Mathematics

C3 Coursework Guide

Numerical Methods

(Using Google Docs & Sheets)

Contents

Introduction

In this coursework you will investigate numerical methods of solving equations.

Your actual coursework will consist of your use of three different methods to find the roots of some equations (and you'll show how each of these methods sometimes fails to find roots), followed by a comparison of how effective these methods are. This means there should be **four main sections** in your work. You must **number the pages** in your work!

What you need to do is outlined on pages 6, 10, 13 and 14 of this guide. There is also an explanation of each method given in this guide (see the Contents list on the front cover for page numbers), but you can also refer to the MEI coursework booklet and any other material (e.g. videos) in the "C3 Coursework" part of the Maths Homepage, and you can also use anything else that your Maths teacher may suggest.

By the end of the coursework you should be able to:

- use the terms: equation, function, root and solution appropriately
- understand that some equations cannot be solved analytically by, for example, factorising
- apply different methods for the numerical solution of such equations, to any degree of accuracy, using a computer
- compare the methods in terms of their efficiency and ease of use
- be able to explain how the methods work, with the help of graphs

The methods you will learn are

- Systematic search for a **Change of Sign** (**Decimal Search**)
- **Fixed Point Iteration** after rearranging the equation f(x) = 0 into the form **x = g(x)**
- **Fixed Point Iteration** using the **Newton-Raphson** method

This coursework counts for 20% of the marks for the C3 module.

Why use numerical methods?

If we want to find the solution to the **equation** $x^3 - x = 0$
we can **factorise** it, so that $x(x+1)(x-1) = 0$
so the three **root**s are $x = 0$, $x = -1$ and $x = 1$.

If we want to find the solution to the equation $x^2 + 10x + 8 = 0$ we can use the **quadratic formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
which gives the two roots, which could be written exactly in surd form, or rounded to some number of decimal places: $x = -0.8769$ (4 d.p.) and $x = -9.1231$ (4 d.p.).

Some equations, however, such as $x^3 - 5x + 3 = 0$, ***cannot*** be solved by algebraic or analytical methods (such as factorising or by a simple formula).
To solve these equations we use **numerical methods**.

You will be asked to investigate **three different numerical methods** and will have to choose your own equations to use, which should be different from those that other students have chosen.

<u>Warning!!</u>

*You will **lose marks** if you choose equations which **can** be solved algebraically or analytically, as we should **<u>only</u> use numerical methods when we <u>cannot</u> solve the equations otherwise.***

<u>Important Note</u>

For all three methods, using the method is when you **do the calculations to find the root** (you will actually get the program Google Sheets to perform them for you in a spreadsheet). Your graphs are just *illustrating* your methods. This means that you must **show full calculations for each method** (including where you demonstrate an example of each method failing) – you can do this **by pasting in your Google Sheets spreadsheets**. When you compare the methods, it is **how efficiently the calculations work** that you are comparing, *<u>not</u>* how easy it was to draw the different illustrations on Autograph!

<u>Terminology</u>

It is important that you understand and use correctly the mathematical language associated with this work. Consequently there is one mark allocated specifically for correct notation and terminology.

Expression
An **expression** is a number of terms that are added or subtracted.
For example, $x^3 - 3x + 1$ is an **expression**.

Function
A **function** of x is usually denoted **f(x)**. (We sometimes use other letters when we have already used f.)
The **function** may be defined, for example, $f(x) = x^3 - 3x + 1$
A **graph** of a **function**, $y = f(x)$, may therefore be drawn on Cartesian axes (labelled x and y).

Equation
An **equation** is an expression set equal to 0, or some other number, or one expression set equal to another. For example, $f(x) = 0$ and $x^3 - 3x + 1 = 0$ are both **equation**s.

The solution of an equation
The terms ***root*** and ***solution*** are often confused. The **equation** $x^3 - x = 0$ has three **root**s, namely $x = 0$, $x = -1$ and $x = 1$. The **solution** of the equation is made up of **all** the **root**s, so it is <u>$x = 0, -1$ or 1</u>.
Consequently, to *solve* an equation, you must find *all* its roots. In your coursework you will *solve* an equation with the Newton-Raphson method, but just *find one root of* an equation with the other two methods.

In your coursework, each **equation** can be written $f(x) = 0$, where you **define your f(x)** each time. The **root**s of this are the x-values of the points where the curve $y = f(x)$ cuts the x-axis. A common mistake among students is to call the *equation* $y = f(x)$, or even just $f(x)$, rather than $f(x) = 0$.

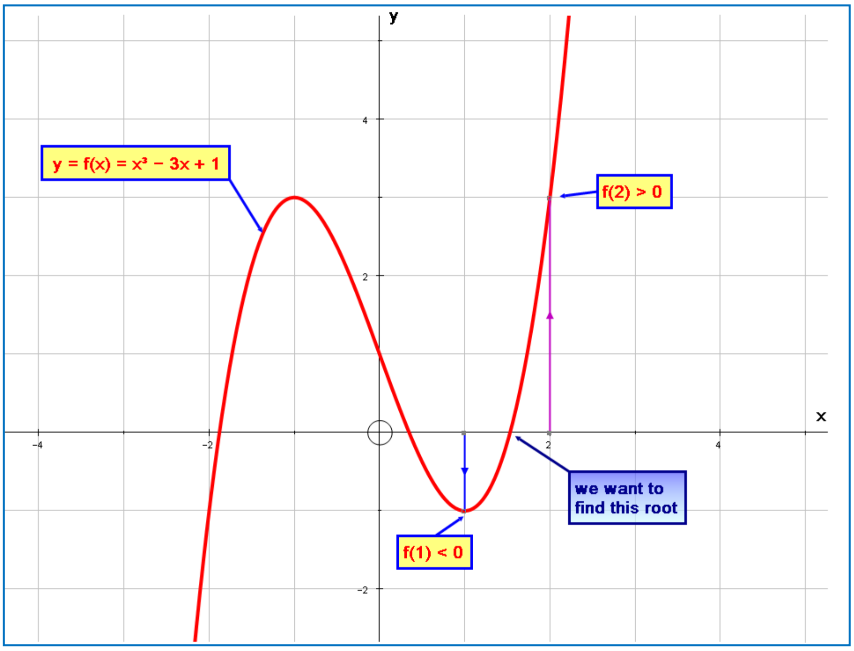
Systematic Search for a Change of Sign (Decimal Search) Method

For example, we might want to find a root of the equation f(x) = 0, where f(x) = x^3 - 3x + 1.

Looking at the graph of y = f(x), we can see that a root lies in the interval [1 , 2].

The y-coordinates to the left of this root are negative and the y-coordinates to the right of this root are positive. So f(x) changes sign from negative to positive somewhere between 1 and 2.

We can find where this happens by evaluating f(x) for x values from 1 to 2, in increments of 0.1. This can be done efficiently using an Google Sheets spreadsheet.



The click-and-drag tool can be used to quickly generate the row of x values.

The f(x) formula only needs to be entered once (in cell B2 in this example), then it can be copied across the row with the click-and-drag tool.

	A	B	
1	x	1	1.1
2	f(x)	=B1^3-3*B1+1	=C1^3

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
f(x)	-1	-0.969	-0.872	-0.703	-0.456	-0.125	0.296	0.813	1.432	2.159	3

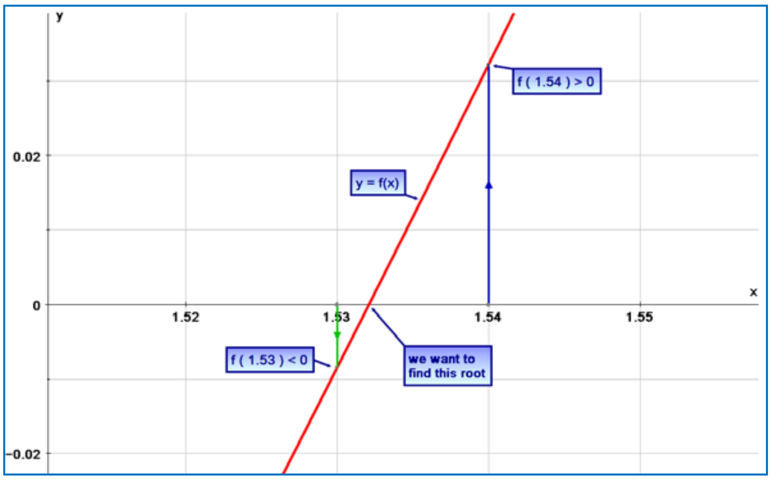
We can see from this table that f(1.5) is negative but f(1.6) is positive, so the root must lie in the interval [1.5 , 1.6]. We can now consider f(x) for values of x from 1.5 to 1.6, in increments of 0.01, again looking for a change of sign in f(x). We can then continue this process, taking smaller increments of x in each row, every time dividing our interval into **ten** parts (hence the name: **Decimal** Search). In your coursework, you should do five rows, so that the x values have 5 d.p. on the last row:

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
f(x)	-1	-0.969	-0.872	-0.703	-0.456	-0.125	0.296	0.813	1.432	2.159	3
x	1.5	1.51	1.52	1.53	1.54	1.55	1.56	1.57	1.58	1.59	1.6
f(x)	-0.125	-0.087049	-0.04819	-0.00842	0.032264	0.073875	0.116416	0.159893	0.204312	0.249679	0.296
x	1.53	1.531	1.532	1.533	1.534	1.535	1.536	1.537	1.538	1.539	1.54
f(x)	-0.00842	-0.0043957	-0.00036	0.003686	0.007741	0.011805	0.015879	0.019961	0.024053	0.028154	0.032264
x	1.532	1.5321	1.5322	1.5323	1.5324	1.5325	1.5326	1.5327	1.5328	1.5329	1.533
f(x)	-0.000359	0.000045	0.000449	0.000854	0.001258	0.001662	0.002067	0.002472	0.002877	0.003281	0.003686
x	1.532	1.53201	1.53202	1.53203	1.53204	1.53205	1.53206	1.53207	1.53208	1.53209	1.5321
f(x)	-0.000359	-0.000319	-0.000278	-0.000238	-0.000198	-0.000157	-0.000117	-0.000076	-0.000036	0.000005	0.000045

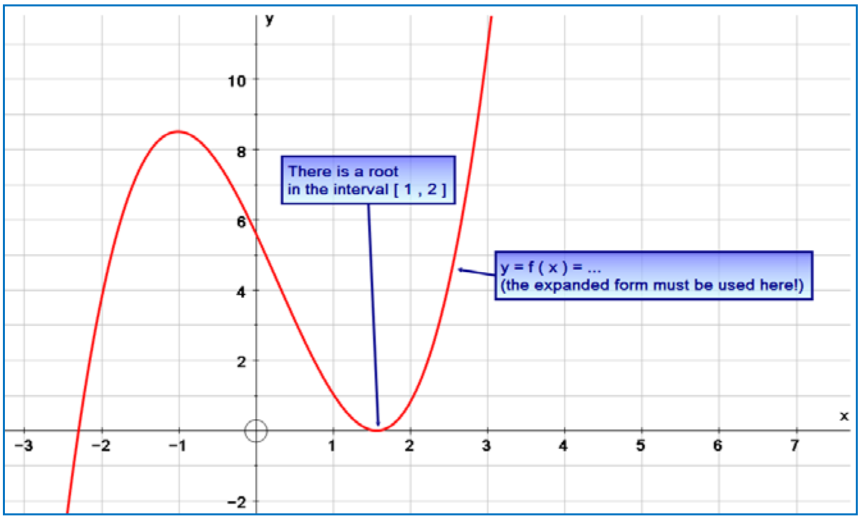
If we take the midpoint of the final interval, we can say that the root we have found is
<u>**x = 1.532085** with a maximum **error of ±0.000005**</u>

Note that we *don't* say how many d.p. the root has, we use the **error bounds** instead.

In your coursework, you will need to include **two** zoomed-in graphs to illustrating this method. These should show x values with one, then two, decimal places. For example, the second zoom should like look like this:



<u>Failure of Systematic Search for a Change of Sign (Decimal Search) Method</u>



This method will sometimes fail to find a root of an equation. An example of how this might happen is when there is a repeated root.

This graph shows a (repeated) root in the interval [1 , 2] so we would try a decimal search in that interval. However, the first row of the table in Google Sheets does *not* give a change of sign of f(x). This is because the (repeated) root is actually between 1.5 and 1.6.

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
f(x)	1.03488	0.71944	0.4536	0.24336	0.09472	0.01368	0.00624	0.0784	0.23616	0.48552	0.83248

The equation used here was f (x) = 0 where **f(x) = x^3 - 0.82x^2 - 4.7424x + 5.59728**

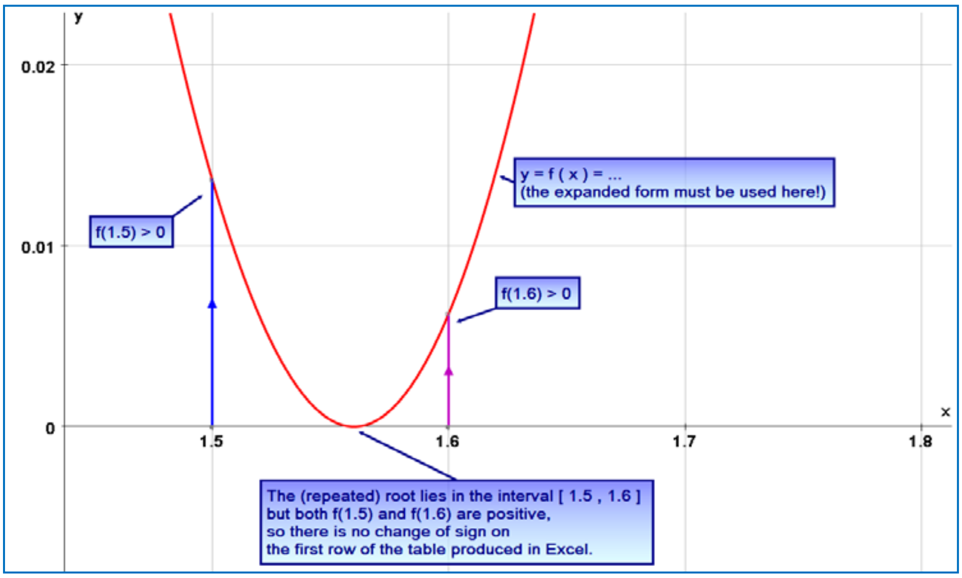
This was created easily using the property that a squared factor gives a repeated root. We started with

not an integer!

the repeated root has 2 d.p.

f(x) = (x+2.3)(x-1.56)^2

then expanded it to give the form above that looks too difficult to solve.



What to include in the

Introduction

State why we sometimes need to use numerical methods to find a root of, or solve, an equation.

Mention the three different methods you will be using.

What to include in the

Systematic Search for a Change of Sign (Decimal Search) Method *section:*

1. State the **equation you are trying to find a root of**,
   which is f(x) = 0, where you define what your f(x) is.
   (There should be a different f(x) used for each method.)

2. Show the **graph of y = f(x)**, indicating the root you are trying to find.

3. Describe **how the method works**, referring to your **Google Sheets** calculations and your zoomed-in graph illustrations.
   - Do this for the **first row** of calculations (with x values to 1 d.p.) with its **corresponding zoomed-in image**,
   - then for the **second row** and the **second zoom**.
   - You then paste in the **final three rows** (up to x values with 5 d.p.).

4. Take the midpoint of the final interval. State the value of the **root** you have found: ***x* = ……** with a maximum **error of ±0.000005**

So, that's some writing, five rows of tables from Google Sheets and 3 graphs from Autograph in this section.

5. Repeat the steps 1. and 2. from above, but this time for an equation where the method **fails** to find the root.

6. Describe **why the method fails to work**, referring to your **Google Sheets** calculations and your zoomed-in **graph** illustration.
   - Do this for the **first row** of calculations (with x values to 1 d.p.) with its **corresponding zoomed-in image**. (You cannot create further rows on the table as there is no change of sign.)

So, that's some writing, one row of a table from Google Sheets and 2 graphs from Autograph in this section.

Fixed Point Iteration, x = g(x) Method

This is a method which involves using an iterative formula. We will rearrange **f(*x*) = 0** to the form ***x* = g(*x*)** to get the *Iterative Formula* $x_{r+1} = g(x_r)$

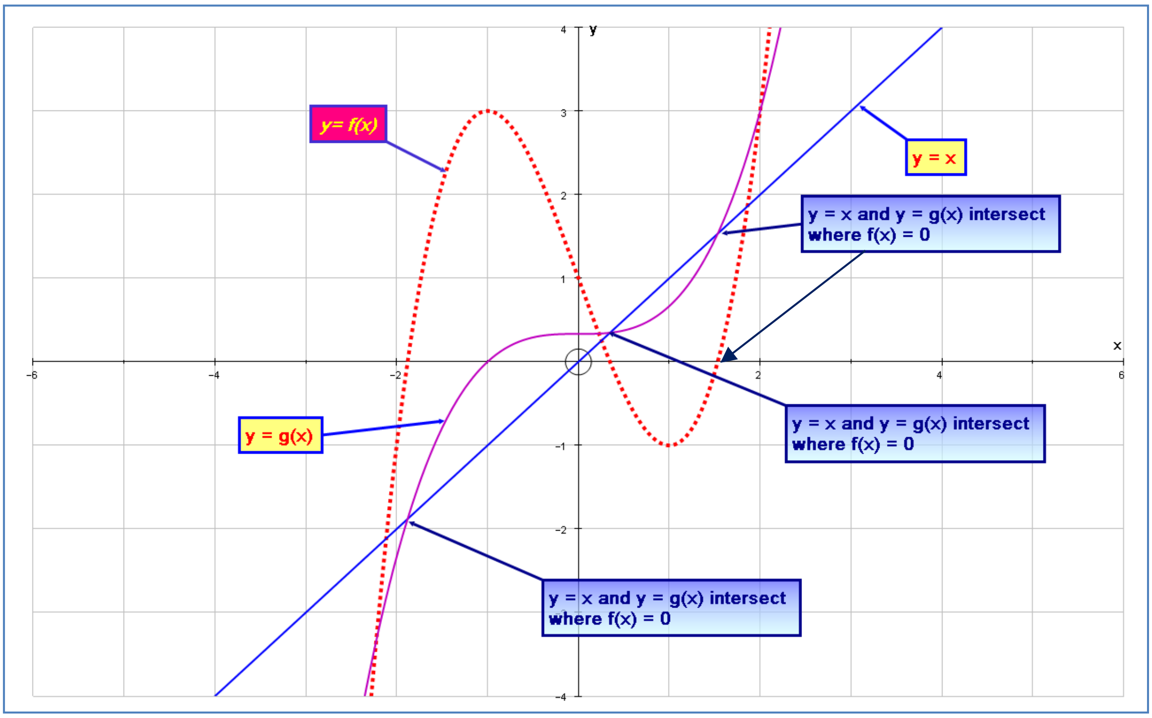
For example, say we want to find a root of the equation **f(*x*) = 0**
where **f(*x*)** = $x^3 - 3x + 1$ so we have $x^3 - 3x + 1 = 0$

We can rearrange this as ***x* = g(*x*)** in several ways:

❶ $x = \dfrac{x^3+1}{3}$ ❷ $x = \sqrt[3]{3x-1}$

❸ $x = \dfrac{3x-1}{x^2}$ ❹ $x = \dfrac{-1}{x^2-3}$

This means that where ***x* = g(*x*)** (so where the graphs of *y* = *x* and *y* = g(*x*) intersect) is a root of **f(*x*) = 0**. The graph below uses $g(x) = \frac{x^3+1}{3}$



The x = g(*x*) rearrangements become the iterative formulae $x_{r+1} = g(x_r)$:

❶ $x_{r+1} = \dfrac{x_r^3+1}{3}$ ❷ $x_{r+1} = \sqrt[3]{3x_r - 1}$

❸ $x_{r+1} = \dfrac{3x_r - 1}{x_r^2}$ ❹ $x_{r+1} = \dfrac{-1}{x_r^2 - 3}$

One of these iterative formulae may be used to find a root of the equation f(*x*) = 0.

(Sometimes this method does not work – you will need to show an example where it fails to work as part of your coursework.)

In each case, we **take x_0 to be an integer near to the root** we are trying to find (we can see this from the graph).

We put $\mathbf{x_0}$ into the RHS of the iterative formula to find $\mathbf{x_1}$, then we put $\mathbf{x_1}$ into the RHS of the formula to find $\mathbf{x_2}$ and keep going until $\mathbf{x_r}$ converges to the root. Google Sheets will do these iterations very quickly for us if we copy the formula down the column (using the click-and-drag tool). If it diverges, then we need to choose a different iterative formula.

It turns out that the iterative formula ❶ $x_{r+1} = \frac{x_r^3+1}{3}$ finds the root near the integer 1,

x_0	1
x_1	0.66667
x_2	0.43210
x_3	0.36023
x_4	0.34891
x_5	0.34749
x_6	0.34732
x_7	0.34730
x_8	0.34730
x_9	0.34730

	A	B
1	x_0	1
2	x_1	=(B1^3+1)/3
3	x_2	=(B2^3+1)/3
4	x_3	=(B3^3+1)/3
5	x_4	=(B4^3+1)/3

The formula was only entered into cell B2 and the click-and-drag tool was then used to copy it down the column.

but fails to find the root near the integer 2.

x_0	2
x_1	3.00000
x_2	9.33333
x_3	271.34568
x_4	############
x_5	############
x_6	############
x_7	############
x_8	#NUM!
x_9	#NUM!

The x_r values are diverging away from the root.

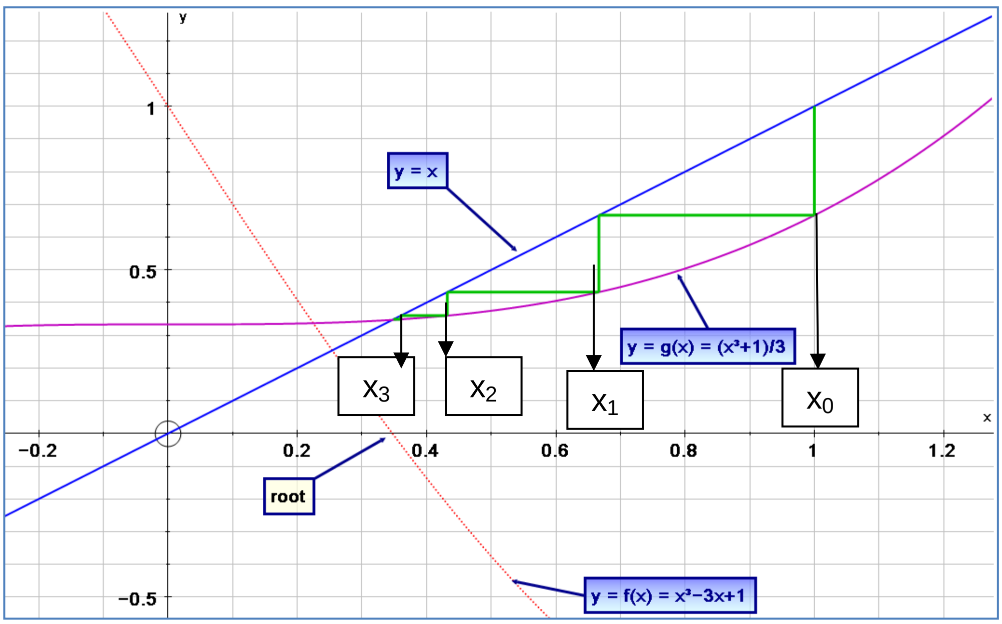
Google Sheets displays #### when the value has too many digits to fit in the cell width.

Google Sheets displays #NUM! when the values are too large (or too small if negative) for it to calculate with.

The x_r values have converged towards the root. We can confirm the root x = 0.34730 by looking for a change of sign in f(x) for x values 0.000005 above and below this root, using Google Sheets.

x	0.347295	0.347305
f(x)	0.000004	-0.000023

This change of signs confirms that we can state that a root is
***x* = 0.34730** with a maximum **error of ±0.000005**



You can show the iterative process on your graph, using Autograph which produces a staircase like this (if gradient of g(*x*) is positive at the intersection)

or a cobweb (if gradient of g(*x*) is negative at the intersection).

You need to label x_0 to x_3 on your graph.

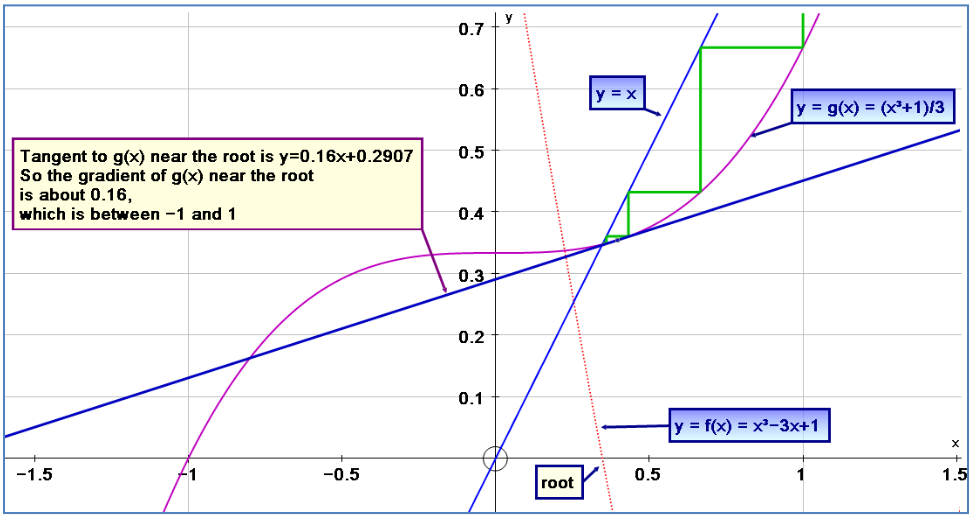
This method can only find a root in situations where the gradient of g(x) near the root satisfies **-1 < g'(*x*) < 1**.
The method will fail to find a root when the gradient of g(x) near the root does not satisfy **-1 < g'(*x*) < 1**.

In your coursework, you need to confirm that the **value of g'(*x*) near the root** is **between -1 and 1** and say that the method only works when this is the case.
In the case where the method fails to find the root, you need to confirm that the **value of g'(*x*) near the root** is **not between -1 and 1** and say that the method does not work when this is the case.

You can do this in several ways, such as *one of* the following:

- draw a tangent to g(*x*) near the root and find the gradient of this tangent.

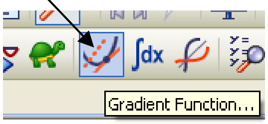
  Autograph will give you the equation of the tangent in the form *y* = m*x* + c, which you can use to find the gradient.

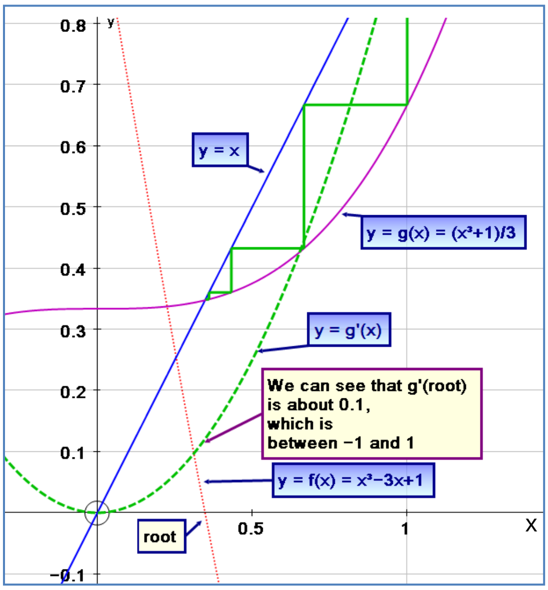


- use the gradient function, *y* = g'(*x*) to find the gradient of g(*x*) near the root.

  The y-coordinate of the gradient function graph gives the value of the gradient of g(*x*) at that value of *x*.

  You can draw this using the icon next to the tortoise on Autograph.





- compare the steepness of the curve *y* = g(*x*) near the root with the steepness of the line *y* = *x* (or *y* = -*x*) at this point.

- evaluate the gradient of g(*x*) near the root by differentiating g(*x*). For example,

  if $g(x) = \frac{x^3+1}{3}$ then $g'(x) = x^2$ and so g'(root) is about $0.34730^2 \approx 0.12$, which is between -1 and 1.

What to include in the

Fixed Point Iteration, *x* = g(*x*) Method *section:*

1. State the **equation you are trying to find a root of**, which is f(*x*) = 0, where you define what your f(*x*) is. (There should be a different f(*x*) used for each method.)
2. Show how you **have rearranged it to the form *x* = g(*x*)**.
3. State the **iterative formula** that you are therefore using. This should be of the form $x_{r+1} = g(x_r)$
4. Show a **graph with your *y* = f(*x*), *y* = g(*x*) and *y* = *x***.
5. Show your **calculations in Google Sheets**, clearly stating the integer near the root that you have chosen to be x_0 and a **zoomed-in graph** showing the staircase or cobweb with the first four x_r values labelled (x_0 to x_3).
6. Describe **how the iterations are performed and the x_r values converge to the root**, referring to both your Google Sheets table and your zoomed-in graph illustration.
7. Perform a **change of sign check** by looking at f(*x*) for *x* values 0.000005 above and below your root, using Google Sheets.
8. State that your **root is *x* = ……** with a maximum **error of ±0.000005**
9. Confirm that the **value of g'(*x*) near the root** is **between -1 and 1** and say that the method only works when this is the case. You can do this in several ways, such as *one of* the following:
   - draw a tangent to g(*x*) near the root and find the gradient of this tangent.
   - use the gradient function, y = g'(*x*) to find the gradient of g(*x*) near the root. Draw this using the icon next to the tortoise on Autograph.
   - compare the steepness of the curve y = g(*x*) near the root with the steepness of the line *y* = *x* (or *y* = -*x*) at this point.
   - evaluate the gradient of g(*x*) near the root by differentiating g(*x*).

So, that's some writing, two tables from Google Sheets and 3 graphs from Autograph in this section.

10. Now repeat all of the steps above (apart from 7. & 8.) for a case where this method fails to find a root you are looking for, using the ***same** equation* **f(x) = 0** with the **same f(x)**. It can be a different rearrangement *x* = g(*x*), but it doesn't have to be. (You will *not* do numbers 7. & 8. as you won't find a root to confirm with change of sign.)

So, that's some writing, one table from Google Sheets and 3 graphs from Autograph in this failure of the method section.

Fixed Point Iteration, Newton-Raphson Method

This is a method which involves using an iterative formula,

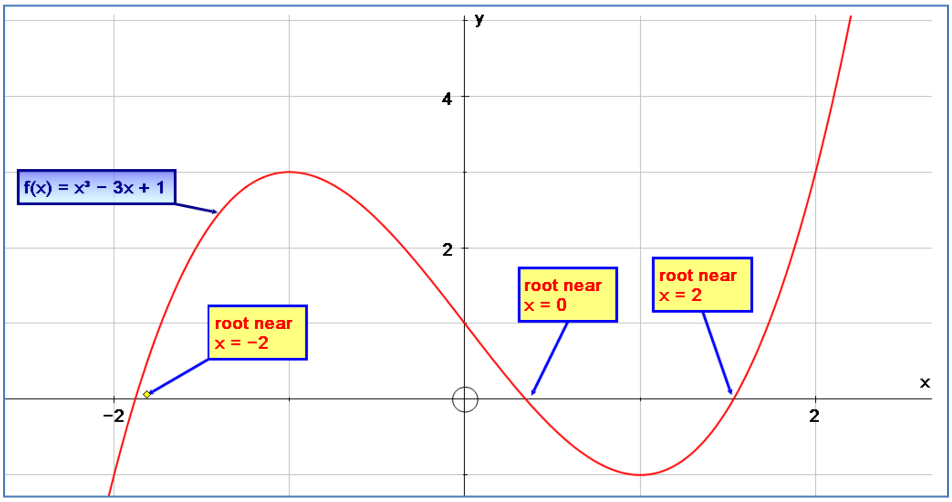
in this case the Newton-Raphson Iterative Formula, which is $x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$

For example, if we want to solve the equation f(x) = 0, where f(x) = x^3 - 3x + 1,
we first need to differentiate f(x) to get f '(x) = 3x^2 - 3 and substitute both f(x) and f '(x) into the Newton-Raphson Iterative Formula,

to give us our iterative formula: $x_{r+1} = x_r - \frac{x_r^3 - 3x_r + 1}{3x_r^2 - 3}$

For each root, we **take x_0 to be an integer near to the root** we are trying to find (we can see this from the graph).

We put $\mathbf{x_0}$ into the RHS of the iterative formula to find $\mathbf{x_1}$, then we put $\mathbf{x_1}$ into the RHS of the formula to find $\mathbf{x_2}$ and keep going until $\mathbf{x_r}$ converges to the root. Google Sheets will do these iterations very quickly for us if we copy the formula down the column (using the click-and-drag tool).



x_0	2
x_1	1.66667
x_2	1.54861
x_3	1.53239
x_4	1.53209
x_5	1.53209

The formula was only entered into cell B2 and the click-and-drag tool was then used to copy it down the column, and copied across to find the other roots.

	A	B
1	x_0	2
2	x_1	=B1-(B1^3-3*B1+1)/(3*B1^2-3)
3	x_2	=B2-(B2^3-3*B2+1)/(3*B2^2-3)
4	x_3	=B3-(B3^3-3*B3+1)/(3*B3^2-3)

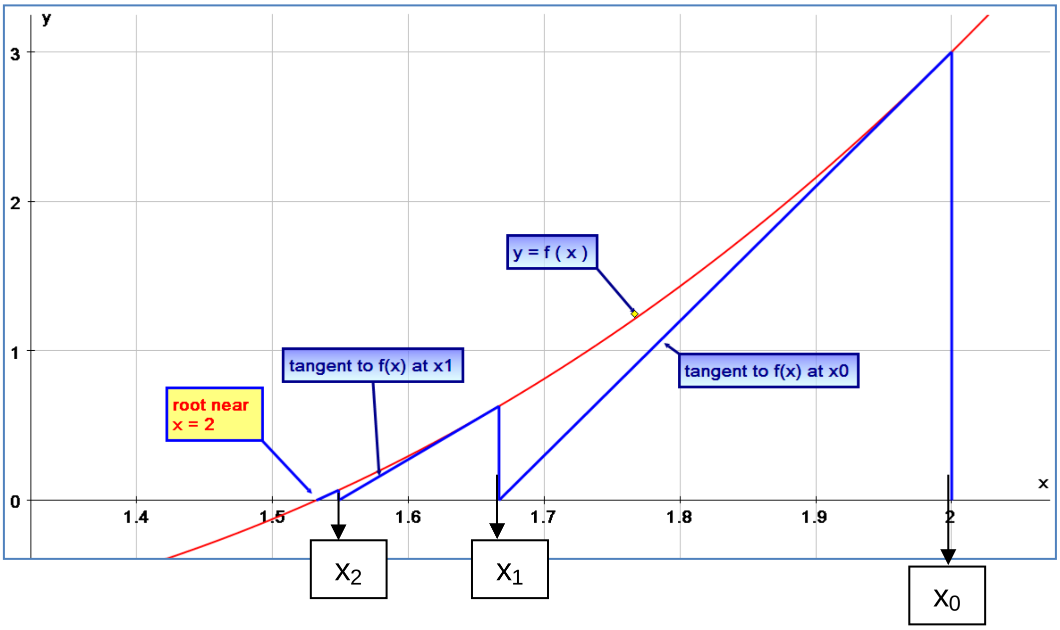
x_0	-2	0	2
x_1	-1.88889	0.33333	1.66667
x_2	-1.87945	0.34722	1.54861
x_3	-1.87939	0.34730	1.53239
x_4	-1.87939	0.34730	1.53209
x_5	-1.87939	0.34730	1.53209

x	1.532085	1.532095
f(x)	-0.000016	0.000025

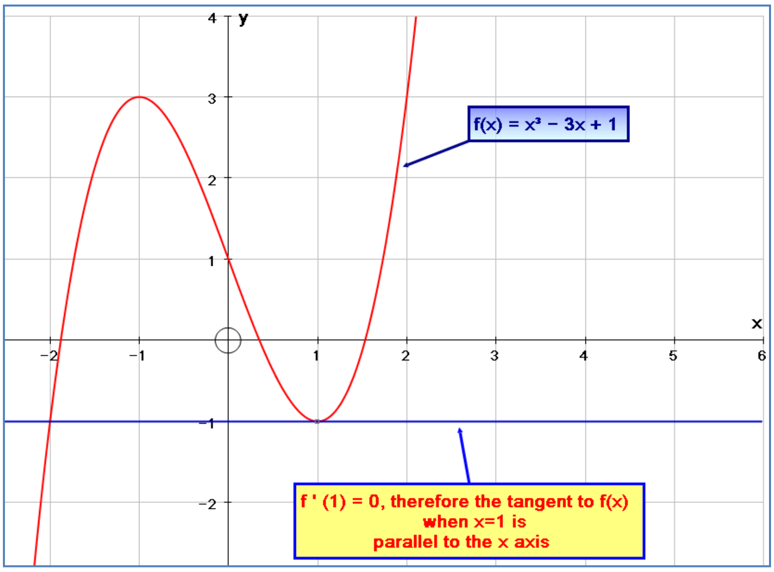
The x_r values have converged towards the root (the column was set to show 5 d.p.). We can confirm the root x = 1.53209 by looking for a change of sign in f(x) for x values 0.000005 above and below this root, using Google Sheets.

You can show the iterative process on your graph, using Autograph, which produces a saw-tooth diagram of tangents.

The Newton-Raphson process works by creating a tangent to f(x) at x_0, then where this tangent cuts the x-axis is x_1, which is closer to the root. A tangent to f(x) at x_1, is then drawn and where this tangent cuts the x-axis is x_2. Tangents continue to be drawn in this way, producing x_r values which converge towards the root. You need to label x_0 to at least x_2 on your graph.



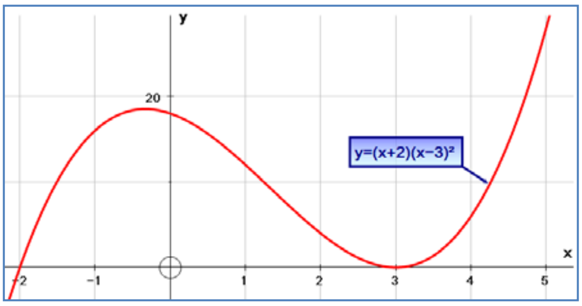
This method will sometimes fail to find a root of an equation. An example of how this might happen is when there is a stationary point at the integer near the root you are trying to find.



The gradient at a stationary point is 0, so in the part of the formula with f '(x) in the denominator, you will be trying to divide by 0, since in this case, f '(x_0) = 0. In the example given, f '(x_0) = f '(1) = $3(1)^2 - 3 = 0$.

x_0	1
x_1	#DIV/0!

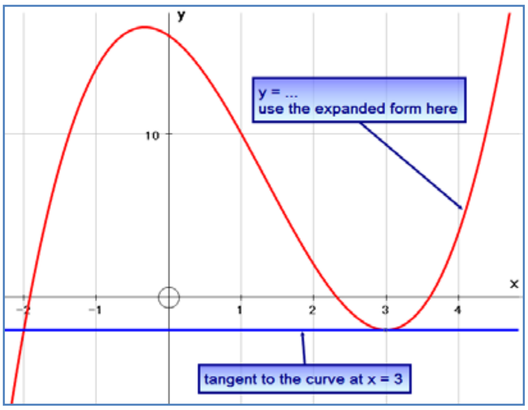
Google Sheets displays #DIV0! If you ask it to divide by zero, and an x_1 value cannot be found to proceed with the iterations.



This failure of the method can be illustrated by drawing the tangent to the minimum (or maximum) point on Autograph.

A suitable equation that will have a stationary point at an integer can be created easily using the property that a squared factor gives a repeated root. For example, we could start with f(x) = $(x + 2)(x - 3)^2$ which has a repeated root at $x = 3$.

This graph with a repeated root can then be translated a little (parallel to the y-axis), to create a curve with roots near to this integer. In the example here, subtracting 2 gives f(x) = $(x + 2)(x - 3)^2 - 2$, which can then be expanded to give the form f(x) = $x^3 - 4x^2 - 3x + 16$, which looks too difficult to solve and is a neat example of a failure case.



What to include in the

Fixed Point Iteration, Newton-Raphson Method *section:*

1. State the **equation you are trying to solve**, which is f(x) = 0, where you define what your f(x) is. (There should be a different f(x) used for each method.) You will find *all* the roots using this method.
2. Show how you **differentiated your f(x)** to get **f '(x)** to put into the **Newton-Raphson iterative formula,** which should be of the form $x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$ but with your f(x_r) and f '(x_r) substituted in the appropriate places.
3. Show a **graph of your y = f(x)**, pointing out the roots that you are trying to find.
4. Show your **calculations in Google Sheets**, and clearly state the integer near the root that you have chosen to be your x_0 values.
5. Show a **zoomed-in graph** showing the tangents used in this method (it looks like a zig-zag, or saw-tooth), with the first three x values labelled (x_0 to x_2).
6. Describe **how the iterations are performed and the x_r values converge to the root**, referring to both your Google Sheets table and your zoomed-in graph illustration.
7. Perform a **change of sign check** by looking at f(x) for x values 0.000005 above and below your root. Explain why you had to do this.
8. State that your **root is x = ……** with a maximum **error of ±0.000005**
9. Repeat steps 4, 7 and 8 for all the other roots. (You do not need to illustrate these, but you must show full calculations.)

So, that's some writing, tables from Google Sheets (showing the x_r values produced for every root and tables showing the change of sign check for every root) and 2 graphs from Autograph in this section.

For a case where this method fails to find a root you are looking for, you need to:

10. Repeat steps 1. to 4. then explain why the iterative formula was not able to produce a value for x_1.
11. Show a **graph of your f(x)** with a **tangent drawn at x_0** (this should be parallel to the x-axis) and use this to illustrate graphically why the method has failed to locate the root in this case.

You will have some writing, one small table from Google Sheets and 2 graphs from Autograph in this failure of the method section.

What to include in the

Comparison of Methods ***section:***

1. State the **equation you are trying to find a root of**, which is f(x) = 0, where you define what your f(x) is. (This should be an equation you have already found a root of with one of the three methods.)

2. Say which method you already used to find this root earlier and paste in your tables of calculations (including the change of sign check and your iterative formula if appropriate). State the root you found: ***x* = ……** with a maximum **error of ±0.000005**

3. Find the same root with the other two methods, using the **same x_0** starting integer for the two iterative methods and using this x_0 as the integer at one end of the first row of the decimal search tables.

4. Show **all the tables you use from Google Sheets** and your **iterative formula**. For each method, **state the root**: ***x*** = …… with a maximum **error of ±0.000005**. *They should all have the same error bounds (the Iterative Formulae will produce x values with 5 digits after the decimal point, the Decimal Search will produce an x value with 6 digits after the decimal point, the last of which is a 5 as you took a midpoint, but all three ways of finding the root should have the same error bounds).*

5. Compare the three methods in terms of **speed of convergence**, that is, how quickly they found the root. You can make a direct comparison of the **number of iterations** for the two iterative methods, but the decimal search does *not* have **iterations**! (You could mention how it has 5 rows of calculations, with 11 calculations on each row.)

6. Compare the three methods in terms of **ease of use** with the available hardware and software (the software means **Google Sheets**, where you actually use the methods, *not* Autograph, which is just our way of illustrating them). You are required to give some detail and depth to your discussion, so you could give examples of some of the formulae you had to type into Google Sheets and how much you could make use of the click-and-drag copying tools for each method.

You are comparing how well the methods performed to find the root of *your* equation, but you may also write about other considerations you may have if you were asked to solve different types of equations, or if a different level of accuracy were required.

Final Checks

When you have finished your coursework, you should read it through carefully to make sure that you have included everything required and that it is all done correctly. You may find it helps to print it out to do this. The final checks we recommend are:

- Make sure you have **numbered the pages**!

- Read through carefully for **typos** and to make sure that what you have written **makes sense** and things are in the **correct order**. It's easy to muddle things up when cutting and pasting a lot. Make sure all the **formulae have printed out correctly**.

- Make sure you have stated the ***equation*** in the form ***…… = 0*** that you are trying to find a root of, for **every** method (**including** in the failure cases)and **again** in the comparison.

- You may refer to your *function*, e.g. $f(x) = x^3 – 3x + 1$ but make sure you *don't* call that an *equation*!

- Make sure that all your graphs are **fully annotated**:
  - **Axes** should be labelled.
  - **Curves** should be labelled. ($y = f(x)$ is fine provided it is clear in the text what your $f(x)$ is.)
  - Positions of $\mathbf{x_0}$ to at least $\mathbf{x_2}$ should usually be labelled. (It may only be possible to show x_0 and x_1 in the failure of $g(x)$.)
  - **Tangents** should be labelled.

- Make sure your **values in the Google Sheets tables match** with the values on your **graphs**.

- You should have **explained** how each method works.

- All roots found should be stated clearly: ***x = …*** with a maximum **error of ±0.000005**. For the iterative methods, the x value has 5 d.p. but for decimal search it has 6 digits after the decimal point, the last of which is a 5 (because you took a midpoint).

- Make sure you have used correct **subscripts** on ***all*** the ***x***'s in your iterative formulae.

- Remember, you can use **pages 6, 10, 13** and **14 of this guide** to check you have included *everything* required to gain all the marks!

Methods for Advanced Mathematics (C3) Coursework: Assessment Sheet

Task: Candidates will investigate the solution of equations using the following three methods:

- Systematic search for change of sign using one of the three methods: decimal search, bisection or linear interpolation.
- Fixed point iteration using the Newton-Raphson method
- Fixed point iteration after rearranging the equation f(x) = 0 into the form x = g(x).

Coursework Title								
Candidate Name						**Candidate Number**		
Centre Number	**1**	**2**	**2**	**9**	**0**	**Date**		

Domain	**Mark**	**Description**	**Comment**	**Mark**
Change of sign method (3)	1 1 1	The method is applied successfully to find one root of an equation. Error bounds are stated and the method is illustrated graphically. An example is given of an equation where one of the roots cannot be found by the chosen method. There is an illustrated explanation of why this is the case.		
Newton-Raphson method (5)	1 1 1 1 1	The method is applied successfully to find one root of a second equation. All the roots of the equation are found. The method is illustrated graphically for one root. Error bounds are established for one of the roots. An example is given of an equation where this method fails to find a particular root despite a starting value close to it. There is an illustrated explanation of why this has happened.		
Rearranging f(x) = 0 in the form x = g(x) (4)	1 1 1 1	A rearrangement is applied successfully to find a root of a third equation. Convergence of this rearrangement to a root is demonstrated graphically and the magnitude of g′(x) is discussed. A rearrangement of the same equation is applied in a situation where the iteration fails to converge to the required root. This failure is demonstrated graphically and the magnitude of g′ (x) is discussed.		
Comparison of methods (3)	1 1 1	One of the equations used above is selected and the other two methods are applied successfully to find the same root. There is a sensible comparison of the relative merits of the three methods in terms of speed of convergence. There is a sensible comparison of the relative merits of the three methods in terms of ease of use with available hardware and software.		
Written communication (1)	1	Correct notation and terminology are used.		
Oral communication (2)	2	Presentation ☐ Interview ☐ Discussion ☐	Please tick at least one box and give a brief report.	
Half marks may be awarded but the overall total must be an integer. Please report overleaf on any help that the candidate has received beyond the guidelines.			**TOTAL**	**/18**