7,1.8] has been found in three steps instead of eight.

Linear Interpolation

In Linear Interpolation not only are the signs of the end points of the interval used but the values of the function there as well. In this example f(3) = –1 and f(4) =11; a straight line drawn between (3, –1) and (4,11) crosses the x-axis at 3.08333 and so this is the next point to try, rather than 3.5 in Bisection and 3.1 in Decimal Search. The same procedure is followed again and again until the required accuracy has been achieved.



$f(x) = x^3 - 3x^2 - 4x + 11$

Advantages and Disadvantages of Change of Sign Methods

The advantages of these methods are:

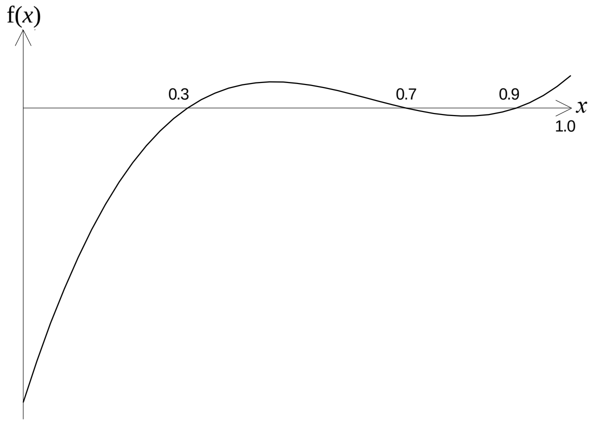
- they are reasonably safe;
- every estimate of the root is accompanied by solution bounds, namely the end points of the smallest interval in which you know it lies.

The disadvantages are:

- they usually take more steps to achieve a given level of accuracy (though with modern technology this is becoming less important);
- the initial search may miss one or more root, for example when the x-axis is a tangent to the curve or when several roots are very close together.

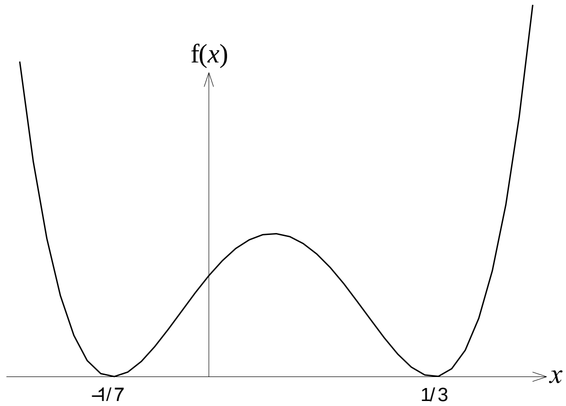
Examples of equations which will cause problems with Change of Sign methods:

(i) $f(x) = x^3 - 1.9x^2 + 1.11x - 0.189 = 0$.
Since $f(0) = -0.189$ and $f(1) = 0.021$, you would conclude that there is a root in the interval [0,1] and if you were using the Bisection method you would evaluate f(0.5). Since this is +0.016, you would conclude that the root is in the interval [0,0.5] and search that interval, eventually arriving at the root $x= 0.3$. This is indeed a root of the equation, but two others, at 0.7 and 0.9, have been completely missed. Similarly Decimal Search would only find the first root.
This problem illustrates the importance of drawing a sketch graph to a suitable scale, since then it is immediately clear that there are three roots rather than one.



$f(x) = x^3 - 1.9x^2 + 1.11x - 0.189 = 0$

(ii) $f(x) = 441x^4 - 168x^3 - 26x^2 + 8x + 1$.
This curve is positive everywhere apart from two points where it touches the x-axis, at $-\frac{1}{7}$ and $\frac{1}{3}$. Any search method is extremely unlikely to find these points and so they will go undetected.
Since there is no change of sign involved, all change of sign methods are doomed to failure on this example.



$f(x) = 441x^4 - 168x^3 - 26x^2 + 8x + 1$

2. *Fixed point iteration - The Newton-Raphson Method*

Fixed point iteration involves finding a single value or *point* as an estimate for the value of *x*, rather than establishing an interval within which it must lie. The point found after sufficient iterations is described as *fixed* because further applications of the function leave it unaltered.
A root of an equation is determined by finding a sequence of estimates and looking at the pattern of convergence. For the Newton-Raphson Method solution bounds are required for one root and, having decided on these, candidates will be expected to show a change of sign between them.
Suppose, for example, that successive iterations give:

2.444 379
2.445 671
2.445 682

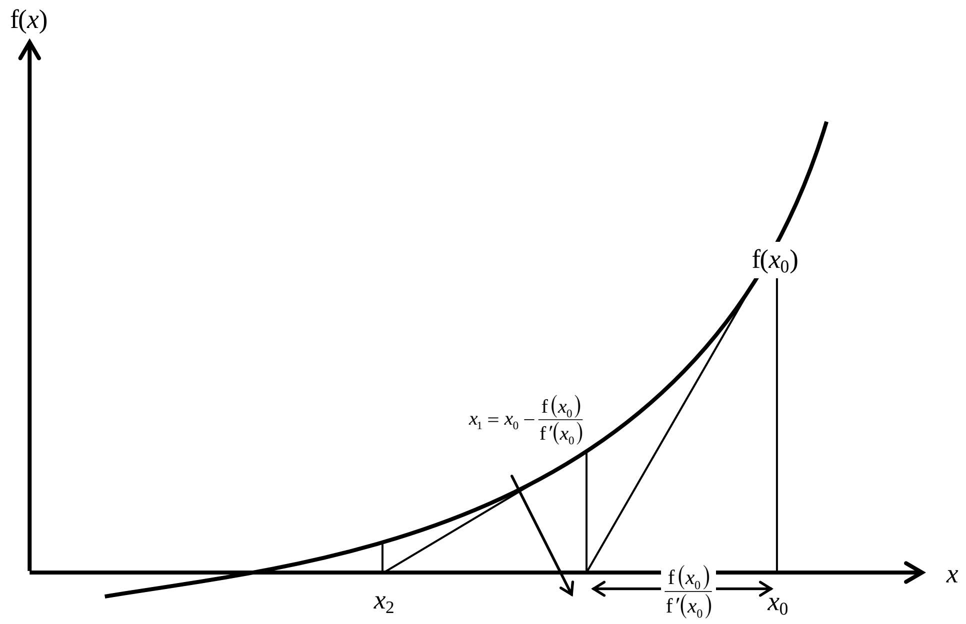
It looks as though the root lies between 2.445 68 and 2.445 70. Just to say that, however, is not adequate; to be safe it is necessary to evaluate f(*x*) at both these points and show that one value is positive, the other negative.

The coursework requirements include two fixed point iteration methods, Newton-Raphson and Rearrangement.

The Newton-Raphson Method
In the Newton-Raphson method, the equation f(*x*)=0 is solved using the iteration

$$x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$$

The most helpful geometrical interpretation of this method is in terms of the intercepts of tangents to the curve with the *x*-axis.

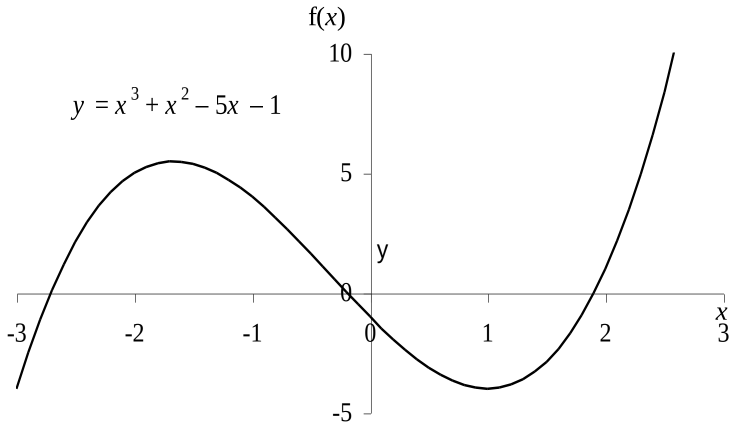


The advantages of the method are that it usually produces convergence, and quickly, provided that the starting point is close to the root being sought.
It can be quite cumbersome to use on a scientific calculator unless the function is rather simple. It requires students to be able to differentiate the function.

Example

Solve $x^3 + x^2 - 5x - 1 = 0$.

A systematic search identifies roots in the intervals [–3, –2], [–1, 0] and [1, 2]



$$x - \frac{f(x)}{f'(x)} = x - \frac{x^3 + x^2 - 5x - 1}{3x^2 + 2x - 5}$$

$$= \frac{2x^3 + x^2 + 1}{3x^2 + 2x - 5}$$

This gives rise to the iterative formula $x_{n+1} = \dfrac{2x_n^3 + x_n^2 + 1}{3x_n^2 + 2x_n - 5}$

Starting with $x_0 = 2$ to find the root in [1, 2] gives $x_1 = 1.909090909$, $x_2 = 1.903235747$, $x_3 = 1.903211926$, $x_4 = 1.903211926$, …..

If this is the root for which error bounds are to be justified to 5 significant figures then it would need to be established that f(1.903205) < 0 and f(1.903215) > 0.

Failure of the Newton-Raphson method is often associated with the choice of initial value. This may need to be close to the (unknown) root and in some cases it is not sufficient to start with an end point of the unit interval containing the root.

For example, $x^3 + 3.7x^2 - 0.2x - 1 = 0$ has roots in the intervals [0,1], [–1,0] and [–4,–3].
Starting with $x_0 = 0$ finds the root –3.68. The root 0.514 can be found by starting with $x_0 = 1$.

It is worth noting that in the example above, using a starting value of $x_0 = 1$ will fail as f ' (1) = 0.

2. *Fixed point iteration - The Rearrangement Method*

Rearranging equations

Any equation $f(x) = 0$ can be rearranged in the form $x = g(x)$ in any number of ways, any of which can be used as a basis for the iteration $x_{n+1} = g(x_n)$.

Thus $\quad f(x) = x^3 - 2x^2 - 4x + 4 = 0$

can be written as

$$x = \frac{1}{4}\left(x^3 - 2x^2 + 4\right)$$

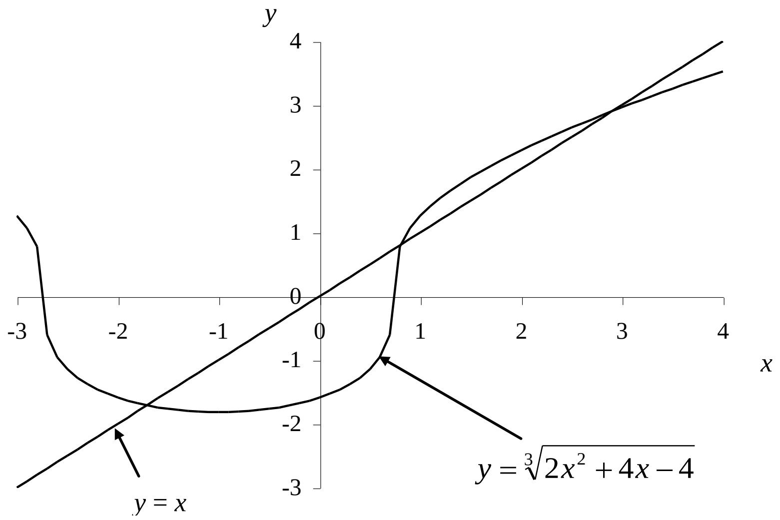
$$x = \sqrt{\frac{x^3 - 4x + 4}{2}}$$

$$x = \sqrt[3]{2x^2 + 4x - 4}$$

and an infinite number of other ways. Some of these will lead to successful, i.e. convergent, iterations, others not.

The rearrangement $x = \sqrt[3]{2x^2 + 4x - 4}$ gives rise to the iteration $x_{n+1} = \sqrt[3]{2x_n^2 + 4x_n - 4}$.

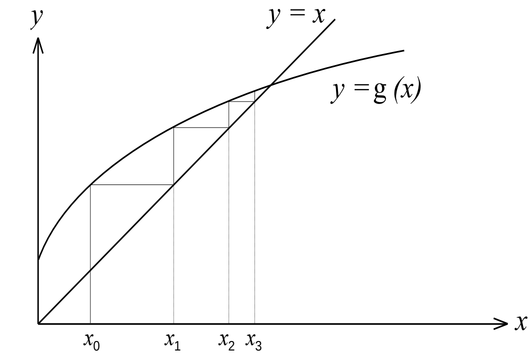
The process is effectively finding the intersection of the curve $y = \sqrt[3]{2x^2 + 4x - 4}$ and the line $y = x$.



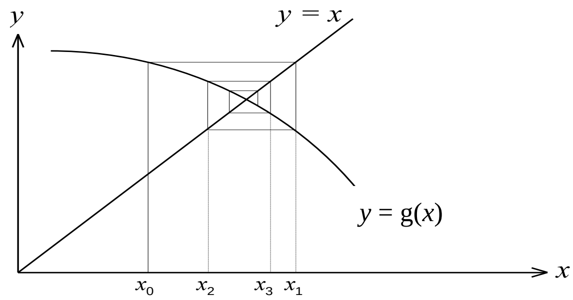
The iteration in this process is represented graphically as either a staircase or a cobweb diagram. Students can think of this as follows:

Choose a value of x	...	Take a starting point on the x-axis
Find the corresponding value of y	...	Move vertically to the curve
Make your value of y into your new value of x	...	Move horizontally across to the line $y = x$
Find the corresponding value of y	...	Move vertically to the curve

and so on

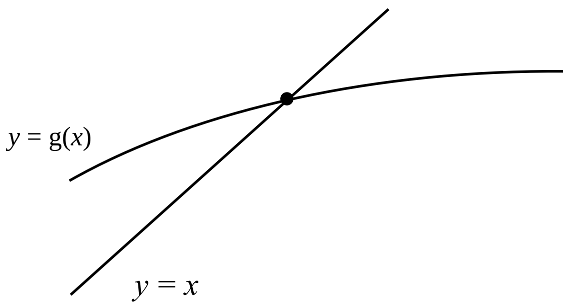


Staircase diagram

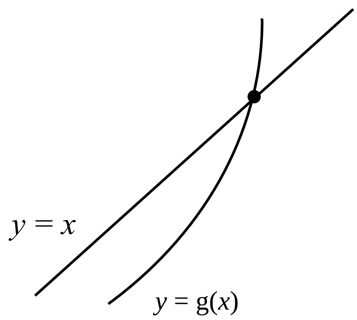


Cobweb diagram

The iteration will converge, given a suitable starting value if the gradient of the curve at the point of intersection is numerically less than 1. Since the gradient of $y = x$ equals 1, this means that, for positive gradients of g(x), convergence occurs when the curve $y = \mathrm{g}(x)$ is less steep than the line $y = x$ at the point of intersection, divergence when it is steeper.

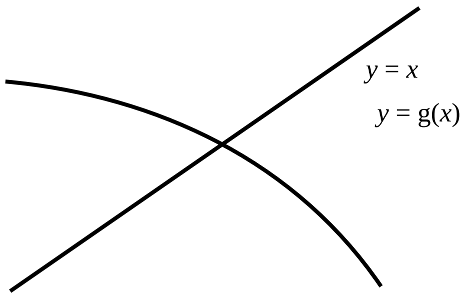


The iteration converges

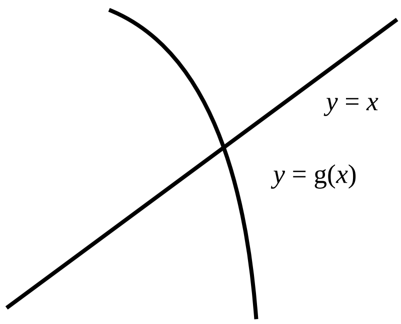


The iteration diverges

If the gradient of g(x) is negative it is less easy to see at a glance whether the iteration will converge or not.

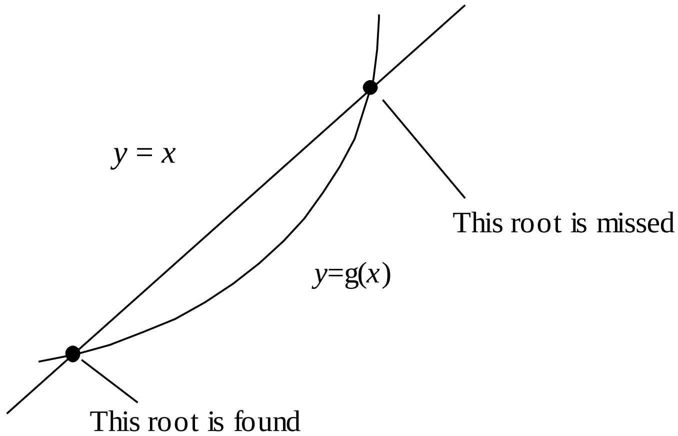


The iteration converges



The iteration diverges

If the equation has more than one root, and f(x) is continuous, this method will usually miss at least one root.



(There are however exceptions to this rule, e.g. equations with repeated roots.)
In the case of using

$x_{n+1} = \sqrt[3]{2x_n^2 + 4x_n - 4}$ to solve $x^3 - 2x^2 - 4x + 4 = 0$

the roots in the intervals [–2, –1] and [2,3] are found, albeit slowly. That in the interval [0, 1] is lost.

There are many ways of making x the subject for any equation, some of which will give more successful iterations than others. Students may need to try several iterations before finding one which converges to the root they are seeking. Consequently the method is more useful in a refined form (e.g. Newton-Raphson). It is a rich source of interest, however, for students with a good software package (e.g. Autograph) to plot a number of different rearrangements to see the shape of the curve and the speed of convergence.

As far as coursework is concerned, it is sufficient for students to work with one rearrangement of one equation, provided that equation has two (or more) roots and the iteration finds one of them but fails to find another. Students will need to illustrate the process with a staircase or cobweb diagram. They should know the condition of convergence to a root a, namely that $-1 < g'(a) < 1$; this may be discussed on the basis of graphical considerations. The use of calculus is another rich source of interest, but is not necessary to fulfil the requirements of the coursework..

The relationship between the starting value and the root found is complicated and a common source of confusion. Students are not expected to know more than that if a root can be found, this will happen when the starting value is sufficiently close to it.

There is a wealth of exciting investigatory work that can be done using this iterative procedure. The logistic equation

$$x_{n+1} = kx_n(1 - x_n)$$

is particularly rich, with different patterns of convergence, oscillation, chaos etc. appearing for different values of k. This is well outside the immediate coursework requirements but it will enthuse many students and illustrates a wider point, that the behaviour of the iteration can be more interesting than the actual solution.

Use of technology

While it is possible to do this coursework with anything from a scientific calculator to a sophisticated mathematics software package, it is recommended that once students understand how a method works, they should use a graphical calculator, spreadsheet or software package such as Autograph.

They will benefit from being confident in the following techniques.

1. Curve Drawing

Although this can be done with a spreadsheet or a graphical calculator, students are likely to find a software package the most effective way to draw graphs due to versatility and ease of operation.

They need to be able to enter a given function and select suitable scales. Use of the zoom and trace facilities will also be helpful.

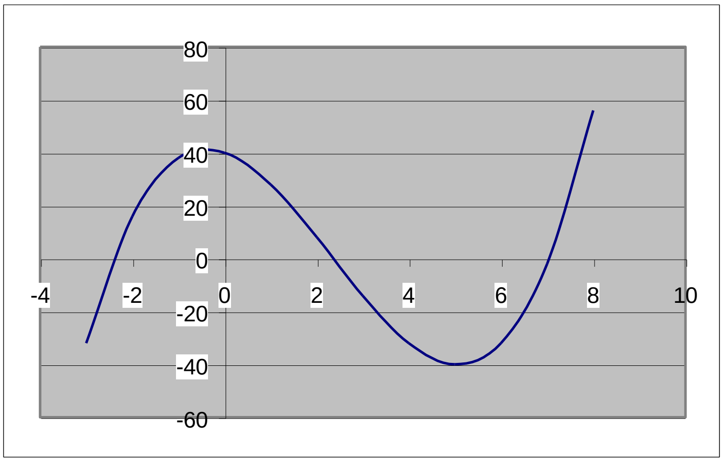
2. Function evaluation

Students should know how to store a given function in a calculator's memory and to evaluate it for particular values of the variable. They will find it helpful to know how to create a table of values on their calculator or spreadsheet.

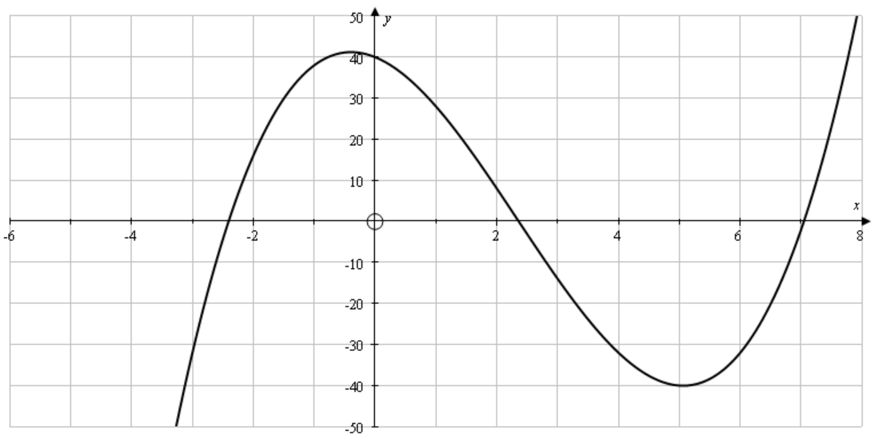
In cases where a spreadsheet is used, the function will be evaluated prior to drawing the curve. At this stage it is essential that a sufficiently large range of values of x is considered in an attempt to locate all the roots. A restricted range for which it is appropriate to draw the graph can then be identified.

This process is shown below for the function $y = x^3 – 7x^2 – 6x + 40$ using a typical spreadsheet. The graph is drawn only for the restricted range shown in the boxes. (In this case three roots of $x^3 – 7x^2 – 6x + 40 = 0$, the maximum possible number for a cubic, have been located.)

x	*y*	
-6	-392	
-5	-230	
-4	-112	
-3	-32	
-2	16	sign change
-1	38	
0	40	
1	28	
2	8	
3	-14	sign change
4	-32	
5	-40	
6	-32	
7	-2	
8	56	sign change
9	148	



On the other hand, entering the function in Autograph will yield the graph immediately.



Students should not swamp their coursework with endless print-outs which show little or no appreciation of what is actually happening, but should edit these in a manner which is informative and reflects their understanding.
The method used for editing will vary depending on the student's IT competence, and in many cases the spreadsheet will be used only for the calculations. Students should state any formulae used in constructing a spreadsheet.
The spreadsheet print-out below shows how a student can present the continuation of the work started above, in this case using decimal search to find the root between –3 and –2.

x	y	
-3	-32	
-2.9	-25.859	
-2.8	-20.032	
-2.7	-14.513	
-2.6	-9.296	
-2.5	-4.375	
-2.4	0.256	sign change
-2.3	4.603	
-2.2	8.672	
-2.1	12.469	
-2	16	

x	y	
-2.5	-4.375	
-2.49	-3.8989	
-2.48	-3.4258	
-2.47	-2.9555	
-2.46	-2.4881	
-2.45	-2.0236	
-2.44	-1.562	
-2.43	-1.1032	
-2.42	-0.6473	
-2.41	-0.1942	
-2.4	0.256	sign change

x	y	
-2.41	-0.1942	
-2.409	-0.1491	
-2.408	-0.1039	
-2.407	-0.0589	
-2.406	-0.0138	
-2.405	0.0312	sign change
-2.404	0.0763	
-2.403	0.1212	
-2.402	0.1662	
-2.401	0.2111	
-2.4	0.256	

At this stage it can be said that $-2.406 < x < -2.405$

3. Iteration

Students should know the most efficient way to enter an iterative procedure into their calculators or spreadsheet. This will save them a lot of time.

4. Programming

It is doubtful whether it is worth the investment of time to learn to program a calculator specifically for this coursework. However those students who already know how to program, or who want to learn anyway, will enjoy using their programs on this work. It is not recommended that students be given complete ready made programs.

Students who wish to use a spreadsheet for a change of sign method, and wish to do so efficiently, will need to know how to program the necessary decision making.

Methods for Advanced Mathematics (C3) Coursework: Assessment Sheet

Task: Candidates will investigate the solution of equations using the following three methods

- Systematic search for change of sign using one of the three methods: decimal search, bisection or linear interpolation.
- Fixed point iteration using the Newton-Raphson method.
- Fixed point iteration after rearranging the equation $f(x) = 0$ into the form $x = g(x)$.

Candidate Name			**Candidate Number**	
Centre Number			**Date**	
Domain	**Mark**	**Description**	**Comment**	**Mark**
Change of sign method (3)	1 1 1	The method is applied successfully to find one root of an equation. Error bounds are stated and the method is illustrated graphically. An example is given of an equation where one of the roots cannot be found by the chosen method. There is an illustrated explanation of why this is the case.		
Newton Raphson method (5)	1 1 1 1 1	The method is applied successfully to find a root of a second equation. All the roots of the equation are found. The method is illustrated graphically for one root. Error bounds are established for one root. An example is given of an equation where this method fails to find a particular root despite a starting value close to it. There is an illustrated explanation of why this has happened		
Rearranging $f(x)$ into the form $x = g(x)$ (4)	1 1 1 1	A rearrangement is applied successfully to find a root of a third equation. Convergence of this rearrangement to a root is demonstrated graphically and the magnitude of $g'(x)$ is discussed. A rearrangement of the same equation is applied in a situation where the iteration fails to converge to the required root. This failure is demonstrated graphically and the magnitude of $g'(x)$ is discussed.		
Comparison of methods (3)	1 1 1	One of the equations used above is selected and the other two methods are applied successfully to find the same root. There is a sensible comparison of the relative merits of the three methods in terms of speed of convergence. There is a sensible comparison of the relative merits of the three methods in terms of ease of use with available hardware and software.		
Written communication (1)	1	Correct notation and terminology are used.		
Oral communication (2)	2	Presentation ☐ Interview ☐ Discussion ☐	Please tick at least one box and give a brief report.	
Half marks may be awarded but the overall total must be an integer. Please report overleaf on any help that the candidate has received beyond the conduct guidelines.			**Total**	/18

Authentication:

Teachers should ensure that an OCR declaration form (CCS160) is completed and signed by every Teacher involved in the assessment and sent with the marks to the Moderator.

Coursework must be available for moderation by OCR.

C3coursework/MEIversion